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Primordial monopoles, proton decay, gravity waves and GUT inflation

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ABSTRACT

We consider non-supersymmetric GUT inflation models in which intermediate mass monopoles may survive inflation because of the restricted number of e-foldings experienced by the accompanying symmetry breaking. Thus, an observable flux of primordial magnetic monopoles, comparable to or a few orders below the Parker limit may be present in the galaxy. The mass scale associated with the intermediate symmetry breaking is 10^{13} GeV for an observable flux level, with the corresponding monopoles an order of magnitude or so heavier. Examples based on $SO(10)$ and E_6 yield such intermediate mass monopoles carrying respectively two and three units of Dirac magnetic charge. For GUT inflation driven by a gauge singlet scalar field with a Coleman–Weinberg or Higgs potential, compatibility with the Planck measurement of the scalar spectral index yields a Hubble constant (during horizon exit of cosmological scales) $H \sim 7\text{–}9 \times 10^{13}$ GeV, with the tensor to scalar ratio r predicted to be $\gtrsim 0.02$. Proton lifetime estimates for decays mediated by the superheavy gauge bosons are also provided.

1. Introduction

The observed quantization of electric charge is elegantly explained by invoking the presence of magnetic monopoles, as shown by Dirac more than eighty years ago [1]. Contemporary unified theories with electric charge quantization based on groups such as $SU(4) \times SU(2)_L \times SU(2)_R$ [2], $SU(5)$ [3], $SO(10)$ or E_6, predict the existence of topologically stable magnetic monopoles [4], and one expects that these monopoles are produced in the early universe.

Despite the presence of fractionally charged quarks, the lightest $SU(5)$ monopole carries a single unit of Dirac magnetic charge. This comes about because the unbroken gauge symmetry $SU(3)_c \times U(1)_{em}$ share a Z_3 symmetry [5]. The $SU(5)$ monopole ends up carrying some color magnetic flux that is screened due to color confinement. The $SU(5)$ monopoles are superheavy with a mass about an order of magnitude larger than $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV.

In non-supersymmetric GUTs such as $SO(10)$ broken to the Standard Model (SM) via $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$, there appears a new scenario for monopole charges and masses. The $SO(10)$ breaking to G_{422} yields, just as in $SU(5)$, a superheavy monopole with a single unit of Dirac magnetic charge [6]. The subsequent breaking of G_{422} at some intermediate mass scale M_I yields monopoles that carry two units of Dirac charge and mass that can be a few orders of magnitude smaller than the mass of the $SU(5)$ monopole [6].

It was argued a long time ago by Lazarides and Shafi [7] that within the framework of GUT inflation driven by a gauge singlet scalar inflaton field [8], these somewhat lighter monopoles may not be entirely inflated away.[1] The superheavy monopoles produced during the first stage of symmetry breaking experience at least the 50–60 e-foldings of observable inflation. The somewhat lighter monopoles, produced during the intermediate symmetry breaking with mass determined by M_I and comparable to the Hubble constant H during inflation, may undergo a significantly reduced number of e-foldings. Therefore, there arises the exciting possibility that these monopoles, lighter than $M_{\rm GUT}$, may be present in our galaxy at an observable number density, comparable to or a few orders of magnitude below the Parker bound [10].

In recent years the WMAP [11] and Planck [12,13] satellite experiments have provided a fairly accurate determination of the scalar spectral index n_s and an upper bound for the tensor to scalar ratio $r \lesssim 0.1$. In the framework of GUT inflation driven by a gauge

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[1] For an earlier discussion of this with cosmic strings, see Ref. [9].

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singlet scalar field, one finds that for $n_s \geq 0.96$, the energy scale during inflation is of order 10^{16} GeV [14,15].

In this brief report we calculate the range of energy scales during non-SUSY GUT inflation such that n_s and r are compatible with the Planck 2015 constraints [12]. This determines the magnitude of H which, in turn, provides an estimate for the range for M_I that is compatible with an observable flux of primordial magnetic monopoles. These monopoles with mass $\sim 10^{14}$ GeV do not necessarily catalyze nucleon decay with a strong interaction rate, and they should be accessible to current and future large scale detectors. Estimates for the proton lifetime are also provided.

2. Inflation with Coleman–Weinberg potential

The first new inflation models [16] were proposed in the early eighties immediately after Guth's seminal paper [17]. They were based on $SU(5)$ GUT, with symmetry breaking due to the Coleman–Weinberg mechanism [18] occurring in the adjoint Higgs field. However, it was shown in Ref. [8] that obtaining sufficiently small density perturbations was only possible if the scalar field ϕ is a gauge singlet. In these Shafi–Vilenkin type models the field ϕ has a quartic potential at tree level, and taking into account radiative corrections the potential becomes (omitting terms that don't play an essential role) [19,20]:

$$V = \frac{\lambda}{4}\phi^4 - \frac{1}{2}\beta^2\phi^2\chi^2 + \frac{a}{4}\chi^4 + A\phi^4\left[\ln\left(\frac{\phi}{M}\right) + C\right] + V_0\,, \quad (2.1)$$

where χ represents the field breaking the GUT group, $A \sim (1/16\pi^2)\beta^4$, M and C are normalization parameters and $V_0 \equiv V(\phi = 0)$ is the vacuum energy density at the origin. The χ field can be replaced by its vacuum expectation value (VEV) $\langle\chi\rangle = (\beta/\sqrt{a})\phi$. The parameter C can be fixed by taking M to be the ϕ VEV at the minimum. Requiring $V(\phi = M) = 0$ fixes $V_0 = AM^4/4$. Also taking $\lambda \ll \beta^4/a$, the effective potential takes the standard form [19,20]:

$$V(\phi) = A\phi^4\left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{AM^4}{4}\,. \quad (2.2)$$

The inflationary predictions of this potential were recently analyzed in Ref. [14] (see also Refs. [21,15]).

The magnitude of A and the inflationary parameters can be calculated using the standard slow-roll expressions. The slow-roll parameters may be defined as (see Ref. [22] for a review and references):

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = \frac{V''}{V}, \quad \xi^2 = \frac{V'V'''}{V^2}\,. \quad (2.3)$$

Here and below we use units $m_P = 2.44 \times 10^{18}$ GeV $= 1$, and primes denote derivatives with respect to the inflaton field ϕ. The spectral index n_s, the tensor to scalar ratio r and the running of the spectral index $\alpha \equiv \mathrm{d}n_s/\mathrm{d}\ln k$ are given in the slow-roll approximation by

$$n_s = 1 - 6\epsilon + 2\eta\,, \quad r = 16\epsilon\,, \quad \alpha = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2\,. \quad (2.4)$$

The amplitude of the curvature perturbation $\Delta_{\mathcal{R}}$ is given by

$$\Delta_{\mathcal{R}} = \frac{1}{2\sqrt{3}\pi}\frac{V^{3/2}}{|V'|}\,, \quad (2.5)$$

which should satisfy $\Delta_{\mathcal{R}}^2 \approx 2.4 \times 10^{-9}$ from the Planck measurement [12] with the pivot scale chosen at $k_* = 0.002$ Mpc^{-1}.[2]

The number of e-folds is given by

$$N_* = \int_{\phi_e}^{\phi_*} \frac{V\,\mathrm{d}\phi}{V'}\,, \quad (2.6)$$

where the subscript "$_*$" denotes quantities when the scale corresponding to k_* exited the horizon, and ϕ_e is the inflaton value at the end of inflation, which we estimate by $\epsilon(\phi_e) = 1$.

For $V_0^{1/4} \gtrsim 2 \times 10^{16}$ GeV, observable inflation occurs close to the minimum where the potential is effectively quadratic ($V \simeq 2AM^2\chi^2$, where $\chi = \phi - M$ denotes the deviation of the field from the minimum). The inflationary predictions are thus approximately given by

$$n_s = 1 - 2/N\,, \quad r = 8/N\,, \quad \alpha = -2/N^2\,. \quad (2.7)$$

For $V_0^{1/4} \lesssim 10^{16}$ GeV, assuming inflation takes place with inflaton values below M, the inflationary parameters are similar to those for new inflation models with $V = V_0[1 - (\phi/\mu)^4]$: $n_s \simeq 1 - (3/N)$, r small, and $\alpha \simeq -3/N^2$.

Note that in the context of non-supersymmetric GUTs, $V_0^{1/4}$ is related to the unification scale M_U, and is typically a factor of $\sim\sqrt{4\pi}$ smaller than the superheavy gauge boson masses due to the loop factor in the Coleman–Weinberg potential. The allowed range of $V_0^{1/4}$ (and hence of M_U) can be calculated by comparing the n_s and r values with the Planck results [12]. Since the resulting constraints depend sensitively on the number of e-folds N, instead of fixing N to a fiducial value, we calculate it using

$$N_* \approx 64.7 + \frac{1}{2}\ln\frac{\rho_*}{m_P^4} - \frac{1}{3(1+\omega_r)}\ln\frac{\rho_e}{m_P^4} + \left(\frac{1}{3(1+\omega_r)} - \frac{1}{4}\right)\ln\frac{\rho_r}{m_P^4}\,. \quad (2.8)$$

Here $\rho_e = (3/2)V(\phi_e)$ is the energy density at the end of inflation, ρ_r is the energy density at the end of reheating and ω_r is the equation of state parameter during reheating, which we take to be constant. For a derivation of eq. (2.8) see e.g. Ref. [23].

To represent a plausible range of N, we consider three cases: In the high-N case ω_r is taken to be 1/3, which is equivalent to assuming instant reheating. In the middle-N case we take $\omega_r = 0$ and the reheat temperature $T_r = 10^9$ GeV, calculating ρ_r using the SM value for the number of relativistic degrees of freedom ($g_* = 106.75$). In the low-N case we take $T_r = 100$ GeV (again with $\omega_r = 0$).[3] The n_s vs. r curve for each case is shown in Fig. 1 along with the contours (at the confidence levels of 68% and 95%) given by the Planck collaboration (Planck TT + lowP + BKP + lensing + ext) [12]. Numerical results for selected values of V_0 and the middle-N case are displayed in Table 1.

3. Inflation with smeared Higgs potential

A generalization of the model considered in section 2 is to take a tree-level Higgs potential $V = -(1/2)m^2\phi^2 + (1/4)\lambda\phi^4$. The effective potential eq. (2.2) then becomes the sum of a Higgs potential and the Coleman–Weinberg potential considered in section 2 [24]:

$$V(\phi) = \left(\frac{m^2M^2}{4}\right)\left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 + A\phi^4\left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{AM^4}{4}\,. \quad (3.1)$$

[2] Note that while the Planck collaboration otherwise uses a pivot scale corresponding to 0.05 Mpc^{-1}, they present their results on r using $k_* = 0.002$ Mpc^{-1}. To facilitate comparison with the Planck results we also take $k_* = 0.002$ Mpc^{-1}.

[3] T_r as low as 10 MeV is consistent with big bang nucleosynthesis, however it is difficult to explain how baryogenesis could occur at such low temperatures.

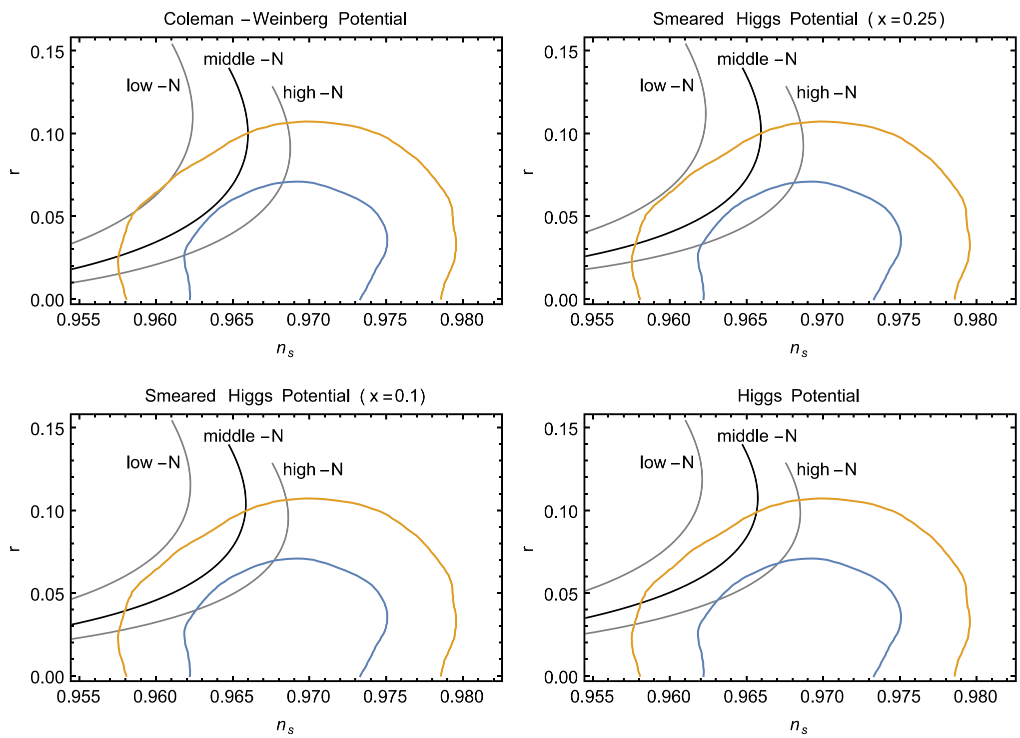

Fig. 1. n_s vs. r curves along with the 68% and 95% confidence level contours given by the Planck collaboration (Planck TT + lowP + BKP + lensing + ext) [12].

Table 1
Parameter values for the middle-N case.

$V_0^{1/4}$ (10^{16} GeV)	$V(\phi_0)^{1/4}$ (10^{16} GeV)	$\log(A)$	m (10^{13} GeV)	M (m_P)	ϕ_* (m_P)	ϕ_e (m_P)	N_*	n_s	r	$-\alpha$ (10^{-4})
Coleman–Weinberg potential										
1.25	1.23	−13.3		15.2	5.13	13.9	55.1	0.955	0.018	6.2
1.5	1.44	−13.4		19.9	8.64	18.5	55.3	0.96	0.034	5.74
1.75	1.61	−13.6		26.2	14.0	24.8	55.5	0.963	0.054	5.74
2.0	1.73	−13.9		34.3	21.4	33.0	55.6	0.965	0.072	5.87
3.0	1.94	−14.7		83.1	68.9	81.7	55.9	0.966	0.112	6.12
6.0	2.03	−16.0		356.0	341.0	354.0	56.0	0.965	0.135	6.23
Smeared Higgs potential ($x = 0.25$)										
1.25	1.23	−13.6	0.88	12.6	2.31	11.3	55.1	0.95	0.019	4.59
1.5	1.45	−13.7	1.00	16.0	4.66	14.7	55.3	0.958	0.035	5.23
1.75	1.62	−13.9	1.04	20.9	8.6	19.5	55.5	0.963	0.055	5.61
2.0	1.74	−14.1	1.04	27.3	14.3	26.0	55.7	0.965	0.074	5.85
3.0	1.94	−14.9	0.97	65.9	51.7	64.5	55.9	0.966	0.114	6.13
6.0	2.03	−16.2	0.91	281.0	267.0	280.0	56.0	0.965	0.135	6.23
Smeared Higgs potential ($x = 0.1$)										
1.25	1.24	−13.9	0.98	12.5	2.0	11.2	55.2	0.946	0.019	3.71
1.5	1.45	−14.0	1.13	15.6	4.05	14.2	55.4	0.957	0.036	4.78
1.75	1.63	−14.2	1.19	20.0	7.59	18.6	55.5	0.962	0.056	5.46
2.0	1.75	−14.4	1.2	26.0	12.9	24.6	55.7	0.965	0.075	5.8
3.0	1.95	−15.2	1.13	62.1	47.9	60.7	55.9	0.966	0.115	6.14
6.0	2.03	−16.5	1.06	264.0	249.0	263.0	56.0	0.965	0.135	6.23
Higgs potential										
1.25	1.24		1.03	12.5	1.85	11.2	55.2	0.943	0.019	3.16
1.5	1.46		1.2	15.4	3.72	14.0	55.4	0.955	0.036	4.45
1.75	1.63		1.29	19.5	6.98	18.1	55.6	0.962	0.057	5.32
2.0	1.76		1.31	25.1	11.9	23.7	55.7	0.964	0.077	5.77
3.0	1.95		1.24	59.6	45.2	58.2	55.9	0.966	0.116	6.14
6.0	2.03		1.17	252.0	237.0	251.0	56.0	0.965	0.136	6.23

Following Ref. [24], we will call $V(\phi)$ in eq. (3.1) the smeared Higgs potential. Depending on the value of m, the inflationary predictions for this potential interpolate between the predictions for the tree-level Higgs potential and for the Coleman–Weinberg potential [24].

The tree-level Higgs potential has been analyzed in several papers, see e.g. Refs. [25,21,15]. The inflationary predictions are similar to the predictions for the Coleman–Weinberg potential. For $V_0^{1/4} \gtrsim 2 \times 10^{16}$ GeV, observable inflation occurs close to the minimum where the potential is effectively quadratic ($V \simeq m^2\chi^2$, where $\chi = \phi - M$ denotes the deviation of the field from the minimum). The inflationary predictions are thus approximately given by eq. (2.7). For $V_0^{1/4} \lesssim 10^{16}$ GeV, assuming inflation takes place with inflaton values below M, a red spectrum is predicted with $n_s \simeq 1 - 8/M^2$. Compared with the Coleman–Weinberg potential, the Higgs potential predicts higher values of r for the same n_s values.

We represent the "smearing" of the Higgs potential by radiative corrections with a smearing parameter x, where $AM^4/4 = xV_0$ and $m^2M^2/4 = (1-x)V_0$. With this definition $x \to 0$ and $x \to 1$ corresponds to the Higgs and Coleman–Weinberg potentials, respectively.

At the end of inflation, the GUT symmetry breaking fields have a VEV $\langle\chi\rangle \sim (\beta/\sqrt{a})M$. Taking $a \sim g^2$, where g is the gauge coupling, the unification scale is given by

$$M_U \sim \beta M \sim \sqrt{4\pi}(xV_0)^{1/4}. \tag{3.2}$$

The n_s vs. r curves for tree-level and smeared Higgs potentials (for $x = 0.25$ and 0.1) are shown in Fig. 1. Numerical results for selected values of V_0 and the middle-N case are displayed in Table 1.

4. Magnetic monopoles and proton decay in non-supersymmetric GUTs

The models discussed in sections 2 and 3 can be realized within the framework of non-supersymmetric GUTs such as those based on $SO(10)$ as well as $SU(5)$, as discussed in Ref. [7]. The breaking of $SO(10)$ to the SM can proceed, for example, via the intermediate group $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$ [2]. The monopoles associated with the breaking at scale M_U of the GUT group to the intermediate group are inflated away. However, the breaking of the intermediate group to the SM gauge symmetry at the intermediate scale M_I yields monopoles (doubly charged in the case of G_{422} [6]), whose mass is an order of magnitude larger than M_I. These may be present in our galaxy at a flux level that depends on the values of V_0 and M_I. Below we will estimate the M_I scale that corresponds to an observable flux level following the arguments in Ref. [7].

First let us consider the potential for the χ fields breaking the GUT group to the intermediate group, at scale M_U. The potential involves a thermal term

$$V \supset \frac{1}{2}\sigma_\chi T_H^2\chi^2 - \frac{1}{2}\beta^2\phi^2\chi^2 + \frac{a}{4}\chi^4, \tag{4.1}$$

where $T_H \equiv H/2\pi$ is the Hawking temperature and the coefficient $\sigma_\chi \sim 1$. Thus symmetry breaking occurs when $\beta^2\phi^2 \gtrsim (H/2\pi)^2$, and topological structures are "frozen" in soon afterwards [26]. It can be easily checked that this happens much earlier than the horizon exit of cosmological scales, so as mentioned above any such topological structures are inflated away.

Let X denote the fields whose VEV breaks the intermediate group to the SM at the scale M_I. This breaking occurs due to the coupling $-(1/2)c^2\phi^2X^2$, where $c \sim M_I/M$. Thus, symmetry breaking occurs and subsequently the monopoles are "frozen" in when

$$\phi \sim \phi_x \equiv \frac{H_x}{2\pi}\frac{M}{M_I}, \tag{4.2}$$

where $H_x = (V(\phi_x)/3)^{1/2}$ is the Hubble constant when $\phi = \phi_x$.

If M is small compared to the Planck scale, the inflaton ϕ is essentially constant until almost the end of inflation, rolling quickly only within the last H^{-1} [20]. This means for substantial dilution of the monopoles, ϕ_x should be very close to ϕ_*. On the other hand, if M is large compared to the Planck scale both ϕ_* and ϕ_x values will be close to M. Since the Planck constraint on n_s is only satisfied for this latter case, we have $\phi_x \approx M$ and therefore $M_I \sim H_x/2\pi \sim 10^{13}$ GeV.

To be more specific, let's consider how much dilution of the monopoles is necessary. $M_I \sim 10^{13}$ GeV corresponds to monopole masses of order $M_M \sim 10^{14}$ GeV. For these intermediate mass monopoles the MACRO experiment has put an upper bound on the flux of 2.8×10^{-16} cm^{-2} s^{-1} sr^{-1} [27]. For monopole mass $\sim 10^{14}$ GeV, this bound corresponds to a monopole number per comoving volume of $Y_M \equiv n_M/s \lesssim 10^{-27}$ [28]. There is also a stronger but indirect bound on the flux of $(M_M/10^{17}$ GeV$)10^{-16}$ cm^{-2} s^{-1} sr^{-1} obtained by considering the evolution of the seed Galactic magnetic field [29].

Direct search bounds stronger than the MACRO bound were obtained in Ref. [30], but these apply to monopoles that catalyze nucleon decay through the Callan–Rubakov process [31]. There are even more stringent indirect bounds from compact astrophysical objects capturing monopoles [32]. However, the monopoles produced during the intermediate symmetry breaking stage do not necessarily catalyze nucleon decay (at least, not with a strong interaction rate) [33]. This improves the chances of directly observing such monopoles in the future, since the bounds from compact astrophysical objects are avoided.

At production, the monopole number density n_M is of order H_x^3 [26,7], which gets diluted to $H_x^3e^{-3N_x}$, where N_x is the number of e-folds after $\phi = \phi_x$. Using

$$Y_M \sim \frac{H_x^3e^{-3N_x}}{s}, \tag{4.3}$$

where $s = (2\pi^2g_S/45)T_r^3$, we find that sufficient dilution requires $N_x \gtrsim \ln(H_x/T_r) + 20$. Thus, for $T_r \sim 10^9$ GeV, $N_x \gtrsim 30$ yields a monopole flux close to the observable level.

Using eq. (4.3), we calculate ϕ_x, H_x and N_x values, denoted with subscripts "$_+$" and "$_-$", corresponding respectively to the flux levels 2.8×10^{-16} cm^{-2} s^{-1} sr^{-1} which is the MACRO bound and 10^{-24} cm^{-2} s^{-1} sr^{-1} which we take as a rough threshold for observability. Then using eq. (4.2), we calculate the corresponding M_I values, which are shown in Fig. 2 and Table 2. For $M_I \gtrsim M_-$, the monopoles are too diluted to be observable, whereas $M_I \lesssim M_+$ is excluded from the bound on the flux.

Another key prediction of GUTs besides magnetic monopoles is proton decay. The proton mean life can be estimated as

$$\tau_p \sim \frac{M_U^4}{\alpha_G^2 m_{pr}^5}, \tag{4.4}$$

where M_U is estimated using eq. (3.2), m_{pr} is the proton mass, and $\alpha_G \sim 1/40$ is the GUT coupling constant. Using eq. (4.4), the experimental bound $\tau_p(p \to e^+\pi^0) > 8.2 \times 10^{33}$ years [34] corresponds to $M_U \gtrsim 4 \times 10^{15}$ GeV, whereas a realistically observable $\tau_p(p \to e^+\pi^0) = 10^{35}$ years [35] corresponds to $M_U \approx 8 \times 10^{15}$ GeV. Since the Planck constraint on n_s is only satisfied for $V_0^{1/4} \gtrsim 10^{16}$ GeV, a glance at eq. (3.2) shows that proton decay is typically too slow to be observed in this class of models, unless the smearing parameter x is close to zero. Proton lifetime estimates are displayed in Fig. 2 and Table 2.

5. Conclusion

In this paper we discussed a class of models where the gauge-singlet inflaton has either a quartic or Higgs potential at tree level. The radiative corrections due to couplings with GUT symmetry

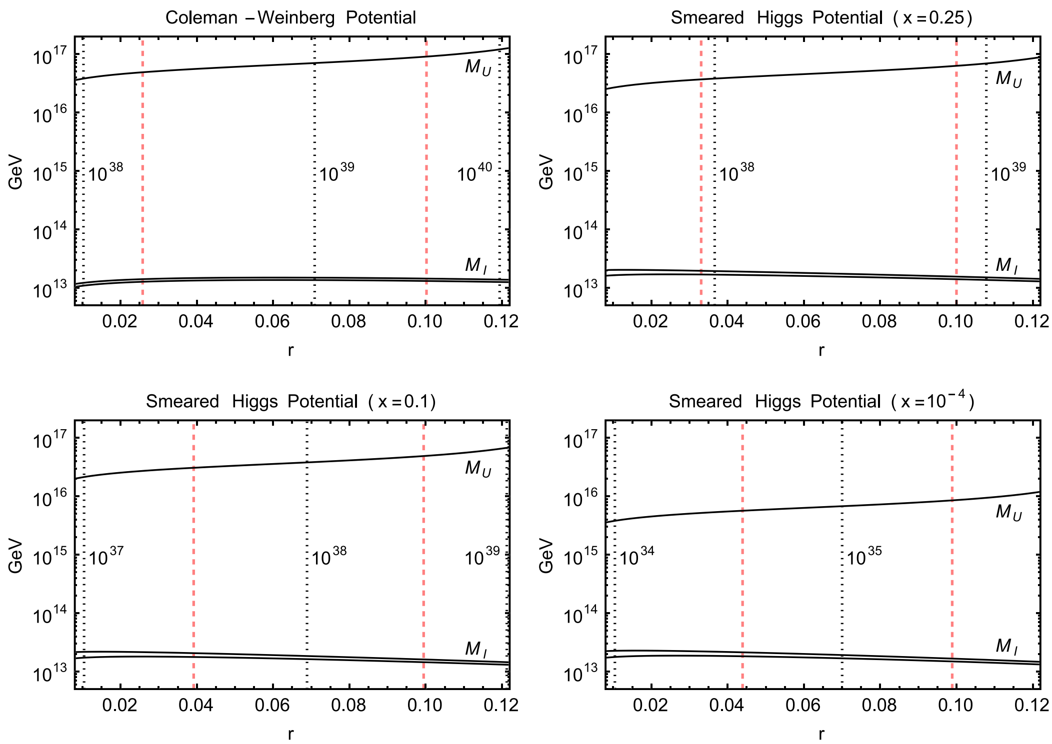

Fig. 2. M_U values and the range of M_I values (corresponding to a thin band around 10^{13} GeV) that give an observable monopole flux (between 2.8×10^{-16} and 10^{-24} cm^{-2} s^{-1} sr^{-1}) are shown as a function of r for the middle-N case. The dotted vertical lines show the proton lifetime in years. The region between the dashed vertical lines correspond to n_s and r values within the 95% confidence level contours given by the Planck collaboration (Planck TT + lowP + BKP + lensing + ext) [12].

Table 2
Parameter values for the middle-N case.

$V_0^{1/4}$ (10^{16} GeV)	ϕ_+ (m_P)	ϕ_- (m_P)	H_+ (10^{13} GeV)	H_- (10^{13} GeV)	N_+	N_-	M_+ (10^{13} GeV)	M_- (10^{13} GeV)	M_U (10^{16} GeV)	τ_p (years)
Coleman–Weinberg potential										
1.25	6.79	6.25	3.38	3.45	29.9	36.4	1.2	1.33	4.43	2×10^{38}
1.5	10.8	10.1	4.46	4.62	30.2	36.7	1.31	1.44	5.32	4×10^{38}
1.75	16.6	15.8	5.37	5.63	30.3	36.9	1.35	1.49	6.2	7×10^{38}
2.0	24.3	23.4	6.0	6.37	30.5	37.0	1.35	1.48	7.09	1×10^{39}
3.0	72.4	71.4	6.96	7.54	30.6	37.2	1.27	1.4	10.6	6×10^{39}
6.0	345.0	344.0	7.31	8.02	30.7	37.2	1.2	1.32	21.3	1×10^{41}
Smeared Higgs potential ($x = 0.25$)										
1.25	4.03	3.46	3.39	3.48	29.9	36.4	1.69	2.01	3.13	5×10^{37}
1.5	6.85	6.16	4.51	4.67	30.2	36.7	1.68	1.93	3.76	1×10^{38}
1.75	11.2	10.4	5.42	5.69	30.4	36.9	1.61	1.82	4.39	2×10^{38}
2.0	17.3	16.4	6.06	6.43	30.5	37.0	1.53	1.71	5.01	3×10^{38}
3.0	55.2	54.2	6.98	7.57	30.6	37.2	1.33	1.47	7.52	2×10^{39}
6.0	270.0	269.0	7.32	8.03	30.7	37.2	1.21	1.34	15.0	2×10^{40}
Smeared Higgs potential ($x = 0.1$)										
1.25	3.77	3.18	3.39	3.48	29.9	36.4	1.79	2.17	2.49	2×10^{37}
1.5	6.28	5.57	4.52	4.7	30.2	36.7	1.78	2.09	2.99	4×10^{37}
1.75	10.2	9.43	5.46	5.74	30.4	36.9	1.7	1.94	3.49	7×10^{37}
2.0	15.8	14.9	6.1	6.48	30.5	37.0	1.59	1.79	3.99	1×10^{38}
3.0	51.4	50.4	7.0	7.6	30.6	37.2	1.35	1.49	5.98	6×10^{38}
6.0	253.0	252.0	7.32	8.03	30.7	37.2	1.22	1.34	12.0	1×10^{40}
Smeared Higgs potential ($x = 10^{-4}$)										
1.25	3.65	3.05	3.39	3.48	29.9	36.4	1.85	2.28	0.44	2×10^{34}
1.5	5.97	5.26	4.53	4.71	30.2	36.7	1.85	2.19	0.53	4×10^{34}
1.75	9.66	8.84	5.48	5.77	30.4	36.9	1.76	2.02	0.62	7×10^{34}
2.0	14.9	14.0	6.13	6.52	30.5	37.0	1.64	1.86	0.71	1×10^{35}
3.0	48.8	47.8	7.02	7.63	30.6	37.2	1.36	1.51	1.06	6×10^{35}
6.0	241.0	240.0	7.33	8.04	30.7	37.2	1.22	1.34	2.13	1×10^{37}

breaking fields modify the tree level potential into a Coleman–Weinberg or smeared Higgs potential. If the GUT symmetry breaking to the SM proceeds via an intermediate group, the breaking of the intermediate group to the SM gauge symmetry at intermediate scale M_I yields monopoles whose mass is an order of magnitude larger than M_I. These may be present in our galaxy at a flux level that depends on the values of V_0 (the vacuum energy density at the origin) and M_I.

For both Coleman–Weinberg and smeared Higgs potentials the Planck constraint $n_s > 0.955$ is only satisfied for $V_0^{1/4} \gtrsim 10^{16}$ GeV, which implies $r \gtrsim 0.02$, a level which can be probed in this decade. Another consequence of the Planck constraint is that an observable level of monopole flux can only occur for $M_I \sim 10^{13}$ GeV, with lower values excluded due to excessive monopole flux. Thus, a smoking gun evidence for this class of models would be the observation of monopoles with masses of order 10^{14} GeV, together with the observation of a B-mode CMB polarization signal corresponding to $r \gtrsim 0.02$.

The lower bound on M_I poses a severe constraint for $SO(10)$ broken to SM via G_{422}, since the typical values obtained from the RG analysis is $M_I \sim 10^{11}$ GeV and $M_U \sim 10^{16}$ GeV [36]. However, taking threshold effects due to the Higgs sector into account, it is possible to achieve M_I as high as 3×10^{13} GeV with $M_U \approx 4 \times 10^{15}$ GeV [36,37].[4] Here the lower bound on M_U follows from proton decay. Although for Coleman–Weinberg potential the Planck constraint on V_0 corresponds to $M_U \gtrsim 4 \times 10^{16}$ GeV, lower M_U values are possible for the smeared Higgs potential. If the smearing parameter x is close to zero, proton decay could also be observed in this class of models.

Finally we note that E_6 breaking via $SU(3)_c \times SU(3)_L \times SU(3)_R$ can yield intermediate mass monopoles carrying three units of Dirac charge [39].

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