CHALLENGES TO TEACHING AUTHENTIC MATHEMATICAL PROOF IN SCHOOL MATHEMATICS

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As pointed out by Stylianides (2007), a major reason that proof and proving have been given increased attention in recent years is because they are fundamental to doing and knowing mathematics and communicating mathematical knowledge. Thus, there has been a call over the last two decades to bring the experiences of students in school mathematics closer to the work of practicing mathematicians. In this paper, I discuss the challenges that a beginning teacher faced as he attempted to teach authentic mathematical proof. More specifically, I argue that his past experiences with proof and the curriculum materials made available to him were obstacles to enacting a practice that was more like what he called “real math.”

INTRODUCTION AND RELEVANT LITERATURE

A major reason that proof and proving have been given increased attention in recent years is because they are fundamental to doing and knowing mathematics and communicating mathematical knowledge (Stylianides, 2007). In describing proof as the “guts of mathematics,” Wu (1996b, p. 222) argued that anyone who wanted to know what mathematics was about needed to learn how to write, or at least understand, a proof. These comments complement the call to bring students’ experiences in school mathematics closer to the practices of mathematicians (Ball, 1993; Lampert, 1992; NCTM, 2000). Thus, many mathematics educators advocate for engaging students in ‘authentic mathematics,’ where they are given opportunities to refute and prove conjectures (Lakatos, 1976; NCTM, 2000; Lampert, 1992).

However, researchers have pointed out that not only do students find the learning of proof to be challenging (Farrell, 1987; Senk, 1985), but teachers also find the teaching of proof to be a difficult endeavor (Knuth, 2002; Wu, 1996a). This is not a problem that is unique to the United States, as similar issues related to difficulties with proof at various school levels have been noted in other countries as well (see, e.g., Furinghetti et al., 2001). At the same time, in recent reforms of school mathematics in the United States (National Council of Teachers of Mathematics [NCTM], 1989, 2000), the Standards documents have reframed the issue of mathematical understanding around constructivist ideas and have served as the impetus for the creation of reformed, understanding-based curricula (Bohl, 2000). More specifically related to proof, the NCTM recommends that teachers at every level “help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures” (NCTM, 2000, p. 3). As a result, the role of curriculum materials has been viewed as increasingly important over the last two decades (Remillard, 2005).
In the case of geometry proof, the curriculum has been noted as a primary cause of students’ poor performance, both in what topics are addressed and how they are communicated (Jaime et al., 1992). Contemporary curriculum materials, even those which purport to be Standards-based, do no necessarily assist teachers in carrying out the goal of teaching authentic proof. Thus, an enormous responsibility is placed on teachers who wish to enact a Standards-based pedagogy (Knuth, 2002) with the aim of teaching mathematical proof that is more authentic.

The work reported in this paper is based on a three-year case study designed to learn more about how a teacher of proof developed in the context of geometry. Due to space limitations, I limit my discussion here to two issues that could serve as obstacles to teaching authentic mathematical proof: past experiences and curriculum materials. In the sections that follow, I first describe the context of the study as well as the data that were collected and analyzed. I then present some findings and discussion before concluding the paper.

CONTEXT OF THE STUDY AND DATA
This study was designed as a longitudinal case study. The secondary school where the teacher, Matt (a pseudonym), taught was located in a city in a Midwestern state in the middle of the United States. Despite the fact that he was a beginning teacher, Matt indicated that he was confident in his knowledge of mathematics. Because his grasp of mathematics (and therefore proof) did not seem limited, I was able to tease out some curricular issues and pedagogical dilemmas that teachers might face as they learn to teach geometry proof.

During each year of the study, Matt selected a focus class for observation that he believed to be fairly typical of 10th-grade (ages 15-16) geometry classes in that particular school. He used a conventional geometry textbook that developed Euclidean geometry as an axiomatic system. The authors either led the reader through the proofs of these theorems or left the proofs as exercises. I refer to the textbook as “conventional” because as with “traditional textbooks,” this book is organized in such a way that it presents the content without much guidance as to what is important to emphasize or how to teach it (Posner, 2004).

The primary data sources for this study were classroom observations and interviews with Matt. I visited the classroom during lessons when the teacher introduced proof and taught triangle congruence proofs. Each lesson was audio and video recorded. All interviews were semi-structured and audio recorded. For more detail about the data collection and analysis, see Cirillo (2008).

FINDINGS AND DISCUSSION
In order to understand how Matt’s prior experiences with proof influenced the ways in which he interpreted his mathematical world, I discuss his early experiences as a student learning proof and then describe his beliefs about “real math.” I hope to impart the beliefs and philosophies that motivated Matt to make
the changes that were described across time (Cirillo, 2008). In the larger study, I argued that the changes found in his teaching of proof were motivated by Matt’s desire to create experiences for his students that were more closely related to “real math” than to the “school math” experience provided through lecturing from his textbook.

**Early Experiences with Proof**

Matt could not recall ever being asked to write a proof during his school mathematics experience, and he did not recall even being shown a proof in high school. As a mathematics major at a university, however, Matt said:

**Example 1**

I was immediately asked to do all sorts of proofs, which...now, looking back at it, I can see as not being so bad, but at the time I’m like, this is a joke. I’m like, this is impossible. You know, you can’t do this...How can people prove things about eigenspace values and all this other kind of stuff?...It was very much like what you thought was real is now no longer true. (Interview, 6/21/06)

The difficult transition that Matt experienced from school to undergraduate mathematics is not uncommon. The paucity of proof in school mathematics coupled with the fact that even in the lower-level university courses (e.g., calculus and linear algebra), few, if any, proofs are required of students (Moore, 1994) helps us understand why Matt felt that doing proofs was “impossible.” During the first year of this study, Matt compared the challenge of doing his first proof (as a student) to walking through a wall. This, he said, caused him to rethink his major in mathematics. When I asked Matt about this experience, he said, “Yeah, actually I really questioned if I was going to keep going to college or not, but yeah...because I was like, this is impossible, I mean, this is a joke” (Interview, 6/21/06). These comments may seem surprising given that Matt was clearly above-average in school mathematics, evidenced by (among other things) his being two years ahead in his studies prior to graduating from high school. As Moore (1994) explained, however, “This abrupt transition to proof is a source of difficulty for many students, even for those who have done superior work with ease in their lower-level mathematics courses” (p. 249). Matt said that even though he did not take any sort of an introductory proof course, eventually he was “able to do it” and “able to understand or believe that this was something that [he] could do” (Interview, 6/21/06). The experiences described here caused Matt to begin to think about mathematics in new ways.

**“Real Math” versus School Math**

The classroom episode described in this section took place during the third year of the study when students were first introduced to proof. After the completion of the first proof of the school year, a brief conversation occurred about the class working on proofs over the course of the next few months. After this conversation, students could be heard saying, “I’m going to get sick of this” and “this is going to
get very tedious very quickly.” In the transcript that follows, Matt responded to his students’ comments by referring to proof as being “real math” several times.1

**Example 2**

Matt: It’s not tedious. It’s exciting….It’s fun
   MS: I like numbers. Numbers are fun. This is harder.
   MS: This takes too long.
Matt: No. No. This is math. Everything that you were taught before with numbers that you thought was math, that’s not really math. Okay. That’s arithmetic. Right? This [gestures toward the proof on the board] is math.
   MS: It’s geometry. It’s not math. Math’s a bigger category. It has all of those in it.
Matt: Okay. This is more like real math.
   MS: How is it more like real math?
   MS: Define real math.
Matt: This [gestures, again, to the proof on the board] is like math.
   MS: Well how?
   MS: Why is this like math?
Matt: Because, because we’re proving, but there’s no numbers. (Transcript, 9/21/07)

As can be seen in this transcript, Matt explained that he saw “this,” meaning proof, as “real math.” When a student challenged him, Matt conceded: “This is more like real math.” So for Matt, it seemed that “real math” had to involve proof. In addition, he told his students that all of the mathematics that they learned before the geometry course was arithmetic. One might question this claim since these 10th grade students would have taken algebra the previous year. Algebra, however, is sometimes described as an extension and a generalization of arithmetic (Leitzel, 1989). In an interview three months later, Matt talked about supplementing the textbook with additional proofs so that his students could experience “real math.” When asked why he believed that it was important to supplement his curriculum with additional proofs, Matt said:

**Example 3**

‘Cause it's actual math. Like it's real. It's like real math. You know…going to college and getting a degree in mathematics, that's the biggest thing is that, you know, computationally, we just don't care….So, I mean it's real math. That's what people really did…it was cutting edge math 3000 years ago, but it was still cutting edge to a certain point, and it can be seen. It's relatively straight-forward to them….they can still draw the picture, they can still see it, and they can see how we can apply the structure that we've agreed upon to reach that conclusion. So in that sense, it's sort of, yeah, it's real mathematics…there's no numbers in it. They actually can write a proof. And they can write a pretty rigorous

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1 In all transcripts, MS is male student, FS is female student, S is a student whose gender was not determined, Ss is a collective group of students, and I is interviewer.
proof...from a pretty well agreed upon set of axioms and definitions. (Interview, 12/15/07)

The statement “computationally, we don’t care” seemed similar to the statement that Matt made in class related to arithmetic not really counting as mathematics. In Matt’s view, since mathematicians would only be concerned with “real math” (i.e., proof), then they could not be bothered with simple computations. Again, Matt points to differences he saw between school math and “real math.” In the examples provided, Matt seems to be saying that, as a student, he did not experience real math, and unless he supplemented the textbook, his students would not experience it either.

CONCLUSION
Lampert (1992) discussed the idea of “authentic mathematics,” which is similar to what Matt called “real math.” Related to proof, the Standards have called for a de-emphasis of the two-column form (NCTM, 1989) and a focus on the logical argument rather than the form of the proof (NCTM, 2000). In his quest toward “real math,” Matt also emphasized the proof process with his students.

An implication related to authentic proof practices involves the presentation style of conventional textbooks. The textbook exercises that emphasized applications of the theorems, rather than the proofs did not engage students in reasoning and proof. If we are serious about promoting more authentic practices in school mathematics classrooms, more attention must be given to teachers’ past experiences with school mathematics, and the curriculum objectives and materials that teachers are provided with to teach proof.

REFERENCES


