

**RANDOM WALK SIMULATIONS OF MASS TRANSPORT IN
COMPLEX CHROMATOGRAPHIC MATERIALS**

by

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A thesis submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Honors Bachelor of Chemical Engineering with Distinction

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ABSTRACT

Chromatographic techniques play a central role in separation science for a wide range of fields. The manufacture and use of devices that operate on the nanoscale as well as porous-layer open-tube (PLOT) systems are becoming more prevalent due to their potential to provide more efficient separations. This thesis attempts to model mass transport phenomena in these systems through simulations utilizing random-walk techniques. The simulations model the flow of finite tracer particles through porous and nonporous capillary systems at varying time and length scales. First, the effect of fluid slip, or a nonzero velocity, at walls is investigated. The impact of retention on the effects of slip flow is considered, and it is determined that the use of slip flow is limited in application but can provide improved efficiency to certain nanoscale systems. Next, PLOT systems are modeled as a central capillary with radially-oriented cylindrical pores. Pore and solute sizes are varied to gain insight into how these variables effect elution times from the capillary. The simulations provide the capability to calculate mass transport rates which would be otherwise impossible using experimental or analytical methods. These findings underscore the potential of a computer-aided approach for the design of improved analytical separation systems.

SYMBOL GLOSSARY

$A(\rho)$	Function in Aris derivation of C_m term
a, b	Numbers for modulus operation
C_m	Mobile phase mass transport coefficient
C_s	Stationary phase mass transport coefficient
C_r	Courant number
d_c	Capillary diameter
D_m	Solute diffusion coefficient in the mobile phase
d_p	Pore diameter
d_s	Solute particle diameter
$\bar{d}$	Average distance traveled in a time step
d'	Distance traveled to wall
e	Distance between segment start and pore perimeter
F_r	Probability distribution for particle being distance r from origin for purely convective flow
F_t	Arrival time probability distribution of particles for purely convective flow case
Gz	Graetz number
H	Height equivalent to a theoretical plate
$\mathcal{H}$	Heaviside unit function
i	Random walk time step iteration number
I_1	First constant in Aris derivation of C_m term
I_2	Second constant in Aris derivation of C_m term

IQR	Interquartile range of arrival time data
K	Size-exclusion distribution coefficient
k	Boltzmann constant
k'	Retention parameter
L	Length of capillary
L_p	Length of pore
L_s	Slip length
M_1	First moment of simulation arrival time distribution
M_2	Second moment of simulation arrival time distribution
N	Number of theoretical plates
N_p	Number of particles
N_r	Number of pores per capillary segment
N_z	Number of segments in the capillary length
$N_{z,L}$	Local segment number
P	Pressure in capillary
P_a	Probability of adsorption
Pe	Peclet number
Q	Volumetric flow rate
R	Retention factor
r	Radial distance from cross section origin
r_c	Capillary radius
r_p	Pore radius
r_s	Solute radius
s_{inc}	Surface curvature of the angle between pores
s'	Arc length (curvature) of the solute
s_L	Local curvature between pores

$s_{L,\phi}$	Arc length of the local curvature due to the phase offset
T	Temperature
$\bar{t}$	Theoretical mean arrival time of solute particles
t_d	Wall retention delay time
$\langle t_d \rangle$	Mean wall retention delay time
t	Arrival time of individual solute particle at end of capillary
t_0	Minimum theoretical arrival time of solute particles
t_j	Arrival time of the j^{th} individual particle
t_s	Time step
t'_s	Truncated time step when wall collision occurs
u	General coordinate of x , y or z
V_c	Capillary volume
V_p	Pore volume
V_R	Retention volume
$v_z(r)$	Fluid velocity at a point
$\bar{v}_z$	Mean velocity in the flow (z) direction
$\bar{v}_z^*$	Mean velocity in the flow (z) direction accounting for finite particle size
W	Histogram bin width
$\bar{x}$	Mean distance of random walk
x,y	Coordinates in the capillary cross section
z	Coordinate in the z flow axis
z_L	Local z coordinate within any segment
γ	Nondimensional variable used for slip flow
ζ_i	Uniformly distributed random number
η	Dynamic viscosity of fluid

θ	Angle of solute location in x and y
θ_c	Contact angle
θ_{inc}	Incremental angle between radial pores
λ	Nondimensional ratio of solute particle radius to pore radius
$\xi_{x,i}, \xi_{y,i}, \xi_{z,i}$	Normally distributed random numbers
ρ	Normalized radial distance from cross section origin
σ	Theoretical standard deviation of arrival time distribution
ϕ	Linear pressure gradient over length of capillary
ϕ_{inc}	Phase angle increment between pores in neighboring segments
ϕ_L	Local phase angle at the local segment

Chapter 1

INTRODUCTION

1.1 Simulation Approach and Motivation

In this thesis, various mass transport simulations are carried out using a capillary chromatography system to gain insight and solve problems relevant to the field of separation science and engineering. The system consists of a cylindrical capillary with idealized cylindrical pores and tracer particles of finite size flowing through. The technique of the random walk is utilized to model the convective and diffusive contributions of fluid flow through the capillary.

This approach results in a system that is complex enough to learn fundamental truths about mass transport in chromatographic systems while also being geometrically simple enough to tailor input parameters and keep computing time low. The benefits of this generality and flexibility are two-fold. First, the conditions at the system boundary could be altered as simulation inputs. In analytical derivations of flow behavior, this would require deriving the relevant equations each time the boundary conditions change and introducing simplifying approximations that may not capture all of the underlying physics. In this simulation system, the boundary condition can be redefined by changing a single input parameter. Second, this simulation system allows access to all time and length scales. Each particle is traced on time scales less than a microsecond, which is impossible to replicate using experiments. In addition, both long and short tubes could be modeled which exhibit drastically different behavior. In particular, short tubes are not well-understood with analytical solutions, so simulations are the ideal method of solving problems on this length scale.

This thesis focuses mainly on two phenomena using this capillary system. The first chapter examines the impact of replacing the traditional no-slip boundary condition of Poiseuille flow with slip at the wall. The second chapter examines a porous-layer open-tube system that provides understanding of the dynamics of mass transport in capillary and size-exclusion chromatography. From these simulations, recommendations regarding the possible design of improved chromatographic materials are made.

1.2 Slip Flow

On the macroscopic level, flow through a capillary is typically treated as having zero velocity at the wall. This so-called “no-slip” boundary condition leads to the classical Hagen-Poiseuille flow with a parabolic velocity profile. In reality, this assumption may not apply, particularly when there is very weak interaction between particles and the wall. For example, this would be the case for a hydrocarbon-coated silica (hydrophobic) and water mobile phase (hydrophilic). This causes particles to “slip” at the wall and leads to a nonzero velocity at the wall.

The phenomenon of slip has been examined on the long tube scale but there is no work in the literature concerning short tubes. The capillary model of this thesis allows simulation of no-slip and slip boundary conditions on both the long and short tube scales by altering the input parameters of the model. The addition of retention is also examined to determine the impact of wall-solute interactions on the zone profiles with slip flow in short tubes.

Slip flow is particularly important on the micro- and nanofluidic scales. Applications utilizing devices on these length scales are becoming more and more prevalent, therefore it is critical to gauge the degree to which slip flow impacts the dynamics of fluid flow in these systems.

1.3 Spiny Model

The capillary model is given a cylindrical pore structure in the second chapter, creating what is affectionately called the “Spiny” model. This allows simulation of fluid

flow through porous-layer open-tube chromatographic systems as well as size-exclusion chromatography systems. For example, the Spiny model can act as a general pore model for SBA-15 silica, implementing an open-tube system that allows calculation of mass transport rates that are impossible to compute using analytical solutions or lumped-kinetic models.

The ability to trace the location of a particle at any given time makes it possible to gain a better understanding of how solutes behave in pores. The relationship between solute size and the residency time in a pore is examined. This is a phenomenon not discussed in the literature, and other models such as partial differential equation-based models cannot provide insight on this. Therefore, these simulations provide the opportunity to gain data and formulate solutions to problems where other approaches fall short. The Spiny model can serve as a testbed for many analytical separation engineering problems which can be assessed using a computer-aided design approach.

Chapter 2

SIMULATION AND THEORY OF OPEN-TUBE DISPERSION IN SHORT AND LONG CAPILLARIES WITH SLIP BOUNDARIES AND RETENTION

Chapter Synopsis

Using random-walk techniques, high resolution simulations of zone shape are conducted in open capillary tubes for short and long tube conditions. Finite size solutes are used as tracers in this treatment. Slip flow boundary conditions and wall retention are utilized as needed. These simulations are able to reproduce previous work in short and long tubes. For the short tube case where dispersion does not asymptotically approach the classic Taylor-Aris and Golay solutions, the effect of slip flow boundaries in the transient region shows zone shapes with abbreviated tails where the larger slip flow values cause zone compression.

The use of slip flow to lower dispersion in capillary-based, wall-coated separations is shown to favor long tube behavior. This is because slip flow is relevant for cases where slip lengths are fractions of small capillary tube diameters. Incorporating slip flow into transport in capillaries favors a very small capillary radius where the cross-sectional diffusion length is very small and sampling times are fast. The purely convective zone shape with slip flow boundaries is derived analytically.

Applications for this type of separation, guided by both analytical theory and simulation, show the potential for nano-sized capillary tubes less than $1\text{ }\mu\text{m}$ in diameter and favor very fast separations.

2.1 Introduction

The study of dispersion in open circular tubes has been the subject of numerous inquiries in basic fluid mechanics leading to insight into coupled convective and diffusive

processes. The original work in this area was the theoretical development of the so-called “Taylor dispersion” mechanism whereby radial diffusion causes a broadening of the axial zone concentration profile as a function of fluid velocity [1, 2, 3]. The theory of Taylor dispersion has been the starting point for more extensive theories of both gas and liquid chromatography [4, 5, 6] and other forms of separation systems, such as field-flow fractionation [7, 8].

In all of these cases, the theories were so-called asymptotic or “long time” theories where it is assumed that the tracer probes, introduced at the head of the tube, sample the velocity field a sufficient number of times so that results are at steady state. This simplifies the mathematical treatment and is essential for many practical applications where dispersion is important and limiting, such as in chromatographic resolution. These long time theories assume that the resulting zone shapes are typically Gaussian or mildly tailed in shape.

A number of years ago, “short time” behavior of zone shape and dispersion was shown to give non-Gaussian zones, with zones forming tails and an unresolved middle-region with a large hump [9]. Essentially, the zone shape looked like multiple components were eluting when only one component was present. This work was unique because not only was this a surprising result, but it required a computer simulation to produce this. It was subsequently shown how this behavior could be reproduced mathematically for short tubes with no wall retention [10] and with retentive short tubes [11]. In both of these cases, the equations were sufficiently complicated to require computational evaluation.

In all of the cases mentioned above, the fluid flow is assumed to be that of Poiseuille flow, where the fluid at the tube wall is assumed to be part of a “no-slip” boundary. A large amount of evidence has been accumulated that suggests that fluid slippage may occur [12, 13, 14], especially for aqueous solutions near hydrophobic walls. Various aspects of this so-called “slip boundary” have been investigated including the correlation of the slip length [15] with fluid wetting. As applications of microfluidic devices [16] increase, the impact of the slip boundary condition becomes important

since certain properties such as zone dispersion have been shown to be affected by the extent of wall slippage [17]. In this case, a larger degree of slippage, as embodied in the slip length which is discussed below, reduces dispersion. This reduction is a desired result in separation applications. It is claimed that the slip flow effect has been utilized in the high resolution separation of biomolecules [18, 19, 20, 21]. Structured or patterned surfaces, which are known to exhibit *superhydrophobicity*, have been made with large surface slippage [22, 23]. However, in many cases, especially those with rough or patterned surfaces, the effect may be due to trapped gas and nanobubbles [24] which can exhibit a substantial slip effect.

Few studies of the fluid motion and dispersion of finite-size colloids have been studied where wall interactions were included with slip flow. In one study, the axial dispersion in microchannels was studied where the colloidal size was similar to the tube dimension [25]. It was found that colloid interactions enhanced the dispersion. Also of interest is the dispersion due to wall interactions in a capillary when electric fields are employed as the driving force for flow [26, 27], however, these studies did not include slip flow or finite-sized solutes. In addition, slow wall adsorption-desorption kinetics were studied in a stochastic simulation of capillary separations of small molecules [28]. This study did not address finite-sized molecules nor slip flow.

In this paper, we study elements of zone dispersion when slip flow, retention, and short capillary tubes are present using random-walk simulation techniques. One of the applications for this is to study very fast separations which can be implemented in short capillary tubes with a wall coating. Finite tracer particle diameters are utilized and wall retention is implemented with a probability-based approach described below. These results suggest that for short tubes, a small amount of wall slip will result in smaller dispersion. We assume in this treatment that particles are present in dilute conditions so that particle-particle interactions are neglected and that particle-wall interactions are assumed to be that of non-interacting hard spheres interacting with solid boundaries. Inertial effects leading to so-called “lift forces” [29] are not included in the model. All flow is modeled under creeping flow conditions, i.e. low Reynolds

number flows where inertial effects are unimportant.

2.2 Mathematical Development

2.2.1 Slip flow

On the macroscopic level, it is typically assumed that fluid located at the walls has zero velocity. With this no-slip boundary condition employed, the theoretical Hagen-Poiseuille velocity profile is parabolic [7, 30] and is described by

$$v_z(r) = \frac{r_c^2}{4\eta} \left[1 - \left(\frac{r}{r_c} \right)^2 \right] \frac{\partial P}{\partial z} \quad (2.1)$$

where $v_z(r)$ is the fluid velocity at a point located radially at r in the direction of flow (z), r_c is the radius of the capillary, η is the dynamic viscosity of the fluid, and P is pressure. In terms of average velocity, this velocity profile can be written as

$$v_z(r) = 2\bar{v}_z \left[1 - \left(\frac{r}{r_c} \right)^2 \right] \quad (2.2)$$

where $\bar{v}_z$ is the mean fluid velocity in the z direction.

The no-slip assumption has proven to be accurate for flows on the macroscopic level, but it is only an assumption that is not rooted in principle [12]. For the no-slip boundary condition to apply, attractive forces between the wall and the fluid molecules must be equal to or greater than the attractive forces between mobile phase fluid molecules [31]. This is certainly not the case when the walls of the capillary are hydrophobic and the fluid is hydrophilic. The result is a “slip” boundary with a nonzero velocity at the wall. A slip boundary can be captured using Navier’s boundary condition,

$$L_s \frac{\partial v_z(r)}{\partial r} \bigg|_{r=r_c} = -v_z(r) \big|_{r=r_c} \quad (2.3)$$

where L_s is defined as the slip length. When replacing the no-slip boundary condition with this new condition in the Navier-Stokes equation, the velocity profile for flow in the capillary with slip becomes

$$v_z(r) = \frac{r_c^2}{4\eta} \left[1 - \left(\frac{r}{r_c} \right)^2 + \frac{2L_s}{r_c} \right] \frac{\partial P}{\partial z} \quad (2.4)$$

[32]. In terms of average velocity, the velocity profile with slip is now expressed as:

$$v_z(r) = \frac{2\bar{v}_z}{1 + 2\gamma} \left[1 - \left(\frac{r}{r_c} \right)^2 + \gamma \right] \quad (2.5)$$

where $\gamma = 2L_s/r_c$ [17].

Figure 2.1 illustrates the no-slip and slip velocity profiles for capillaries with equal radii and flows with equal average velocities. The slip length in the figure is equal to one-half the radius of the capillary for illustrative purposes. While the chosen slip length was arbitrarily chosen for the figure, it highlights the fact that slip length must be on the order of r_c to have an appreciable impact on the velocity profile relative to that of no-slip conditions. A slip length such that $L_s \ll r_c$ results in γ approaching zero and the velocity profile converging to no-slip conditions.

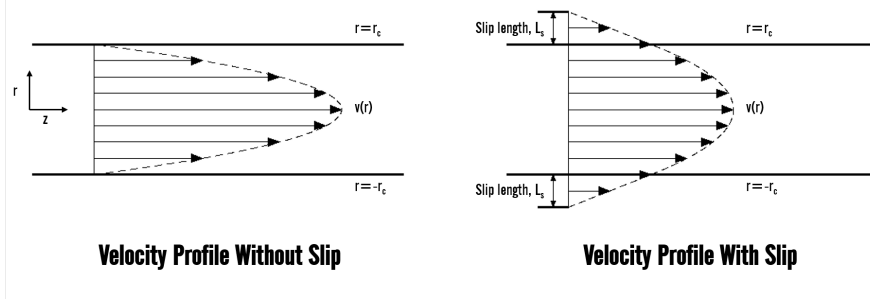


Figure 2.1: Velocity profiles for both no-slip and slip conditions with equal average fluid velocity. Left: Hagen-Poiseuille pressure-driven flow with zero velocity at the capillary walls. Right: Slip flow with nonzero velocity at walls. A slip length of $L_s = 0.5r_c$ is used for visualization.

Slip lengths for various surface/liquid boundaries reported in the literature are typically on the order of tens of nanometers [14]. For this reason, the phenomenon of slip becomes significant on the nanoscale using devices such as carbon nanotubes with radii on the order of nanometers through tens of nanometers [32, 33, 34] and with sub-micron silica particles [19, 31]. It also follows that slip would be negligible for macroscopic flows resulting in the no-slip boundary condition being a valid assumption.

The contact angle between surface and fluid directly impacts slip length as contact angle is a measurement of attraction between different materials. Huang et

al. developed a relationship between slip length and contact angle, θ_c , using molecular dynamics simulations stating $L_s \sim (1 + \cos \theta_c)^{-2}$, which was in accord with experimental results and supports slip lengths less than 20 nm up to a contact angle of approximately 150° [35]. Thus, larger slip lengths will require very weak interactions between surface and fluid such as water and a highly hydrophobic material.

An enhancement in flow through a capillary due to the nonzero fluid velocity at the wall is known to take place in nanoscale fluid mechanics. When a constant pressure gradient is applied, this enhancement is directly related to γ by

$$\frac{Q_{slip}}{Q_{no-slip}} = \frac{\bar{v}_{z,slip}}{\bar{v}_{z,no-slip}} = 1 + 2\gamma \quad (2.6)$$

[14] where Q is the volumetric flow rate through the capillary. This increase in flow rate could potentially improve the throughput and efficiency of devices used for separation [32]. At the wall, the fluid velocity is

$$v_z(r)|_{r=r_c} = 2\gamma\bar{v}_{z,no-slip} \quad (2.7)$$

For the simulations involving slip flow, γ was limited to $\gamma \leq 0.4$ or a flow enhancement of $\bar{v}_{z,slip} \leq 1.8\bar{v}_{z,no-slip}$. This limit was chosen based on studies summarized by Lauga and Stone [36] where reported slip lengths ranged from the nanometer scale to the micron scale and system size ranged from the nanometer to the millimeter scale. Despite this wide range, values of L_s/r_c differed by a factor of 40 from 0.005 to 0.2, corresponding to $0.01 \leq \gamma \leq 0.4$ [36].

Figure 2.2 and Table 2.1 summarize the above concepts by illustrating how increasing L_s will increase γ which leads to an increase in $\bar{v}_z$ and $v_z(r)|_{r=r_c}$. Figure 2.2 shows that the slip velocity profile is equivalent to the no-slip profile shifted up by a value equal to the slip velocity at the wall given by Eq. 2.7.

2.2.2 Plate height theory

Comparisons of simulation results to long tube theoretical behavior are made through the theoretical plate height. The general relationship,

$$H = \frac{L\sigma^2}{\bar{t}^2} \quad (2.8)$$

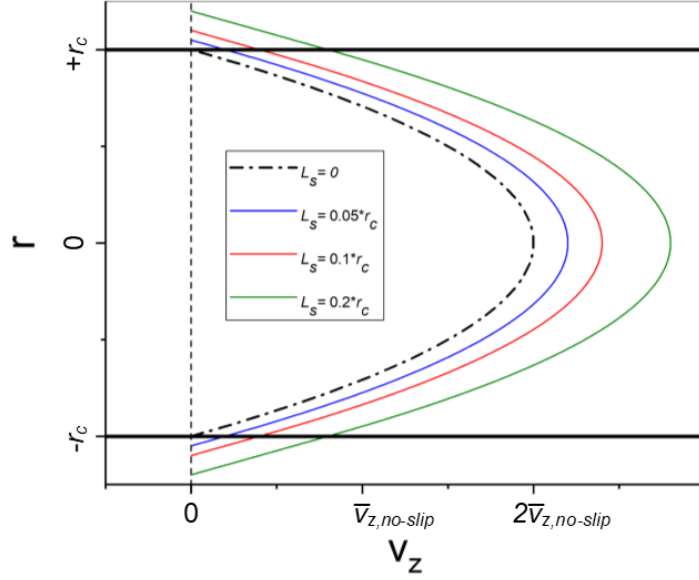


Figure 2.2: Velocity profiles for varying slip lengths relative to capillary radius. $\bar{v}_z$ is enhanced when slip flow occurs relative to no-slip conditions. The profiles with slip are equivalent to shifting the no-slip profile to the right by the velocity at the wall given by Eq. 2.7.

Table 2.1: Average velocity with slip flow and velocity at capillary wall for varying slip lengths as a fraction of average velocity in the axial direction under no-slip conditions.

L_s	γ	$\frac{\bar{v}_z}{\bar{v}_{z,no-slip}}$	$\frac{v_z(r) _{r=r_c}}{\bar{v}_{z,no-slip}}$
0	0	1	0
$0.05r_c$	0.1	1.2	0.2
$0.1r_c$	0.2	1.4	0.4
$0.2r_c$	0.4	1.8	0.8

is used, where H is the height equivalent to a theoretical plate, L is the length of the capillary, $\bar{t}$ is the mean arrival time of solute particles at the end of the capillary, and σ is the standard deviation of the Gaussian concentration density profile in time units. The simulations provide outputs of the arrival time for each individual particle. For a given particle size, the first and second moments of the elution profiles are found so that Eq. 2.8 is equivalently represented as

$$H = \frac{LM_2}{M_1^2} \quad (2.9)$$

where M_1 is the first moment of the arrival time distribution and M_2 is the second central moment.

The theoretical mean arrival time is simply

$$\bar{t} = \frac{L}{\bar{v}_z} \quad (2.10)$$

To relate the second moments from simulation to theory, the theoretical plate height for open-tube systems with the slip flow boundary condition is determined using the expression,

$$H = \frac{2D_m}{\bar{v}_z} + C_m \frac{r_c^2 \bar{v}_z}{D_m} + C_s \bar{v}_z \quad (2.11)$$

[7, 37] where r_c is the capillary radius, and C_m and C_s are the mobile and stationary phase mass transport coefficients, respectively. D_m is the solute diffusion coefficient in the mobile phase and is calculated using the Stokes-Einstein equation,

$$D_m = \frac{kT}{3\pi\eta d_s} \quad (2.12)$$

[38] where k is the Boltzmann constant, T is temperature assumed to be 25°C, η is the dynamic viscosity of the fluid assumed to be 1 cP, and d_s is the effective solute diameter.

In the case of the open-tube system with no separation media, there is no stationary phase mass transport resistance and thus $C_s = 0$. For no-slip boundary conditions used in this ideal capillary column, it is established that [7]:

$$H = \frac{2D_m}{\bar{v}_z} + \frac{1}{24}(6R^2 - 16R + 11)\frac{r_c^2 \bar{v}_z}{D_m} \quad (2.13)$$

For the limiting case of no wall retention, the retention factor, $R = 1$, and Eq. 2.13 reduces to the well-known relationship:

$$H = \frac{2D_m}{\bar{v}_z} + \frac{r_c^2 \bar{v}_z}{24 D_m} \quad (2.14)$$

For a system with input parameters of r_c , d_s , L , and $\bar{v}_z$, the approach to validate the simulation results with long tube theory is as follows: $\bar{t}$ is calculated from Eq. 2.10, D_m is calculated from Eq. 2.12, the plate height is calculated from Eq. 2.13, and the standard deviation of the Gaussian elution profile is calculated from Eq. 2.8. M_1 and M_2 from the simulation data are compared with $\bar{t}$ and σ .

A similar approach to comparing simulation results with long tube theory is employed with the slip flow boundary condition. C_m is determined via the method laid out by Martin and Guiochon [37] using the generalized dispersion theory of Aris [2]. Based on this theory,

$$C_m = I_1 - 2RI_2 + \frac{R^2}{4} \quad (2.15)$$

where I_1 and I_2 are defined as,

$$I_1 = \int_0^1 \frac{A^2(\rho)}{\rho} d\rho \quad (2.16)$$

$$I_2 = \int_0^1 \rho A(\rho) d\rho \quad (2.17)$$

where $\rho = r/r_c$ and

$$A(\rho) = \int_0^\rho 2\rho' \frac{v_z(\rho')}{\bar{v}_z} d\rho' \quad (2.18)$$

Beginning with Equation 2.5 in terms of ρ [17],

$$\frac{v_z(\rho)}{\bar{v}_z} = \frac{2}{1+2\gamma}(1-\rho^2+\gamma) \quad (2.19)$$

which yields

$$A(\rho) = \int_0^\rho 2\rho' \frac{2}{1+2\gamma}(1-\rho'^2+\gamma) d\rho' = \frac{\rho^2}{1+2\gamma}[2(1+\gamma)-\rho^2] \quad (2.20)$$

and performing the integrations in Eqs. 2.16 and 2.17 results in

$$I_1 = \frac{24\gamma^2 + 32\gamma + 11}{24(1 + 2\gamma)^2} \quad (2.21)$$

$$I_2 = \frac{2 + 3\gamma}{6(1 + 2\gamma)} \quad (2.22)$$

This leads to an expression for C_m under the conditions of slip flow by substituting I_1 and I_2 into Eq. 2.15:

$$C_m = \frac{24\gamma^2 + 32\gamma + 11}{24(1 + 2\gamma)^2} - 2R \left[\frac{2 + 3\gamma}{6(1 + 2\gamma)} \right] + \frac{R^2}{4} \quad (2.23)$$

Finding a common denominator and grouping terms with like orders of γ ,

$$C_m = \frac{24(R^2 - 2R + 1)\gamma^2 + 8(3R^2 - 7R + 4)\gamma + (6R^2 - 16R + 11)}{24(1 + 2\gamma)^2} \quad (2.24)$$

When $L_s = 0$, this relation simplifies to the equivalent no-slip expression for C_m in Eq. 2.13. For the limiting case of no wall retention, Eq. 2.24 simplifies to

$$C_m = \frac{1}{24(1 + 2\gamma)^2} \quad (2.25)$$

Using the same approach for validating the long tube simulation results with the no-slip boundary condition presented above, long tube simulation results with the slip boundary condition are compared to the theoretical Gaussian distribution moments. The only additional input required is L_s .

2.2.3 Purely convective limit with slip flow

One limit of the arrival time distribution for flow with the slip boundary condition is when diffusion is neglected, or that the movement of fluid is purely due to convective forces and particles remain on the same streamline from the beginning to the end of the capillary. Following the approach laid out by Bosworth for Poiseuille flow in cylindrical vessels [39], a similar derivation is performed for the purely convective limit with slip flow in the open-tube cylindrical capillary system. Beginning with

the velocity profile given by Eq. 2.4, and assuming a linear pressure gradient so that $\frac{\partial P}{\partial z} = \frac{\Delta P}{L} = \phi$, the flow rate is calculated as

$$Q = \int_0^{r_c} 2\pi r v_z(r) dr = \frac{r_c^4 \pi \phi}{4\eta} \left(2\gamma + \frac{1}{2}\right) \quad (2.26)$$

The expected arrival time for a particle located at r is given by

$$t(r) = \frac{L}{v_z(r)} = \frac{4\eta L}{r_c^2 \phi \left[1 + \gamma - \left(\frac{r}{r_c}\right)^2\right]} \quad (2.27)$$

Considering the element of a hollow cylinder of fluid with inner radius r and outer radius $r + dr$, the fraction of the total volume of fluid that enters this element in unit time is denoted as $F_r dr$ and is equivalent to

$$F_r dr = \frac{2\pi r v_z(r) dr}{Q} = \frac{2r \left[1 + \gamma - \left(\frac{r}{r_c}\right)^2\right]}{r_c^2 \left(\gamma + \frac{1}{2}\right)} dr \quad (2.28)$$

Similarly, $F_t dt$ is defined as the fraction of total molecules with an arrival time between $t(r)$ and $t(r) + dt$ and is represented as

$$F_t dt = F_r \frac{1}{\frac{\partial t}{\partial r}} dt = \frac{r_c^2 \phi \left[1 + \gamma - \left(\frac{r}{r_c}\right)^2\right]^3}{4\eta L \left(\gamma + \frac{1}{2}\right)} dt = \frac{1}{t^3} \frac{16\eta^2 L^2}{r_c^4 \phi^2 \left(\gamma + \frac{1}{2}\right)} dt \quad (2.29)$$

More completely, there is some minimum theoretical arrival time corresponding to the fastest moving fluid that is located at $r = 0$ which is denoted as t_0 . From Eq. 2.27,

$$t_0 = \frac{4\eta L}{r_c^2 \phi (\gamma + 1)} \quad (2.30)$$

Substituting in t_0 and adding a Heaviside function, $\mathcal{H}$, to Eq. 2.29 to more accurately capture the fact that no particles have arrival times less than t_0 ,

$$F_t = \mathcal{H}(t - t_0) \frac{t_0^2 (\gamma + 1)^2}{t^3 \left(\gamma + \frac{1}{2}\right)} \quad (2.31)$$

This solution reduces to the no-slip arrival time distribution when $\gamma = 0$ [39]. It is worth noting that in the no-slip case, the distribution depends only on the minimal

arrival time and not any geometric parameter of the capillary. In other words, two cylinders with differing dimensions can have the same arrival time distribution if they are tuned to have the same t_0 . In the case of slip flow, the presence of γ in the distribution introduces a dependence on the slip length and capillary radius so that this geometric independence noted by Bosworth no longer applies.

2.3 Numerical Methods

2.3.1 Random walk principles

The random walk technique used for these simulations is based on combining the principles of convection and diffusion to model fluid transport through the capillary. It is a common technique used in molecular and Brownian dynamics simulations [40, 41, 42, 43, 44, 45].

The process consists of adding convective and diffusive contributions to the x , y , and z Cartesian coordinates with each time step. Typically, a time step, t_s , is on the order of 1×10^{-7} seconds or smaller. The new radial and z positions are calculated with each time step. If at a particular time step $z_i \geq L$, where i is the i^{th} time step of the random walk, the particle has reached the end of the capillary. That time is recorded as the arrival time of each particle, and combining the arrival times of all particles of a specific diameter yields the elution profile for the given solute size. The radial position of the particle, r , at the exit position, $z_i \geq L$, is also recorded.

2.3.2 Starting conditions

For each simulation, at least 20,000 particles and typically 100,000 particles are used with all particles starting at $z = 0$. Each particle is placed randomly along the front face of the capillary radial position. This is performed using a random number generator to choose numbers between 0 and the boundary of the capillary accounting for the size of the solute, or $(r_c - r_s)$, for both x and y starting coordinates. This ensures that all particles are initialized within the bounds of the capillary so that $r \leq \sqrt{x_0^2 + y_0^2}$ is satisfied.

2.3.3 Convection

Convection is limited to only the z direction for the purpose of these simulations. Velocity is considered as a function of radial position alone in Hagen-Poiseuille flow. Therefore, the distance in the z direction a particle moves in a given time step due to convection is solely dependent on r_i , or the velocity streamline the particle resides on at that particular time. Mathematically,

$$z_{i+1} = z_i + v_z(r_i) \cdot t_s \quad (2.32)$$

The velocity profile, $v_z(r)$, is given by Eqs. 2.2 and 2.5 for no-slip and slip boundary conditions, respectively. Due to the restriction of convective flow to the z direction, there are no convective contributions to the x and y coordinates of a particle.

2.3.4 Diffusion

The diffusive contribution of each time step involves the addition of a randomly distributed number to each of the x , y , and z components of the location of a particle. This distribution is determined by the Einstein equation [7, 46] which describes the average distance traveled by a particle in each coordinate, $\bar{x}$, as $\bar{x} = \sqrt{2D_m t_s}$. A particle is equally likely to move in either direction for a given coordinate, so a random number is generated for each Cartesian coordinate from a normal distribution with a mean of zero and a standard deviation of $\bar{x}$. The diffusive contribution is then represented as

$$\begin{aligned} x_{i+1} &= x_i + \xi_{x,i} \\ y_{i+1} &= y_i + \xi_{y,i} \\ z_{i+1} &= z_i + \xi_{z,i} \end{aligned} \quad (2.33)$$

where $\xi_{x,i}$, $\xi_{y,i}$ and $\xi_{z,i}$ are normally distributed random numbers added to each coordinate at the i^{th} time step.

2.3.5 Stability

The Courant-Friedrichs-Lewy (CFL) condition is used as a measure of stability for the simulations [47]. This condition states that the Courant number, C_r , defined as

$$C_r = \frac{\bar{v}_z \cdot t_s}{\bar{x}} \quad (2.34)$$

should meet the requirement $0 \leq C_r \leq 1$ for stability. C_r is the ratio of convective and diffusive contributions with each time step, and a small enough time step must be chosen to ensure that the CFL condition is satisfied.

2.3.6 Retention

The probability of adsorption, P_a , controls whether a time delay should occur when a particle collides with a wall. This situation is analogous to an interfacial hopping model [48] where adsorption does not occur on every wall collision. This is implemented with a uniformly-distributed random number, ζ_i , so that if the wall collision occurs and $\zeta_i \leq P_a$, then the particle is considered adsorbed. When adsorbed, a Poisson (exponentially distributed) deviate is chosen with mean delay time $\langle t_d \rangle$ and this is added to the current time counter of the particle. The exponentially distributed number is obtained using the inverse transform sampling theory [49] such that: $t_d = -\langle t_d \rangle \ln(\zeta_i)$.

2.3.7 Particle-wall algorithm

One difficulty in implementing stochastic simulations is what to do when a particle undergoing random walks moves through a boundary. This subject has been previously addressed [50] and the three types of actions that were discussed include rejection, interruption, and specular reflection. In the rejection method, a trial trajectory is rejected when a particle moves through a boundary. New particle coordinates are chosen until the particle no longer moves through the boundary. This technique is simple to program but neglects the space nearest to the boundary. Interruption allows the particle surface to touch the wall and places the particle on the wall along the trial

trajectory. The modified time step, t'_s , is calculated as $t'_s = t_s \cdot (d'/\bar{d})^2$ where d' is the distance traveled to the wall and $\bar{d}^2 = 6Dt_s$, where $\bar{d}$ is the average distance traveled in one time step. The last method, specular reflection, which is the recommended method to implement the boundary encounter [50], involves calculating the specular reflection vector component utilizing the surface normal and the dot product of the incident vector [51]. A general algorithm for this, in the context of stochastic mass transport simulations, is discussed by Maier et al. [52]. All three methods are evaluated for error and tabulated below in section 2.4.4.

2.3.8 Assumptions

Assumptions of this method and model include utilizing a fully-developed flow profile throughout the column length with no entrance effects. The solute is introduced as a perfect sheet in the column cross section. Most important, particle-particle interactions and collisions are absent; this is a zero-concentration model. The presence of both point and finite-size molecules do not perturb the bulk fluid flow. The retention of molecules on the wall surface is assumed to not affect the slip flow at the wall nor the flow in the bulk inner regions of the tube.

2.3.9 Programming tools

FORTRAN-95 was used to write all software for the simulations with parallel programming performed using the message-passing interface (MPI)[53, 54]. Analysis of arrival time data was performed in MATLAB (Mathworks, Natick, Massachusetts).

2.3.10 Data presentation

The widths of histogram elements used when displaying arrival time curves were based on the rule described by Freedman and Diaconis [55]. The bin width, W , was chosen as

$$W = 2(IQR)N_p^{-1/3} \quad (2.35)$$

where IQR is the interquartile range of the arrival time data and N_p is the number of particles used in the simulation.

2.4 Results

2.4.1 Nondimensional characterization of long and short tubes

Simulations were performed to ensure that the algorithm and assumptions used in the code produced results that were consistent with theoretical behavior. First, the well-defined behavior of Poiseuille flow with no-slip boundary conditions were examined and compared to theoretical predictions. This provided validation as well as confirmation of baseline behavior to which results with the slip flow boundary condition could be compared.

In order to differentiate between long and short tube systems, the nondimensional Graetz number, Gz , was used [56] to characterize these systems. It is the ratio of characteristic times for diffusion in the radial direction to flow in the z direction and is defined as

$$Gz = \frac{r_c^2/D_m}{L/\bar{v}_z} = \frac{r_c^2\bar{v}_z}{D_m L} \quad (2.36)$$

$Gz \ll 1$ suggests radial diffusion takes place on a much smaller characteristic time scale than axial flow. Solute particles have time to sample all radial positions within the capillary due to diffusion allowing full access to all of the Poiseuille velocity profile shown in Figure 2.1. Therefore, when $Gz \ll 1$, the plate height theory described in section 2.2.2 should allow an accurate method of determining the moments of the arrival time distribution for these long tube systems. When $Gz \approx 1$, deviations from the long tube Gaussian behavior in the form of tailing [26] typically takes place. $Gz \gg 1$ suggests solute particles flow too quickly in the axial direction to sample the entire velocity profile before reaching the end of the capillary. The result is the failure of plate height theory and arrival time distributions with non-Gaussian distorted peak shapes for these short tube systems. In addition, the more familiar Peclet number, $Pe = \bar{v}_z r_c / D_m$, is used to tabulate the ratio of convective and diffusive contributions and is included for comparison with the Graetz number.

2.4.2 Validation of long and short tube behavior with no-slip boundary conditions

Figure 2.3 presents the peak shapes for simulations run with no-slip boundary conditions and no retention ($R = 1$) at a range of Gz values. Each row corresponds to a particular Gz value. The first column only varies in d_s while holding all other system input values constant as “control inputs”: $L = 10$ cm, $\bar{v}_z = 0.1$ cm/s, and the capillary diameter, $d_c = 41.8$ μm . The second column alters one of the control input values and adjusts d_s accordingly to hold Gz constant. A visual comparison of Figures 2.3b with 2.3c, 2.3d with 2.3e, and 2.3f with 2.3g demonstrates that peak shape is indeed retained when Gz is held constant. In other words, Gz is a valid method to quantify how long or short a tube is despite size and velocity scales that may vary by orders of magnitude.

Figures 2.3a, 2.3b, and 2.3c are the results of long tube simulations. The first and second moments of each case are produced in Table 2.2 along with the theoretical prediction of the moments determined using the plate height theory for no-slip boundary conditions laid out in section 2.2.2. The simulation results of cases a and b show agreement with theoretical prediction for both first and second distribution moments within 1.0%.

Table 2.2: Long tube simulations with no-slip boundary condition and comparison to theoretical distribution. Simulation parameters: $L = 10$ cm, $\bar{v}_z = 0.1$ cm/s, and $N_p = 20,000$ for all cases. The “Figure” column corresponds to the label of the elution profile in Figure 2.3.

		Inputs		Results		Theory			Percent Difference	
Figure	Gz	d_c (μm)	d_s (nm)	M_1 (s)	M_2 (s^2)	$\bar{t}$ (s)	H (μm)	σ^2 (s^2)	In Arrival Time (%)	In Variance (%)
a	0.01	41.8	1	99.99	4.21	100	42.5	4.25	-0.01	-1.02
b	0.1	41.8	10	100.18	41.30	100	416.8	41.68	0.18	-0.89
c	0.1	13.2	100	98.74	38.68	100	416.7	41.67	-1.26	-7.18

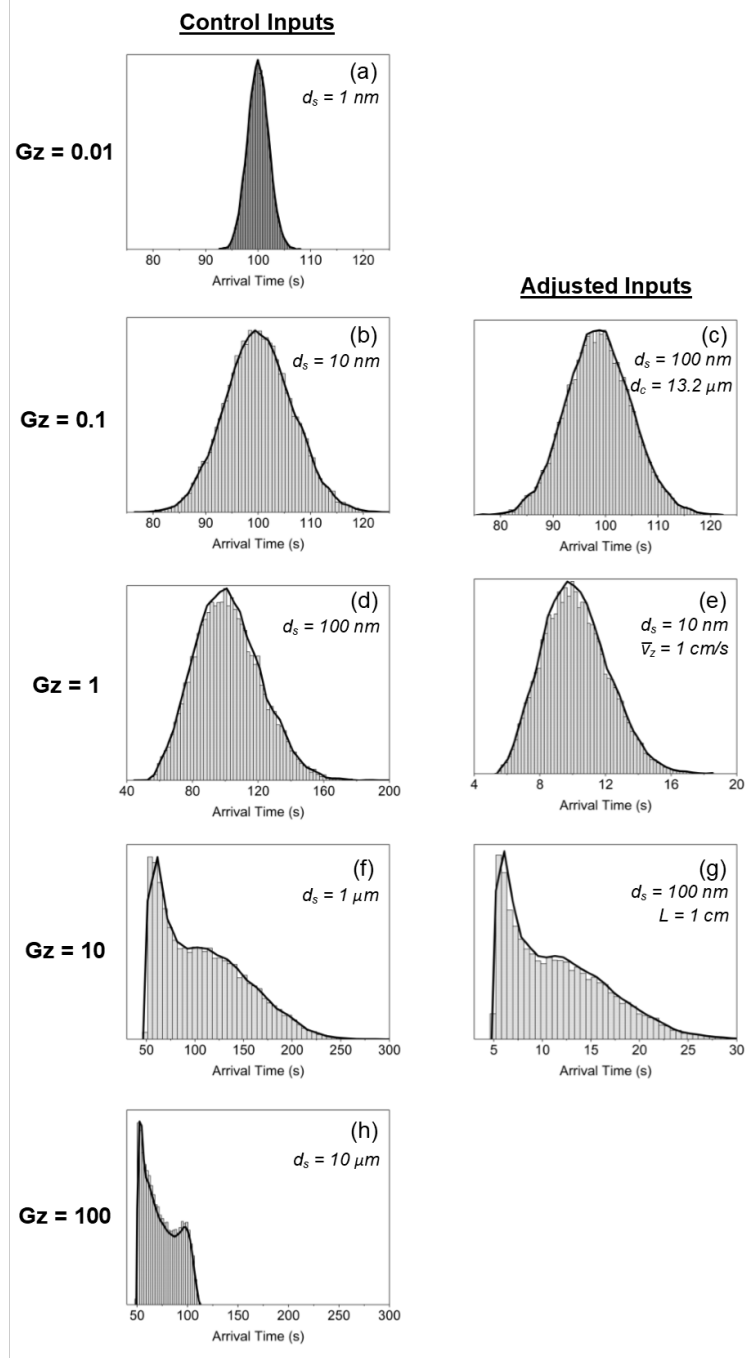


Figure 2.3: Arrival time distributions for long tube and short tube systems with no-slip boundary condition. Each row corresponds to a constant value of Gz . The simulations of the first column have constant inputs of $L = 10 \text{ cm}$, $\bar{v}_z = 0.1 \text{ cm/s}$, $d_c = 41.8 \text{ } \mu\text{m}$, and $N_p = 20,000$, with varying values of d_s . Pe values for each case are a) 4.78×10^1 ; b) 4.78×10^2 ; c) 1.51×10^3 ; d, e, g) 4.78×10^3 ; f) 4.78×10^4 ; h) 4.78×10^5 .

The simulation resulting in Figure 2.3c shows a larger difference between simulation and theory; both the first and second moments from simulation are smaller than those predicted by theory. This can be explained by the finite size of the particles, which both the definition of Gz as well as plate height theory fail to account for. A particle that is smaller but is considerable relative to the capillary radius will result in solutes that cannot sample the entire velocity profile as the finite size prevents the center of mass from reaching $r > (r_c - r_s)$. The impact of this is a lesser velocity gradient and a larger average velocity than would be predicted by theory leading to smaller first and second moments as supported by the parameters used in making Figure 2.3c. To check this, the average velocity for the same velocity profile used in case c was calculated for a capillary where particles are limited to $r \leq (r_c - r_s)$ as

$$\bar{v}_z^* = \frac{\int_0^{r_c - r_s} v_z(r) \cdot 2\pi r \, dr}{\int_0^{r_c - r_s} 2\pi r \, dr} \quad (2.37)$$

where $\bar{v}_z^*$ is the new average velocity. For no-slip conditions, $v_z(r)$ is defined by Eq. 2.2 and Eq. 2.37 becomes

$$\bar{v}_z^* = \bar{v}_z \left[2 - \left(\frac{r_c - r_s}{r_c} \right)^2 \right] \quad (2.38)$$

Plugging in the values from case c yields $\bar{v}_z^* = 0.1015 \text{ cm s}^{-1}$ and a new theoretical arrival time of 98.52 s. With this adjustment, the first moment of the simulation of the simulation only varied by +0.23% compared to the theoretical prediction. Therefore, the finite size of solute particles is an aspect that needs to be considered in addition to Gz when fully describing the behavior of a system. Because particles have finite sizes, it is also a phenomenon that these simulations are able to capture while both theory and simulations that treat particles as points fail to.

The arrival time distributions of the shorter tube cases in Figures 2.3d-g resemble the expected behavior shown by Figure 1 in the paper of Atwood and Golay [9]. In their figure, the distributions are computed for various numbers of theoretical plates, N , which they define as

$$N = \frac{24\pi D_m L}{Q} \quad (2.39)$$

Since $Q = \bar{v}_z \pi r_c^2$, $N = 24/Gz$.

The plots in Figure 2.3 correspond to $N = 24$, $N = 2.4$, and $N = 0.24$ for $Gz = 1$, $Gz = 10$, and $Gz = 100$, respectively. Figures 2.3d and 2.3e correspond to $Gz = 1$ and exhibit the anticipated behavior marked by roughly Gaussian peak shapes but with noticeable tailing. Figures 2.3f and 2.3g correspond to $Gz = 10$ and exhibit the characteristic two-humped distributions shown in Figure 1 of the Atwood and Golay paper by the $N = 3$ curve. The two humps occur at roughly the expected average arrival time and at one-half of this value which corresponds to theory [17].

Figure 2.3h does not resemble the expected behavior which would be a single peak initially followed by hyperbolic decay down to zero as the purely convective limit is approached. Our simulation shows two humps still which is due to the fact that the particle diameter is one-quarter of the capillary diameter so that a large fraction of the velocity gradient is not accessed.

2.4.3 Long tube behavior with slip flow boundary conditions and no wall retention

Simulations were performed with the slip flow boundary condition now employed at Gz values on the order of 10^{-5} and smaller. In these simulations γ was varied between 0 and 0.4 and the moments of the arrival time distributions were compared to the theoretical moments now using Eq. 2.25. Note that the input for $\bar{v}_z$ was varied according to Eq. 2.6. In addition, Eq. 2.5 was substituted into Eq. 2.37 to account for the finite size of the particle restricting the velocity gradient near the walls. The result for slip flow is

$$\bar{v}_z^* = \frac{\bar{v}_z}{1 + 2\gamma} \left[2(1 + 2\gamma) - \left(\frac{r_c - r_s}{r_c} \right)^2 \right] \quad (2.40)$$

The system used was a tube of $L = 1$ cm and $r_c = 200$ nm with solute particles of $d_s = 1$ nm and $\bar{v}_{z,no-slip} = 0.1$ cm/s. This corresponds to $Gz = 9.2 \times 10^{-6}$ for no-slip conditions. Although the value of capillary radius chosen here appears very small, this is in the range of values where the slip length is a significant fraction of the capillary radius. It is in this range that slip lengths are particularly viable and effective.

Larger slip lengths may be obtainable, but as mentioned in the introduction, these are probably due to trapped gas and this situation would make for a very irreproducible system.

Capillary inner diameters as small as 200 nm are commercially available such as capillaries offered by the Polymicro Technologies division of the Molex Corp. (Lisle, IL, USA). This scale is also conducive to chip-based microfluidic separation systems. Alternatively, the technique of electrospinning could be utilized to produce hollow nanofibers that would function as capillary systems on this size scale. This method uses a high voltage electrical driving force which forces a polymer melt or solution through a spinneret and is capable of producing fibers ranging from several nanometers to several microns [57]. A wide range of polymers and ceramics can be used which allows the ability to tailor aspects such as fiber diameter and surface properties [58, 59].

Figure 2.4 displays M_1 and M_2 for a range of γ values along with the theoretical expectations from Eq. 2.40. The simulation data closely traces the long tube theory over the entire range of γ values: M_1 and M_2 vary in magnitude at most by 0.0078% and 0.77%, respectively. M_1 and M_2 are reduced to 55.7% and 17.4% of the no-slip values, respectively, when γ reaches 0.4. The potential benefits of slip flow are highlighted by the decrease in both moments as shorter arrival times result in higher throughput and smaller variance results in sharper elution peaks.

2.4.4 Comparison of particle-wall algorithms

The same system in section 2.4.3 was used to compare the three wall collision methods detailed in section 2.3.7. Table 2.3 summarizes the percent differences between simulation and theory with each of the three methods for a range of γ values. In addition, Table 2.3 compares how each collision method compares to theory both when the finite size of the particle is accounted for (velocity profile given by Eq. 2.40) and when it is not (velocity profile given by Eq. 2.5).

The rejection method resulted in M_1 and M_2 consistently lower than the theoretical prediction. Both the interruption and specular reflection methods resulted

in moments that were closer to theory, and specular reflection was slightly closer to theory than the interruption method when the particle size was accounted for. For all three values of γ using specular reflection and particle size accounted for, M_1 varied from theory in magnitude by only 0.01% and M_2 varied from theory at a maximum of 0.77%. Therefore, the simulations in section 2.4.3 and all subsequent sections utilize the specular reflection method for handling particle-wall collisions.

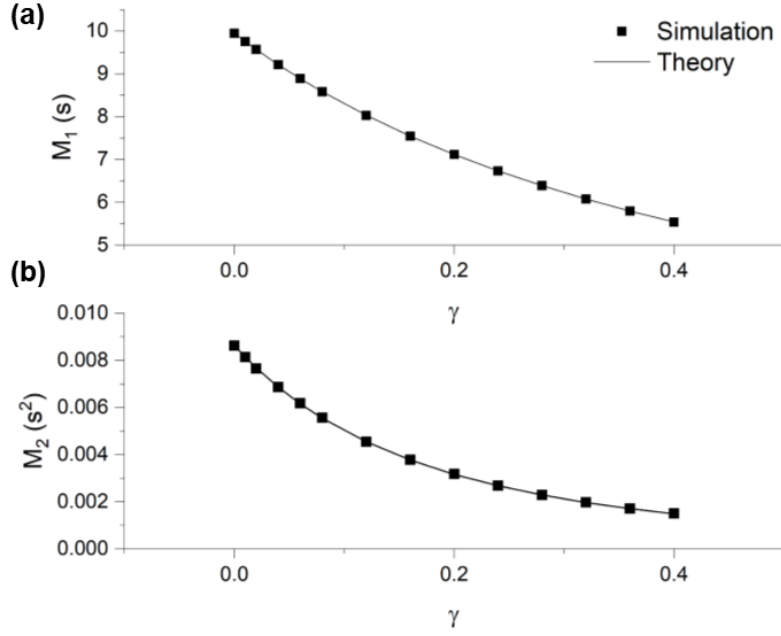


Figure 2.4: First (a) and second (b) moments of simulation results for long tube system with slip boundary condition and comparison to theory. Simulation parameters: $L = 1$ cm, $\bar{v}_{z,no-slip} = 0.1$ cm/s, $r_c = 200$ nm, $d_s = 1$ nm, and $N_p = 100,000$. This corresponds $Gz = 9.2 \times 10^{-6}$ and $Pe = 0.46$ for no-slip conditions.

Table 2.3: Results of wall collision method testing for long tube capillaries with no-slip and slip conditions. The simulations are compared to theoretical moments for both the case of accounting for finite size and not accounting for it. For all trials, $L = 1$ cm, $\bar{v}_{z,no-slip} = 0.1$ cm/s, $r_c = 200$ nm, $d_s = 1$ nm, and $N_p = 100,000$. This corresponds to $Gz = 9.2 \times 10^{-6}$ and $Pe = 0.46$ for $\gamma = 0$, $Gz = 1.2 \times 10^{-5}$ and $Pe = 0.60$ for $\gamma = 0.16$, and $Gz = 1.7 \times 10^{-5}$ and $Pe = 0.82$ for $\gamma = 0.40$. All values reported are the percent differences in moments between simulation and theory.

		$\gamma = 0$		$\gamma = 0.16$		$\gamma = 0.40$	
Collision Method	Size Accounted?	M_1	M_2	M_1	M_2	M_1	M_2
Rejection	No	-3.78	-11.1	-2.89	-8.43	-2.14	-5.59
	Yes	-3.30	-9.72	-2.52	-7.39	-1.87	-4.81
Interruption	No	-0.01	-0.53	0.00	-0.23	0.00	0.02
	Yes	0.49	0.96	0.37	0.91	0.27	0.85
Reflection	No	-0.50	-1.60	-0.38	-0.86	-0.28	-0.06
	Yes	-0.01	-0.12	-0.01	0.27	-0.01	0.77

2.4.5 Long tube behavior with slip flow boundary conditions and wall retention

Having verified that the simulation and theory are in agreement for long tube behavior with slip flow boundary conditions, we can use the theory developed in section 2.2.2 and specifically Eq. 2.24 to evaluate the effect of slip flow with retention in the long tube limit. The results are shown in Figure 2.5 at different values of the retention parameter k' noting that $k' = (1/R) - 1$. The retention parameter k' indicates the number of eluted column volumes minus 1; hence the void peak gives a $k' \approx 0$.

A number of features are highlighted from this figure. At zero retention conditions, the reduction in C_m as a function of γ is large compared to the no-slip condition. However, this reduction in the liquid phase plate height, via Equation 2.11, shows significant reduction at moderate γ values. For example, when $\gamma \approx 0.1$, C_m shows $\approx 30\%$ reduction in mobile phase plate height. At the same slip length, more retained zones show a much smaller effect. For example, for $k' = 10$, the reduction in C_m is

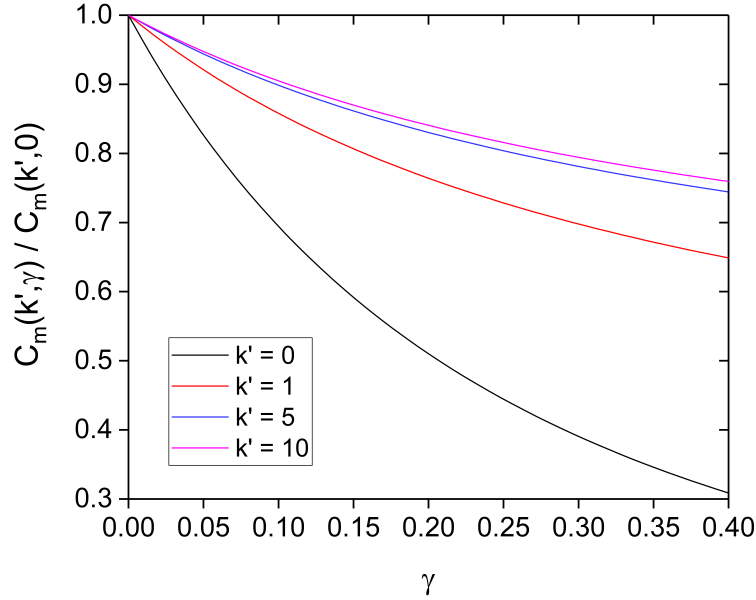


Figure 2.5: The mobile phase mass transport coefficient normalized to $\gamma = 0$ for 4 cases of retention at differing k' values.

$< 10\%$ and this is about the same reduction as that found for $k' = 5$. These results suggest that the main reduction in plate height by utilizing slip flow takes place in the region where retention is in the range $0 \leq k' \leq 1$. This suggests that utilizing slip flow for chromatographic separations is not routinely applicable to all regions of the chromatogram.

2.4.6 Short tube behavior with slip flow boundary conditions and no wall retention

Short tube conditions were simulated for the same $r_c = 200$ nm as in section 2.4.3. In order to reach $Gz > 10$, parameters of $L = 1$ μm , $d_s = 20$ nm and $\bar{v}_{z, no-slip} = 1$ cm/s were chosen. Note that the length of the capillary here is ultra-short and would be expected from a thin membrane, micro- or nano-channel, or even a pore in a chromatographic particle. Further aspects of this will be presented in

the discussion section below. These parameter values correspond to Gz ranging from 18.3 for no-slip conditions to 33.0 for $\gamma = 0.4$. The elution profiles varying γ while holding all parameters constant are displayed in Figure 2.6. As expected, the velocity enhancement provided by the increase in γ leads to the distribution shifting towards shorter elution times and becoming narrower in shape. The second hump of the profile becomes more pronounced as γ increases.

The dashed lines in Figure 2.6 correspond to no-slip boundary conditions but with the same enhancement in $\bar{v}_z$ as if there was slip occurring. For small values of γ , slip and enhanced no-slip curves nearly overlap, meaning that any shift in the profile is due to the velocity enhancement opposed to the conditions at the wall. As shown throughout Figure 2.6, as γ gets larger, these two curves diverge, and the slip conditions result in narrower profiles at earlier times with a more pronounced second hump. Therefore, the presence of a finite wall velocity impacts zone shape under short tube conditions, in addition to the overall velocity enhancement that results from slip flow. This is reminiscent of the reduced dispersion in capillary electrophoresis with electroosmotic flow [60, 61] where the velocity distribution of fluid is essentially flat across the capillary as electroosmosis moves the fluid at constant velocity.

A comparison of the decrease in M_1 and M_2 is provided in Figure 2.7. Similar to long tube conditions, M_2 decreases more than M_1 relative to the corresponding no-slip values. M_1 drops to 53.2% of its no-slip value when $\gamma = 0.4$ while M_2 drops to 10.5% of its no-slip value. These are comparable to the drops seen under long tube conditions.

Figure 2.7 also illustrates the difference between the enhanced no-slip flow and flow with slip boundary condition. The difference in M_1 between the two cases is small across the entire range of γ . Conversely, the difference in M_2 becomes large as γ increases. The effect of slip at the wall (opposed to just increased velocity) is therefore larger for M_2 compared to M_1 under short tube conditions.

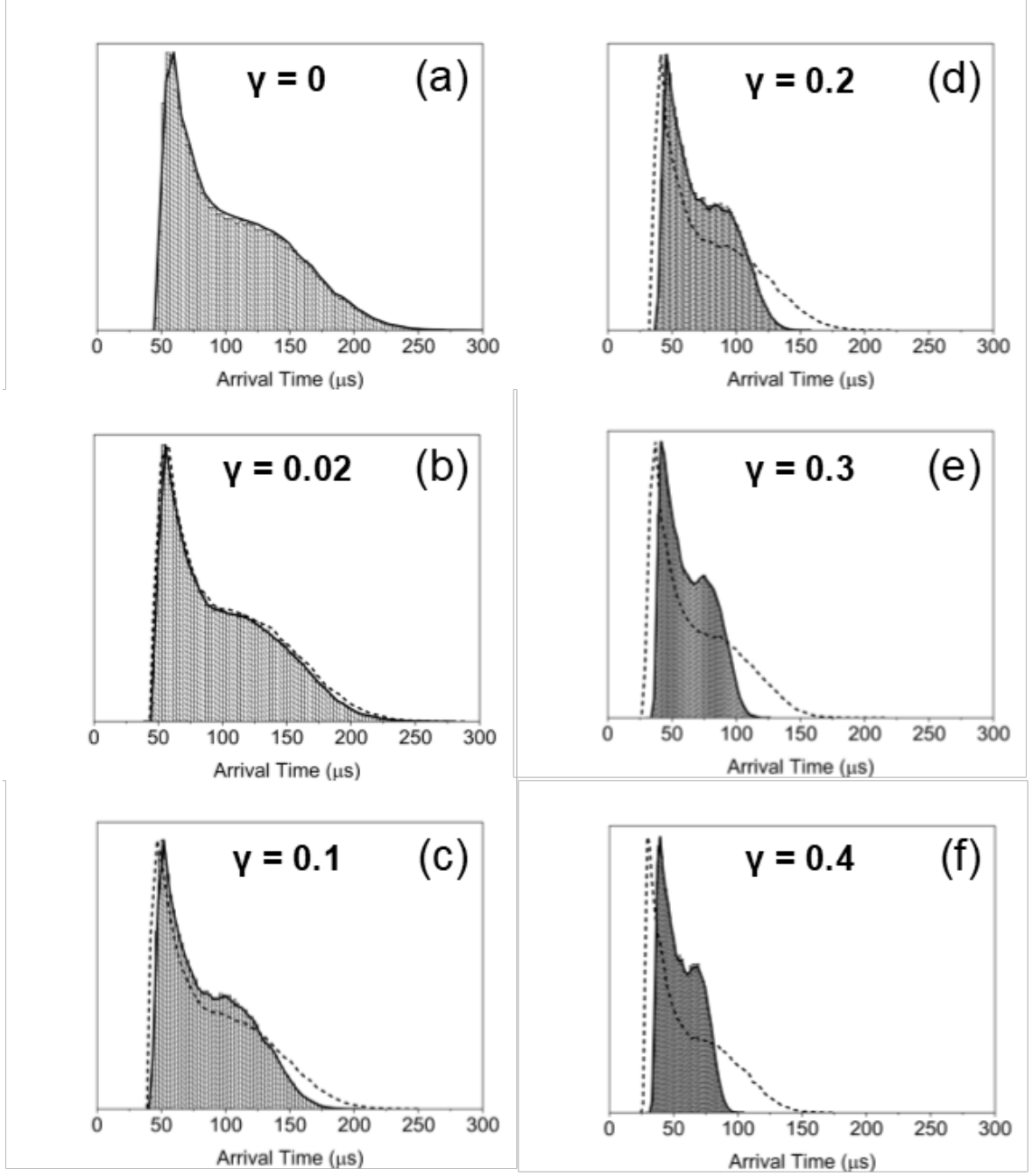


Figure 2.6: Zone shapes for varying slip lengths under short tube conditions without retention. Solid lines: slip boundary condition. Dashed lines: no-slip boundary condition but with the same velocity enhancement as the corresponding γ value. Simulation parameters: $L = 1 \mu\text{m}$, $\bar{v}_{z,\text{no-slip}} = 1 \text{ cm/s}$, $r_c = 200 \text{ nm}$, $d_s = 20 \text{ nm}$, and $N_p = 100,000$. Gz and Pe range from 18.3 and 91.6 for $\gamma = 0$ to 33.0 and 165 for $\gamma = 0.4$.

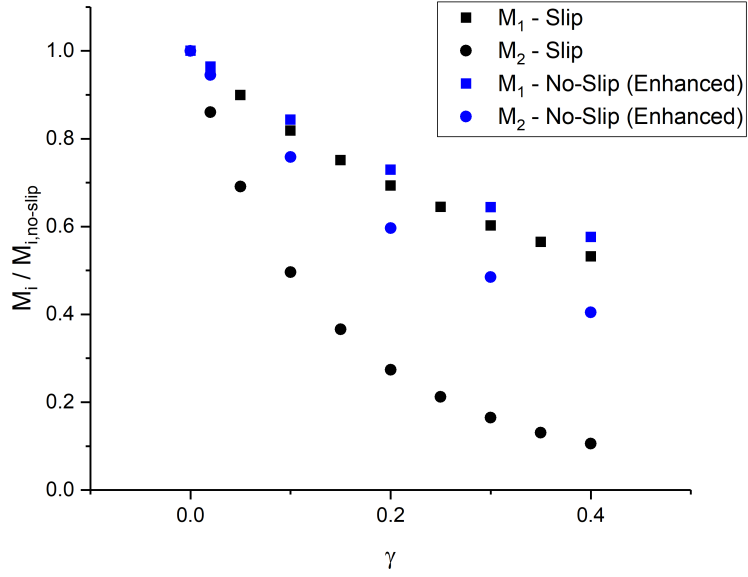


Figure 2.7: First (squares) and second (circles) moments for varying slip lengths under short tube conditions without retention. Black: slip boundary conditions. Blue: no-slip boundary condition but with the same velocity enhancement as the corresponding γ value. Simulation parameters: $L = 1 \mu\text{m}$, $\bar{v}_{z,no-slip} = 1 \text{ cm/s}$, $r_c = 200 \text{ nm}$, $d_s = 20 \text{ nm}$, and $N_p = 100,000$. Gz and Pe range from 18.3 and 91.6 for $\gamma = 0$ to 33.0 and 165 for $\gamma = 0.4$.

2.4.6.1 Purely convective limit

To test the derivation performed in section 2.2.3, short tube conditions must be achieved so that convection dominates diffusion. Small tube radii where slip flow applies is not conducive to these conditions as it allows for fast sampling of the entire velocity field. This means that diffusion will always be a factor unless velocity is increased greatly to increase $Gz \gg 1$. In addition, $L \gg \bar{x}$ for the theory to apply. At these velocities and capillary lengths, the Reynolds number becomes greater than 100 where these simulations using laminar flow no longer apply.

Figure 2.8 illustrates the theoretical impact of slip flow on the purely convective limit if these short tube conditions could be met. In practice, these conditions cannot be met because, as stated previously, the capillary diameters where slip flow is a significant part of the velocity profile are small and this promotes long tube behavior even with short-length tubes. This is reflected in the r_c^2 dependency of the Graetz number where the radius of the capillary is the controlling parameter as contrasted to all other parameters in Eq. 2.36 which have a linear dependence. In attempts to compare simulation and the pure convective theory with slip flow, it was found that for small-length capillaries, axial diffusion was significant enough to render the comparison poor even if the radial Taylor dispersion was not included. From the viewpoint of practical separations which could use slip flow to reduce dispersion, the length scales involved favor long tube behavior and this can be a positive driving force for using slip flow; i.e. very short tubes can give fast and efficient separations without the zone development experienced with short tubes.

2.4.7 Short tube behavior with slip flow boundary conditions and wall retention

As mentioned previously, it is difficult to find conditions where slip flow and short tube behavior are possible. This is due to the dueling requirements of a small capillary radius to accommodate slip flow and a large enough capillary to promote the short tube behavior. This is an advantageous situation from an experimental viewpoint

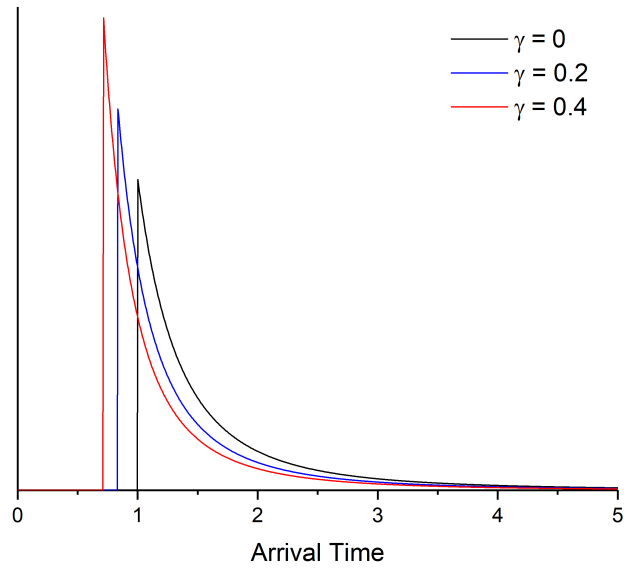


Figure 2.8: Theoretical impact of slip flow on pure convective limit. The derivation for this limit is produced in section [2.2.3](#). A no-slip t_0 value of 1 time unit is used for these curves.

since short tube zone profiles are undesirable from a high resolution analytical sense. However, there are examples where a binary separation is desirable and zone shape is important. We used the same set of conditions from section 2.4.6 here to determine what happens in the case of an extremely short, extremely fast set of conditions with and without slip flow for illustrative purposes. The results are shown in Figures 2.9 ($P_a = 0.1$) and 2.10 ($P_a = 1.0$).

In all cases shown in Figures 2.9 and 2.10, slip flow is helpful in compressing the tails of these types of zones. This is due to the limited amount of low velocities which the solute molecules sample. Short tube behavior emphasizes both low and high velocity regions because molecules cannot diffuse out of these regions fast enough. However, slip flow limits the extent of available low velocity regions.

When comparing Figures 2.9a and 2.9d with Figures 2.9b and 2.9e, it is seen that a small amount of retention with a limited probability of adsorption will extend the tail, and slip flow counteracts this tail extension. This may be helpful where a fast affinity separation is to be utilized at trace concentration levels and under extreme speed conditions. It must be realized that this is extremely difficult to implement because it is assumed that the zone is introduced as a perfect plug of infinitely thin axial extent; any variation in sample introduction would cause spillover to the column outlet. At longer retention conditions, shown in Figures 2.9c and 2.9f, the same effect is shown; zone compression by slip flow boundaries yield zone shapes that do not include as large a tail. However, when the probability of adsorption is increased to 1.0 in Figure 2.10, a more extreme tailing is shown, especially at the longest average adsorption time. Under these conditions, slip flow is ineffective at preventing long tails that resemble column bleed. This is a similar situation to adsorption in capillary electrophoresis where even small amounts of adsorption completely destroy resolution [26].

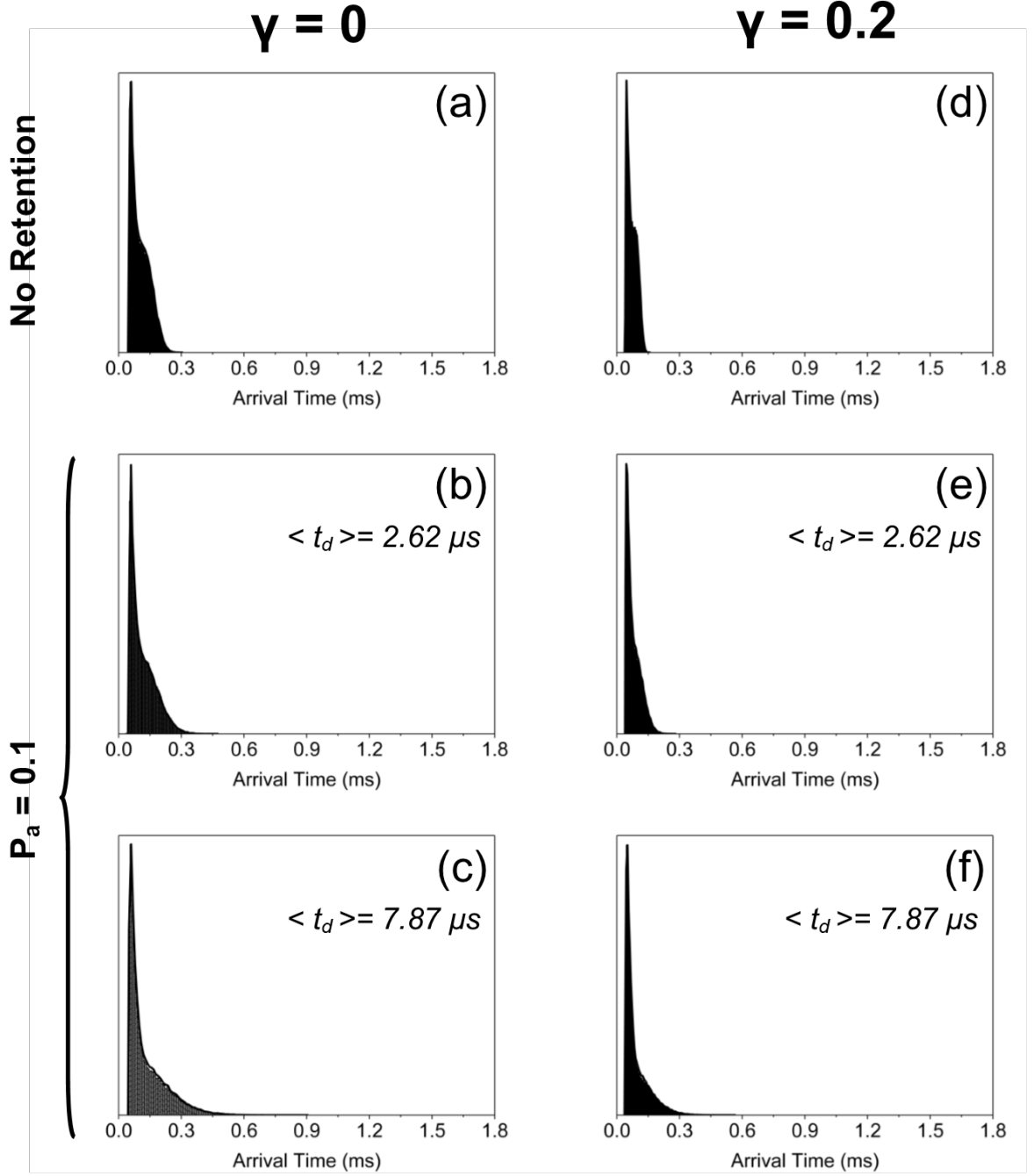


Figure 2.9: Simulation results for slip flow with retention for $P_a = 0.1$ and multiple $\langle t_d \rangle$ values. The $\langle t_d \rangle$ values correspond to 2.5% and 7.5% of M_1 from Figure 2.9a (no-slip and no retention). Simulation parameters: $L = 1 \mu\text{m}$, $\bar{v}_{z,\text{no-slip}} = 1 \text{ cm/s}$, $r_c = 200 \text{ nm}$, $d_s = 20 \text{ nm}$, and $N_p = 100,000$. $\gamma = 0$: $Gz = 18.3$ and $Pe = 91.6$; $\gamma = 0.2$: $Gz = 25.6$ and $Pe = 128$.

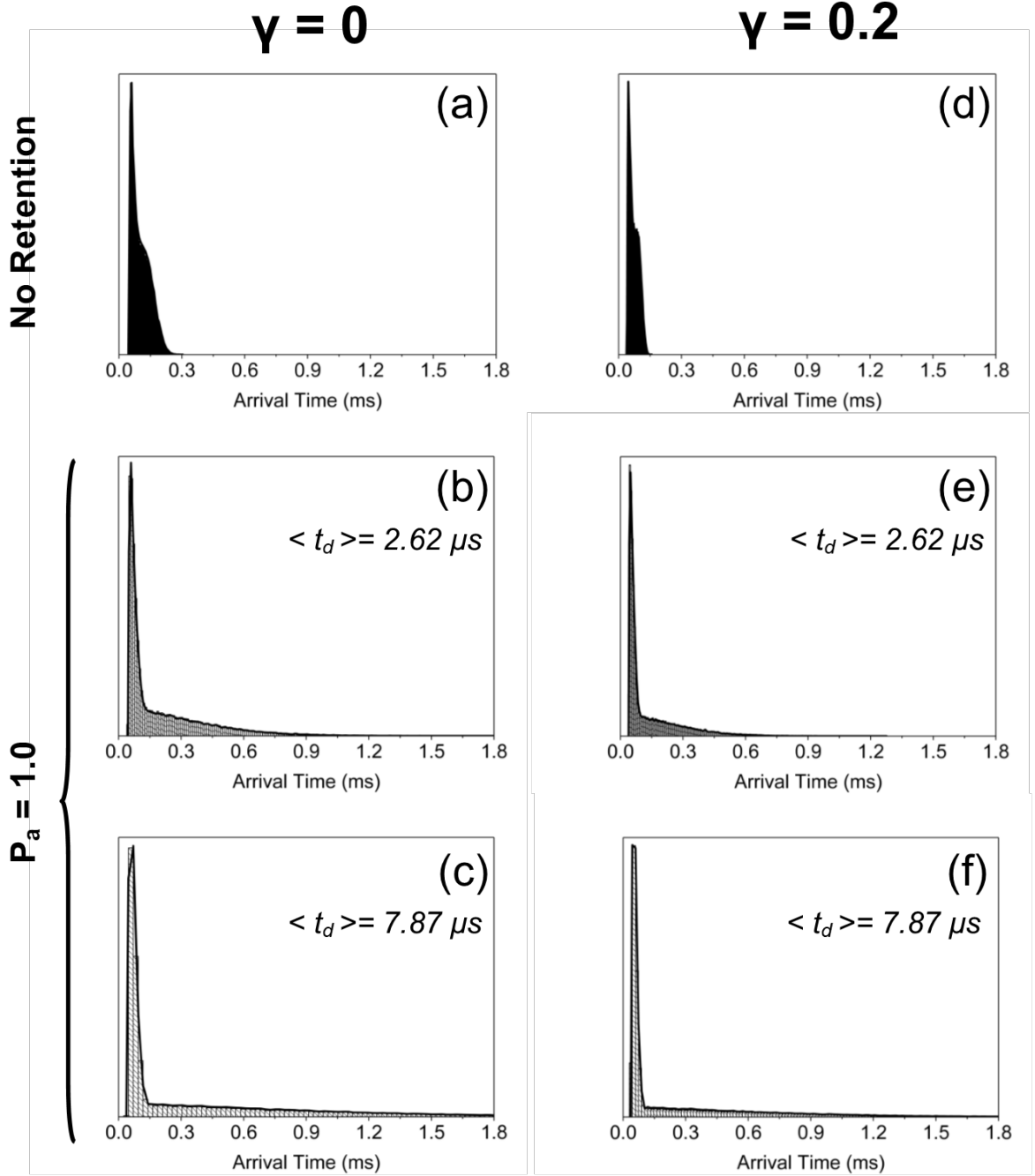


Figure 2.10: Simulation results for slip flow with retention for $P_a = 1.0$ and multiple $\langle t_d \rangle$ values. The $\langle t_d \rangle$ values correspond to 2.5% and 7.5% of M_1 from Figure 2.10a (no-slip and no retention). Simulation parameters: $L = 1 \mu\text{m}$, $\bar{v}_{z,\text{no-slip}} = 1 \text{ cm/s}$, $r_c = 200 \text{ nm}$, $d_s = 20 \text{ nm}$, and $N_p = 100,000$. $\gamma = 0$: $Gz = 18.3$ and $Pe = 91.6$; $\gamma = 0.2$: $Gz = 25.6$ and $Pe = 128$.

2.5 Discussion

We have surveyed a theoretical and simulation approach to understanding zone shape and dispersion in capillaries where slip flow boundary conditions are imposed. A number of important points were determined from these calculations and simulations, and these will be discussed here. As is now accepted and demonstrated in a number of cases in this paper, the use of slip flow boundaries to reduce chromatographic dispersion is limited to very small tube diameters. Very small particles have been recommended for packed-bed chromatography incorporating slip flow [62]. It is impossible in these cases to isolate out the source of fluid-surface interaction from surface roughness, which is also known to be a source of fluid boundary slippage [12, 13, 14, 22, 23]. However, capillaries with surface modification by hydrophobic coatings used for the preparation of reversed-phase liquid chromatography particles may facilitate slip flow. In this regard, long tube behavior is facilitated by slip flow boundaries and this favors the use of short capillaries when high plate count is necessary.

A number of other effects come out in the analysis. In Figure 2.5, where the reduction in dispersion is shown as a function of the nondimensional slip length γ and the retention factor k' , moderate reduction in dispersion is limited to the early part of the chromatogram where $k' \leq 1$. The larger is γ , the larger is the reduction in plate height compared to no-slip boundaries. The liquid phase typically would be pure water to maximize γ . However, gradient elution chromatography, the mainstay of high resolution chromatography, will lessen γ as organic solvents are introduced into the mobile phase. This will increase the width of zones as γ is reduced. This brings into question the general use of slip flow as one that can function in a narrow range of solvents and probably cannot be used with gradient elution. Even with isocratic operation, the curves in Figure 2.5 suggest a limited elution range in improving the dispersion and lowering plate heights for general usage. Nonetheless, the use of slip flow for very fast, high resolution separations of a limited number of components with small retention values may be a feasible method for implementing fast capillary-based separations.

Chapter 3

PORE-SCALE MASS TRANSPORT SIMULATION OF POROUS-LAYER OPEN-TUBE (PLOT) COLUMNS WITH RADIALY-ORIENTED PORES FOR SIZE-EXCLUSION CHROMATOGRAPHY

Chapter Synopsis

A stochastic simulation of porous-layer open-tube (PLOT) liquid chromatography with radially-oriented cylindrical pores is presented. This simulation facilitates the study of pore length and pore density for spherical finite-size solutes, allowing the computer-aided design of these columns for size-exclusion chromatography (SEC) and bonded-phase chromatography. The entrance effects of finite-size solutes diffusing into radial pores are probed over a wide range of molecule to pore size ratios.

A fast algorithm is described which is able to locate the solute position within radially-oriented pores using binning techniques; this algorithm does not require searching. The implementation allows the parallel computing of detailed pore diffusion trajectories while computing the macroscopic peak elution profiles obtained from experiment.

Zone shape, statistical moments, and the number of theoretical plates are computed and described in detail for this column geometry. This allows the evaluation of radially-oriented pore PLOT columns which can now be made due to recent synthesis innovations. The algorithm and code are highly parallel and allow for fast execution on large parallel computers. The use of this approach alleviates having to assume the mass transport rates for radially-oriented pores, as has been previously reported. The calculation of plate count allows the study of finite-sized solutes which are difficult if not impossible to explore through chromatographic theory. Size-based separations affected by varying pore length, average velocity, and efficiency are able to be explained over

a wide-range of parameters. The choice of pore length is shown to be a compromise between kinetics (small pore lengths) and thermodynamics (larger pore lengths).

3.1 Introduction

A large effort has been undertaken in the literature for over 50 years in describing the mechanism and transport properties which take place in gas and liquid chromatography [4, 6, 63, 64]. The physical description of chromatography is typically presented through two distinctive effects: phase equilibria, which describes the change in distribution coefficient with solute molecular structure, and mass transport effects which dominate the zone width and control chromatographic efficiency and resolution. Many questions are still open regarding both of these aspects in relationship to chromatographic materials and chromatographic separations.

In the case of phase equilibria, detailed molecular simulations have shed light on how solute molecules interact with the stationary phase in both gas chromatography [65, 66] and liquid chromatography [67, 68, 69]. For liquid chromatography, the long established theoretical interpretation, the so-called “solvophobic theory” introduced by Horváth and coworkers [70], was found to be incorrect [68] as simulation methods were able to map the thermodynamics of retention across the solvent range of interest.

Particle simulation methods have also been introduced for studying mass transport effects related to chromatography [71]. These simulation methods use particles as tracers to determine where convective and diffusive forces move solutes. Applications of these numerical techniques to separation science research include open tubes [26], packed beds [43], and systems with force fields [72, 73] for colloid and polymer separations.

In this chapter we show that a model of a porous-layer open-tube (PLOT) column can capture the essential physical dynamics of mass transport processes which define this mode of capillary chromatography [74] and size-exclusion chromatography (SEC) [75]. The model we study here extends naturally to a recently published radially-oriented pore system [76] which was implemented with packed-bed particles. This type

of pore system was formulated in the context of a pillar-array column [77] and was solved numerically for a differential equation-based model [77]. In this study, empirical values were utilized for the pore mass transport rate. Because we are able to model the transport at the pore level, we can track individual rates of finite-sized particles in and out of the radially-oriented pore system and calculate the mass transport in and out of discrete pores using random-walk simulation technology without assuming empirical transport rates. Pore-scale modeling was recently demonstrated for packed-bed particles with detailed fluid mechanics [78] and for a high resolution mass transport model which was able to explain the movement of finite-size tracers in pore diffusion with convection [52]. We extend that study here with pores specified in a radially-directed and ordered configuration utilizing random-walk tracers in a PLOT column configuration. This allows a number of observations to be made prior to synthesis of this style of column and furthers the idea of using computer resources to *a priori* promote the design and engineering of analytical chromatography columns *in silico*.

3.2 Computational Method

3.2.1 Model and particle location

The model is composed of two parts. The first is a cylindrical capillary, which is the main transport region with convection and diffusion. Perpendicular to this path are radially-projected cylindrical pores where only diffusion can occur. This is shown schematically in Figure 3.1 and a visualization of this system is shown in Figure 3.2. The parameters and the corresponding symbols used to generate and describe this model are summarized in Table 3.1, and a further description of the pore system is had below.

Random walks are used in this work to model diffusion. This technique is widely known and is often referred to as Brownian dynamics [40, 41, 42, 43, 44, 45]. This technique is easy to use, especially when boundaries make up a large portion of the available space. However, collision detection of solute tracers with solid boundaries must be fast or the method becomes inefficient. The remainder of this section details

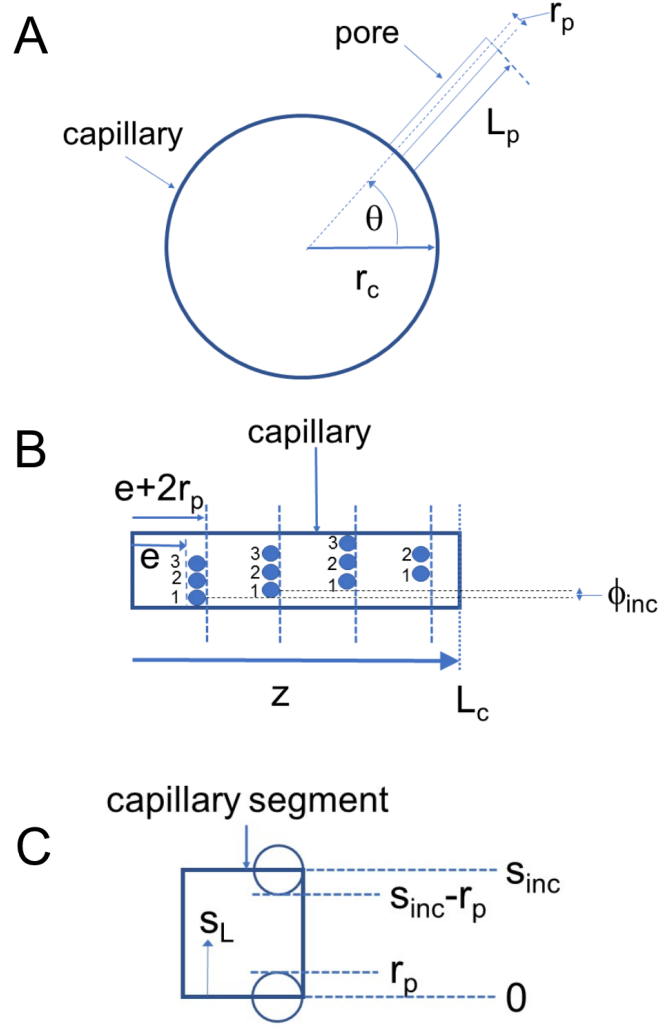


Figure 3.1: Coordinate system of the model. A: x-y plane cross section showing a radial pore with length L_p and pore radius r_p . B: z-axis coordinates where the pore-capillary interface is shown as filled circles and the incremental phase, ϕ_{inc} , is shown between successive z segments. C: Radial pore coordinates between two pores in one radial segment. The pore-capillary interface is shown as open circles.

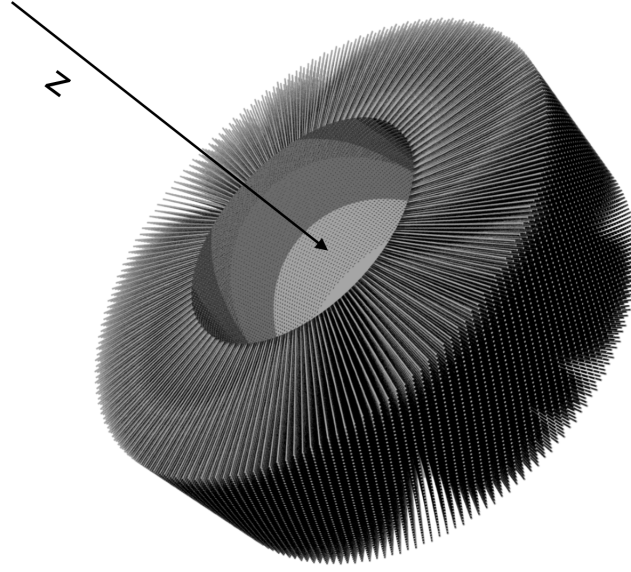


Figure 3.2: The pores and central cylinder comprising the model.

Table 3.1: Parameters needed to generate the model.

Symbol	Units	Explanation
e	m	Distance between segment start and pore perimeter
L	m	Length of capillary
L_p	m	Length of pore
N_r		Number of pores per capillary segment
r_c	m	Capillary radius
r_p	m	Pore radius
ϕ_{inc}	$radians$	Pore placement phase increment

an efficient scheme for wall collision recognition and describes the solute convection and diffusion mechanics.

The phase factor, ϕ_{inc} , is used to move the locations of each successive group of pores in the axial direction. This helps reduce correlation effects between pores to some extent. This parameter will be shown to be easily incorporated into the pore detection scheme described below; the staggered pores are shown in Figure 3.1B.

The collision detection of solute particles with the walls of the model is required for the random-walk simulation and is performed as follows. To detect whether the solute is in the capillary or the pores, the distance from the capillary cross section origin to the solute is calculated as $r = \sqrt{x_i^2 + y_i^2}$ where x_i , y_i and z_i are the coordinates of the solute particle at the i^{th} iteration of convection and diffusion. In order for the particle to be in either the capillary or pores, it must be within the length of the capillary, or $0 \leq z_i \leq (L - r_s)$, where L is the length of the capillary.

The solute particle of radius r_s is inside the capillary when $r \leq (r_c - r_s)$. The requirements for the solute being in a pore is more complicated and will be developed here noting that pore occupancy requires that $r > (r_c - r_s)$ as the first criterion for pore penetration.

The pores are located at specific locations along the capillary perimeter. As shown in Figure 3.1B, the capillary has radial pores which emanate from the capillary perpendicular to the z (flow) axis. These pores are axially spaced in z to occur in segments of length $e + 2r_p$. The concept of segment number is central to the pore location scheme described here. The total number of these segments, N_z , is

$$N_z = \left\lfloor \frac{L}{e + 2r_p} \right\rfloor + 1 \quad (3.1)$$

where e is the distance in z from the start of each segment to the left side of the pore (referring to Figure 3.1), r_p is the pore radius, and $\lfloor \cdot \rfloor$ is the floor function which for positive values eliminates the fractional value of the number.

Next, the z location within the present local segment, z_L , is determined as

$$z_L = z_i \bmod (e + 2r_p) \quad (3.2)$$

where $a \bmod b$ is the modulus function which returns the remainder of the division of a by b . The particle may be in the pore when $e + r_s \leq z_L \leq e + 2r_p - r_s$. If this inequality is not satisfied, the particle cannot be in a pore. Whether or not the particle is in a pore requires that the specific pore location needs to be identified. This is performed once the first two criteria described above are satisfied.

The third criterion for pore detection is based on finding the arc location of the pore within a segment. This scheme requires the calculation of the local segment number, $N_{z,L}$, so as to determine the phase factor value present in the local segment. This is calculated as

$$N_{z,L} = \left\lfloor \frac{z_i}{e + 2r_p} \right\rfloor + 1 \quad (3.3)$$

The arc angle of the solute particle location, θ , is $\arctan 2(y_i, x_i)$ where $\arctan 2$ is the arc tangent function used to compute angles [79]. The incremental angle between pores within the same segment is $\theta_{inc} = 2\pi/N_r$. The arc curvature s , the surface distance as a function of θ , is the product θr_c when θ is in radians. The curvature between pore centers is $s_{inc} = \theta_{inc} r_c$.

One wants to find the local curvature between pores, s_L , if the z range is correct, so as to test whether the solute is within the pore cross section. This is shown in Figure 3.1C. First, one must determine the local phase of the curvature offset in the $N_{z,L}$ segment. This is simply

$$\phi_L = \phi_{inc} \cdot (N_{z,L} - 1) \quad (3.4)$$

and can be expressed as a curvature by $s_{L,\phi} = \phi_L r_c$. In addition, the arc length of the tracer, $s' = r_c \theta$.

The arc length due to the phase offset, $s_{L,\phi}$, must be subtracted off of the tracer arc length, s' , and normalized with one of the radial segments shown in Figure 3.1C so that

$$s_L = (s' - s_{L,\phi}) \bmod s_{inc} \quad (3.5)$$

Hence, the third criterion for pore occupancy is $s_L < r_p - r_s$ or $s_L > s_{inc} - r_p + r_s$. If these criteria are met and the particle did not come out of the pore end, so that $r \leq r_c + L_p - r_s$, then the particle is within the pore. If the particle is not in the capillary or in a pore, the trial trajectory is rejected and a new trajectory using new random numbers is generated, as described below.

3.2.2 Random walk procedures

3.2.2.1 Starting conditions

Each particle is started in the cross section of the capillary tube using uniform random number generators for $0 \leq x_0 \leq (r_c - r_s)$ and $0 \leq y_0 \leq (r_c - r_s)$ so that $r \leq (r_c - r_s)$; if this criterion is not met new starting conditions are generated. All particles are started at $z_0 = 0$. Typically $\geq 20,000$ particles are utilized for each simulation.

3.2.2.2 Diffusion

The time step can be specified using the Einstein equation [46, 7] which relates the average distance ($\bar{x}$) traveled by a particle with diffusion coefficient D_m in time step t_s as $\bar{x} = \sqrt{2D_mt_s}$ so that $t_s = \bar{x}^2/2D_m$. This allows the easy implementation of diffusion in all coordinates

$$\begin{aligned} x_{i+1} &= x_i + \xi_{x,i} \\ y_{i+1} &= y_i + \xi_{y,i} \\ z_{i+1} &= z_i + \xi_{z,i} \end{aligned} \quad (3.6)$$

where the subscripts indicate the iteration number and $\xi_{u,i}$ is a normally-distributed random number with zero mean and standard deviation $\bar{x}$ in coordinates x , y and z .

3.2.2.3 Convection

The convective contribution of a particle in a complex flow field has been previously obtained [73, 43, 44]. For the model used here, convection is restricted to the flow (z) axis as

$$z_{i+1} = z_i + v_z(r) \cdot t_s \quad (3.7)$$

where $v_z(r)$ is the Hagen-Poiseuille velocity [80]

$$v_z(r) = 2\bar{v}_z \left[1 - \left(\frac{r}{r_c} \right)^2 \right] \quad (3.8)$$

and $\bar{v}_z$ is the average fluid velocity in the capillary due to pressure-driven flow.

3.2.2.4 Stability

The Courant number [47], C_r , gives the nondimensional ratio of the distance traveled by convection to the average diffusion distance $\bar{x}$ in time step t_s as

$$C_r = \frac{\bar{v}_z \cdot t_s}{\bar{x}} \quad (3.9)$$

and is used as a simulation stability metric; simulations are typically run at low Courant number with $0 < C_r < 1$. In addition, the choice of t_s is also dictated by the pore radius.

3.2.2.5 Analysis of arrival time data

The first and second moments of the arrival time of N_p particles, M_1 and M_2 , are obtained from the particle arrival time data, t_j , as

$$M_1 = \frac{1}{N_p} \sum_{j=1}^{N_p} t_j \quad (3.10)$$

and

$$M_2 = \frac{1}{N_p} \sum_{j=1}^{N_p} (t_j - M_1)^2 \quad (3.11)$$

The number of theoretical plates, N , is calculated as

$$N = \frac{M_1^2}{M_2} \quad (3.12)$$

3.2.2.6 Assumptions

The method and model assumptions include utilizing a fully-developed flow profile throughout the capillary length with no entrance effects. The solute is introduced on an infinitely-thin cross section at the head of the column. Particle-particle interactions and collisions are absent. The presence of both point and finite-size molecules do not perturb the bulk fluid flow. There is no retention in these models although that could easily be added as could slip flow boundaries. Furthermore, the radial pores are assumed to possess no convection; the fluid in the pores is stagnant. The region where pores connect to the main capillary have no recirculation or convective entrance effects; the stick boundary of the capillary is extended into the radially-oriented pores so that transport in the pore region is purely diffusive.

3.2.2.7 Programming

All simulation software is written in FORTRAN-95. Parallel operation is implemented using the message-passing interface (MPI) [53, 54]. Data analysis is conducted with MATLAB (Mathworks, Natick, Massachusetts). Visualization of the model is conducted using the POVRay scripting language.

3.3 Results

3.3.1 Elution as a function of solute size and pore length

The study of a size-based separation utilizing a PLOT column with radially-oriented pores was conducted with a pore model similar to that shown in Figure 3.2.

We focus in on simulations where the solute diameter is studied over the range of the pore diameter. The nondimensional ratio $\lambda = r_s/r_p$ and is varied so that $0 < \lambda < 1$. Two capillary diameters are studied while varying the pore length in the range of 500 to 2000 nm. The parameters chosen for this simulation are given in Table 3.2 and represent reasonable parameters if one was to actually implement this system in a laboratory PLOT column.

Also shown in Table 3.3 is the lag time due to solute particles diffusing into pores. This is calculated as the difference between the first moment from simulation and the arrival time from a capillary without pores. The average time of zone elution from a capillary without pores is given as $L/\bar{v}_z$. Note that there is no compensation in $\bar{v}_z$ due to the restriction of the particle's center of mass exclusion from the full capillary volume. The number of pores in the capillary perimeter is kept constant for both capillary diameters used in this study. The parameters chosen for these simulations are listed here in Table 3.2.

Table 3.2: The input parameters used to model the PLOT column.

Parameter	Symbol	Value
Number of particles	N_p	5000
Spacer distance	e	10 nm
Pore radius	r_p	10 nm
Length of capillary	L	3 cm
Length of pore	L_p	500, 1000, 2000 nm
Capillary diameter	d_c	5, 10 μm
Phase factor	ϕ_{inc}	2°
Number of radial pores	N_r	750
Pore diameter	d_p	20 nm
Time step	t_s	4.0×10^{-8} s
Average velocity	$\bar{v}_z$	0.2 cm s^{-1}
Solute diameter	d_s	2, 6.33, 10.67, 15 nm

The arrival time results are shown as histograms in Figure 3.3 at two different bin sizes for each solute particle diameter d_s . The four solute diameters are given

explicitly in Table 3.3. As can be seen from the first moment of the arrival time, given in Table 3.3, and with peak centers shown in Figure 3.3, the pore system exhibits a pronounced size-exclusion effect with larger particles eluting first, followed by smaller and smaller particles. This is, to our knowledge, the first discrete particle simulation that explicitly shows SEC in a pore system.

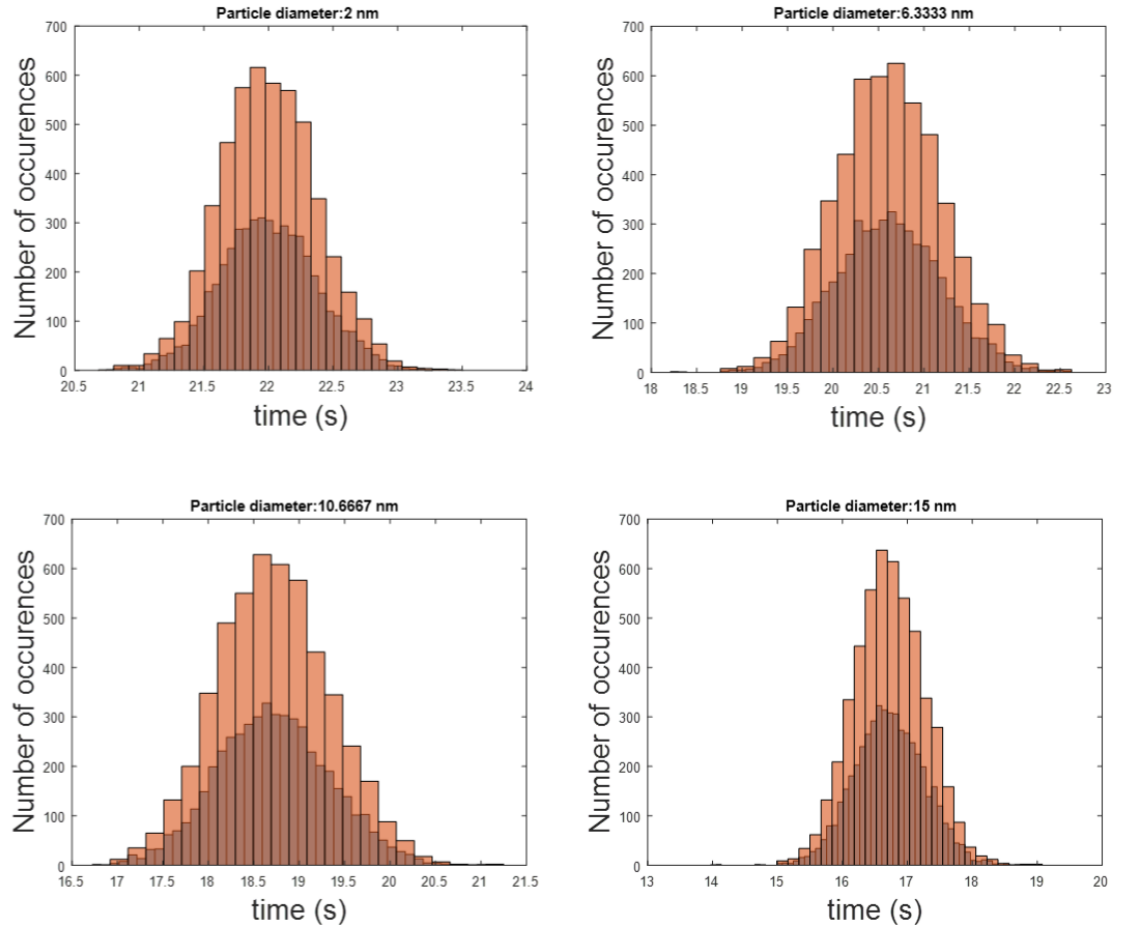


Figure 3.3: The arrival time histograms shown in two different gradations for $L_p = 1000$ nm and $d_c = 5$ μm .

A plot of the first moment, M_1 , and the second moment, M_2 , both as functions of pore length L_p , is given in Figure 3.4. A plot of the plate count, also as a function of pore length L_p , is shown in Figure 3.5. More of the size-exclusion effect is shown

Table 3.3: The first and second moments, M_1 and M_2 , from the pore length and solute diameter study.

d_c μm	d_s nm	λ	L_p nm	M_1 s	M_2 s^2	Plates per meter	Pore lag time s	V_p/V_c
5	2.00	0.10	500	18.19	0.055	201607	3.19	0.20
5	2.00	0.10	1000	22.00	0.139	116356	7.00	0.40
5	2.00	0.10	2000	31.38	0.633	51882	16.38	0.80
5	6.33	0.32	500	17.55	0.140	73484	2.55	0.20
5	6.33	0.32	1000	20.61	0.332	42609	5.61	0.40
5	6.33	0.32	2000	28.16	0.180	147123	13.16	0.80
5	10.67	0.53	500	16.67	0.180	51562	1.67	0.20
5	10.67	0.53	1000	18.72	0.383	30507	3.72	0.40
5	10.67	0.53	2000	23.78	1.595	11812	8.78	0.80
5	15.00	0.75	500	15.74	0.181	45533	0.74	0.20
5	15.00	0.75	1000	16.72	0.296	31533	1.72	0.40
5	15.00	0.75	2000	19.15	0.991	12333	4.15	0.80
10	2.00	0.10	500	16.52	0.124	73145	1.52	0.10
10	2.00	0.10	1000	18.21	0.211	52302	3.21	0.20
10	2.00	0.10	2000	22.03	0.543	29779	7.03	0.40
10	6.33	0.32	500	16.23	0.354	24761	1.23	0.10
10	6.33	0.32	1000	17.57	0.563	18285	2.57	0.20
10	6.33	0.32	2000	20.65	1.347	10558	5.65	0.40
10	10.67	0.53	500	15.79	0.518	16052	0.79	0.10
10	10.67	0.53	1000	16.73	0.715	13047	1.73	0.20
10	10.67	0.53	2000	18.75	1.488	7879	3.75	0.40
10	15.00	0.75	500	15.37	0.614	12815	0.37	0.10
10	15.00	0.75	1000	15.80	0.737	11287	0.80	0.20
10	15.00	0.75	2000	16.78	1.208	7771	1.78	0.40

in Figure 3.4 with respect to the first moment. As pore length (and pore volume) is increased for the 5 μm capillary, the separation in elution time between zones increases.

The 2000 nm length pore results show a much larger separation between zone centers as opposed to the 1000 nm length pores. This figure also shows a diminished separation span for the 500 nm length pores. For the 10 μm capillary diameter simulations, the range of the elution times is much more compressed in range, compared to the 5 μm capillary. This is due to the ratio of the pore volume to the capillary volume, denoted as V_p/V_c in Table 3.3, which shows this ratio to be twice that for the 5 μm capillary as the 10 μm capillary.

SEC is controlled by the solute distribution between the pore volume and the interstitial volume for packed beds, which is a thermodynamic quantity. Here, the interstitial volume is the same as the capillary volume. The larger this ratio, the more is the separation time between different solute particle sizes. This can be shown easily through the equation that governs retention in SEC, adapted to the symbols and geometry of the PLOT column

$$V_R = V_c + K V_p \quad (3.13)$$

where V_R is the retention volume and K is the size exclusion distribution coefficient such that $0 \leq K \leq 1$. For $K = 0$, the solute is fully excluded from the pores and for $K = 1$, the solute has full access to capillary and pores. Dividing Eq. 3.13 by V_c gives

$$\frac{V_R}{V_c} = 1 + K \frac{V_p}{V_c} \quad (3.14)$$

which shows that the larger is the term V_p/V_c , the larger will be the relative retention volume V_R/V_c at constant K noting that K is a complex function of λ [75]. $V_R/V_c - 1$ is just the chromatographic retention parameter k' and V_p/V_c is equivalent to the chromatographic phase volume. For these separations, Eq. 3.13 reveals that $V_c \leq V_R \leq V_c + V_p$, illustrating the restricted elution range characteristic of SEC.

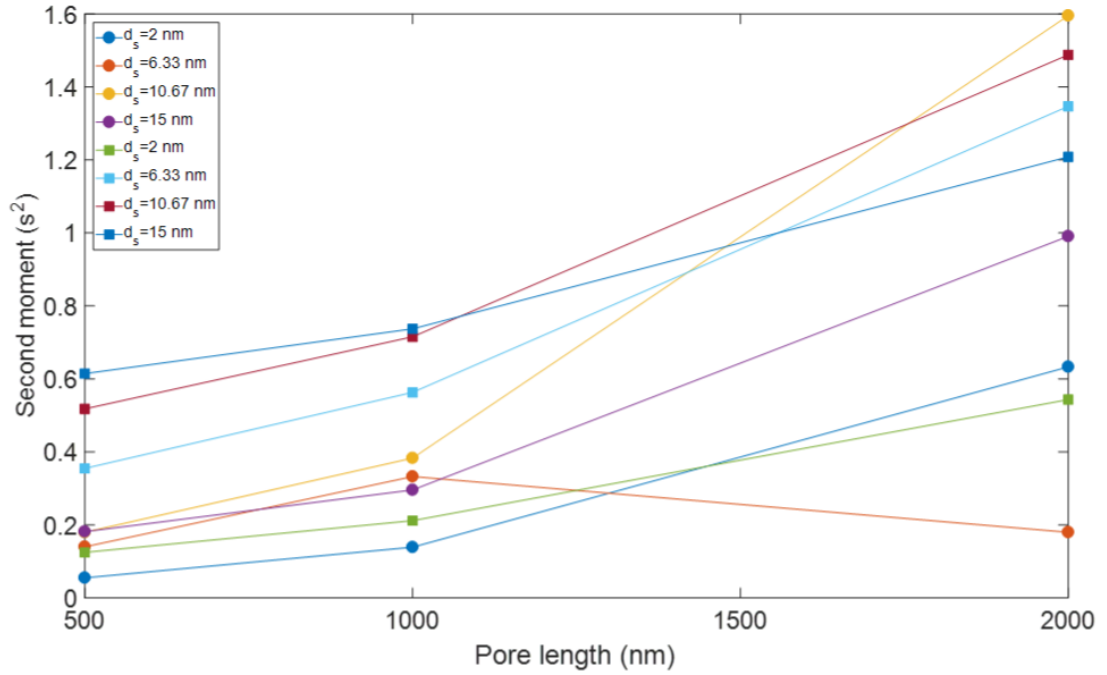
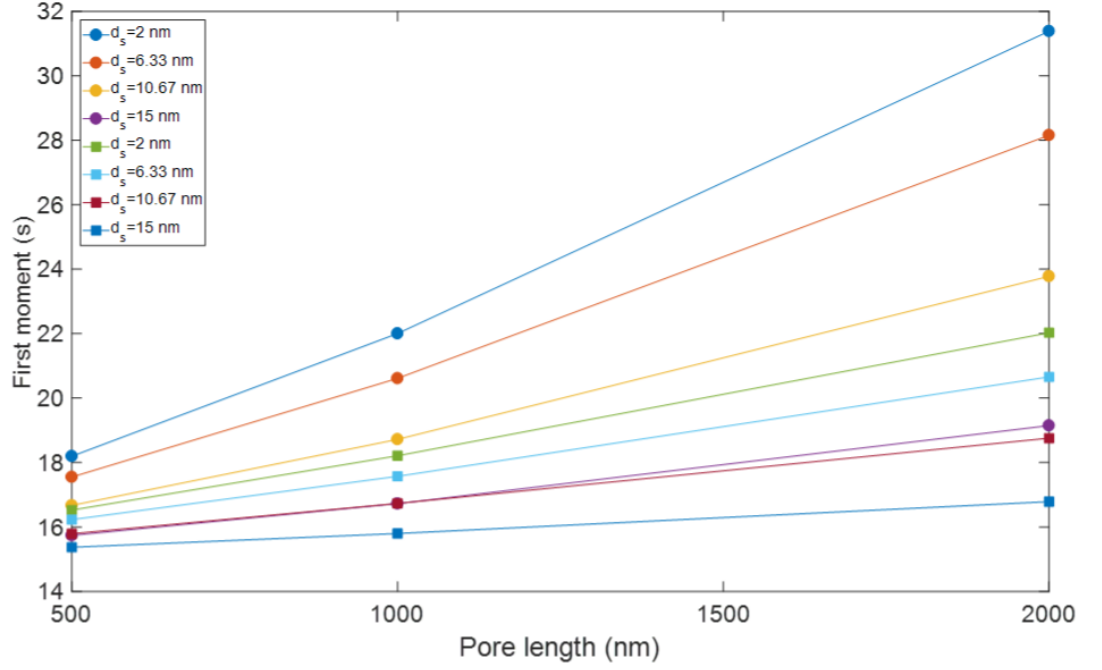


Figure 3.4: The first and second moments from the pore length and solute diameter study. Two capillary diameters are utilized in this study: circles represent $d_c = 5 \mu\text{m}$ and squares represent $d_c = 10 \mu\text{m}$.

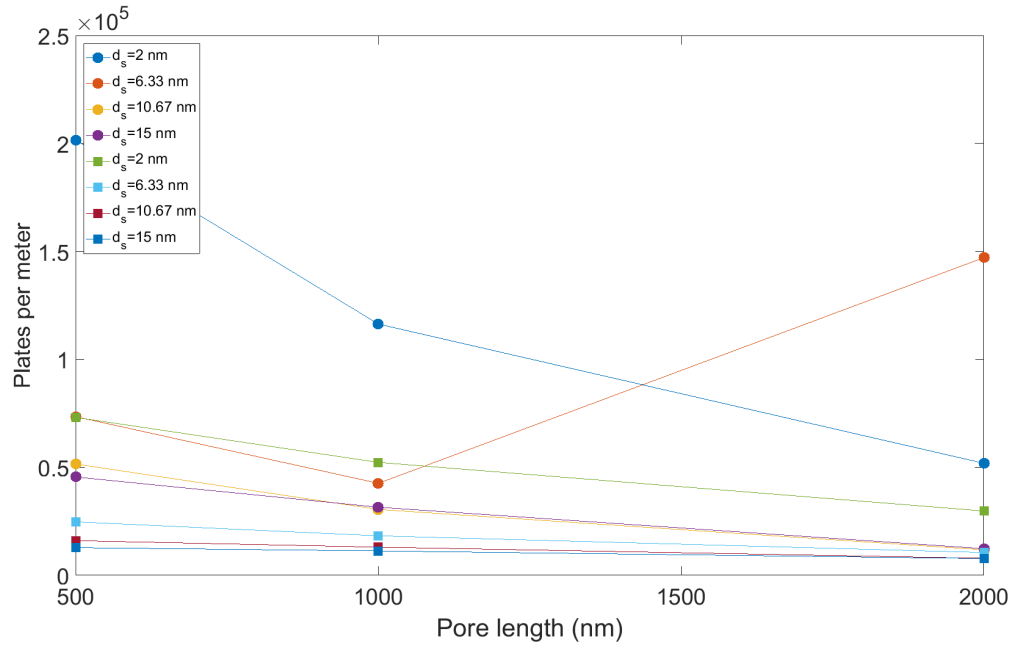


Figure 3.5: Plates per meter from the pore length and solute diameter study. Two capillary diameters are utilized in this study: circles represent $d_c = 5 \mu\text{m}$ and squares represent $d_c = 10 \mu\text{m}$.

The separation of these particles is also dependent on mass transport kinetics. A good example of the balance between thermodynamics and kinetics was illustrated for SEC with superficially porous particles (SPPs) and with fully porous particles (FPPs). This compromise was the subject of a recent study looking at SEC with SPPs and FPPs [81] where it was shown that SPPs have faster kinetics. The thermodynamic advantage of more pore volume, contained in FPPs, can be compensated by the faster diffusion time of the thin shells present in SPPs.

A very similar situation exists here with respect to optimizing the pore length in this system. Longer pores would clearly favor the thermodynamics and shorter pores would favor kinetics. However, longer pores also have longer analysis time.

As can be seen in Table 3.3, many of the plate count numbers are very high and are highly dependent on the conditions chosen for simulation. For the smallest solute, $d_s = 2$ nm, and the smallest pore length, $L_p = 500$ nm, the plate count is over 200,000 plates per meter, a very high number for any SEC-based separation. However, the plate count drops almost linearly with increasing pore length. This suggests that smaller pore lengths are advantageous with respect to fast diffusional exchange with the capillary, albeit at a thermodynamic penalty due to loss of pore volume. This also highlights that there is an optimum pore length, at fixed capillary diameter, which can balance the two competing effects here. However, this is a multiple-objective optimization problem because one effect loses when the other gains and some overall objective function needs to be established other than a subjective optimization.

3.4 Discussion

The findings here have highlighted the types of results that are possible with this class of separation system. The PLOT column has seen a resurgence of usage in proteomics analysis but is hindered in a number of ways that are now discussed. In a practical sense, the PLOT column can run with extremely low flow rates that are possible and necessary when mass spectrometry is used as a detection scheme as ionization is most efficient at extremely low volumetric flow rates.

The mass transport resistance in the fluid phase will always be a limitation in the PLOT geometry. This is due to the relatively large capillary inner diameters that have been utilized in PLOT column design. We have used a small capillary diameter of $5\mu\text{m}$ and this could be reduced, albeit with practical limitations in polymerizing the radially-oriented pore system. However, with chromatographic particle media now available in $\leq 2\mu\text{m}$ diameters, the reduction in liquid phase mass transport resistance heavily favors packed beds. In the packed-bed format, columns with particles of these diameters are exceedingly difficult to pack efficiently without voids into cylindrical columns. Since the application of the pore systems in a PLOT column is mostly chemical, there are distinct advantages to this configuration besides having a very uniform pore diameter.

Although the PLOT column with radially-oriented pores appears to be an interesting separation geometry, there is a problem with respect to the efficiency of phase utilization. As the pore length increases, the volume of the region containing the pores becomes more vacant and less occupied with pores. For short pores this is not a problem. However, short pores have limited mechanical stability because with this type of pore, the region between pores can be so small as to be fragile. This is somewhat alleviated with longer pores but then the space utilization of the pores becomes less efficient. This is never a problem with packed beds, even with beds of SPPs, because the pore density remains constant within any volumetric element of the column. However, this is not the case with radially-oriented pores and questions remain about the efficient utilization of the space in this configuration. PLOT columns with layers of porous particles keep the phase volume constant within the cylindrical volume element of the chromatographic phase.

Many so-called “lab on a chip” designs used in microfluidic environments have the same issue whereby the effective use of the chip does not promote effective phase utilization because of the reduced spatial height dimension. This is one of the issues whereby the microfluidic devices are closer to two-dimensional systems than three-dimensional albeit they are three-dimensional with a reduced length scale in their

height.

Rectangular channels can easily be modeled with the particle locator algorithm described here. This geometry is actually simpler because the dimensions are straight and not curved. Nonetheless, this geometry would be difficult to effectively utilize in all three dimensions; it is envisioned that pores would be laterally-oriented and not above or below the rectangular channel. The flow velocity profile in rectangular channels has been known for a long time [82].

These types of random-walk simulations allow probing the nanoscale details such as pore diffusion in almost-molecular dimensions and at the same time computing at length scales that are realistically large enough to mimic real-world devices like chromatography columns. In some respects this is due to the ease of parallelization. In this regard, this approach is similar to packed-bed simulation with periodic boundary conditions [43, 44, 83, 52], but this simulation does not use these boundary conditions. It is not necessary as the pore structure can be calculated exactly at any point along the column length. Nonetheless, due to the extremely small pore diameters that are possible with this problem, exceedingly small time steps must be employed to resolve the pore structure; this is a limitation of any method that attempts to compute over such a large multi-length scale range. This is common to chromatography where resolution of pores and macroscopic lengths must be computed simultaneously [78, 52]. Nonetheless, this type of problem is computable with high-performance parallel computers and can offer insight without the limitations of assuming near-equilibrium conditions [6]. This is one step closer to providing the capabilities of designing analytical separation systems via a computer-aided design approach.

Chapter 4

CONCLUSIONS

The work included in this thesis provides fundamental insight into mass transport within porous and nonporous materials using highly parallel supercomputer simulations. Both chapters examine a phenomenon that is applicable to real-world devices. These include nanoscale devices where slip may improve efficiency as well as SBA-15 silica and SEC columns which are modeled effectively by the Spiny model.

The results of these simulations are able to solve problems related to separation science and engineering which would otherwise be impossible using experimental or theoretical techniques. One major conclusion from this work is that improvements in dispersion due to slip flow are limited to a narrow range of applications and are counteracted by retention at boundaries. The Spiny model was then able to produce a clear size-exclusion effect in a pore system. These results are the first of their kind for a discrete particle simulation.

The findings of this thesis extend past the realm of the theoretical and demonstrate the potential of computer-aided design for improved chromatographic materials and devices. For example, these simulations could allow optimization of parameters such as wall coatings in the case of slip and pore length in the case of PLOT systems. These models are easily tailored based on the particular system so that a more informed design can be implemented prior to actually developing such devices. Therefore, a computer-aided approach using simulations is a powerful tool that can be wielded in the future of separation system engineering.

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