THE HABITABILITY AND STABILITY OF EARTH-LIKE PLANETS IN BINARY STAR SYSTEMS

by

Nicholas W. Troup

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Nicholas W. Troup

Approved: ____

John E. Gizis, Ph.D.
Professor in charge of thesis on behalf of the Advisory Committee

Approved: ____

James MacDonald, Ph.D.
Committee member from the Department of Physics and Astronomy

Approved: ____

James Glancey, Ph.D.
Committee member from the Board of Senior Thesis Readers

Approved: ____

Michael Arnold, Ph.D.
Director, University Honors Program

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This thesis is dedicated to the glory of God,

the creator of the Heavens and the source of my passion.

The heavens declare the glory of God;
the skies proclaim the work of his hands.
Day after day they pour forth speech;
night after night they reveal knowledge.
- Psalm 19:1-2

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ABSTRACT

In 2011, NASA's Kepler spacecraft observed the first planet to be detected in a known binary star system, Kepler 16-b. Its discovery has sparked new discussion on the potential of binary systems supporting habitable Earth-like planets. In this study, by using relatively simple computation models, we are able to shed new light and place new limitations on this discussion. Habitable Zone geometry in binary systems is discussed as a part of this study, in which we are able to classify binary habitable zones into two major classes of merged and unmerged, with important special cases of each class presented. We were able to learn a good deal about the behavior of planets in S-type binary orbits, and the possibility of life on those planets. The effect of the orbit of the planet on the climate as well as how the planet experiences day and night are discussed, and we also present a lower limit for the binary separation at which a binary system would fail to have a habitable S-type orbit. In addition, we explore the possibility that a planet could remain habitable in a transfer orbit between the system's two stars or in orbit around one of the system's Lagrange Points. In this endeavor, we derive an analytical expression for the surface temperature of an Earth-like planet at a binary system's stable L4 and L5 Lagrange Points, which closely match simulated results. We find definite possibilities for habitable L4 and L5 Lagrange Points in binary systems over a wide range of stellar masses and mass ratios. Finally, we propose many extensions and improvements to this study that would be worthwhile to pursue.

Chapter 1

INTRODUCTION AND BACKGROUND

Over half of the stars in our galaxy are expected to be binary or other multiple star systems, in which two or more stars are orbiting around a mutual center of gravity, known as the system's barycenter. NASA's Kepler mission is revealing that systems harboring planets are far more common than previously thought, and with the discovery of Kepler 16-b, we now have observational proof that planets do indeed exist in binary systems. With these discoveries, astronomers are beginning to contemplate whether or not these types of systems can create habitable terrestrial planets that could support complex life.

1.1 Criteria For Long-Term Habitability

First we must discuss the necessary criteria for a planet to be considered habitable. A number of factors can effect the habitability of a planet, but the most important among these is the presence of liquid water on the planet. The surface temperature of the planet will act as a proxy for this criteria, and this is the criteria we will use to determine planet habitability. We will also discuss what is necessary for a planet to remain habitable over the time spans needed for life to develop.

1.1.1 The Habitable Zone

The Habitable Zone is defined as the region in a stellar system where a planet could have liquid water on its surface. In other words, it is the region where the average surface temperature of a planet would be between 273K and 373K, the freezing and boiling points of water. The habitable zone of a single star is characterized by a ring around the star with a warmer inner radius and cooler outer radius. The location and size of a star's habitable zone is directly related to the temperature of the star. The inner radius of a hotter star's habitable zone will be further away from the star than a cooler star's. Also the difference between the outer and inner radii will be greater for the hotter star, making its habitable zone larger overall. This means, up to a certain extent[1], warmer stars have a higher probability of harboring a habitable planet.

1.1.2 Dynamical Stability

It is not sufficient that a planet occasionally finds itself in the habitable zone of a star. The planet must both keep its orbit and consistently remain in the system's habitable zone over a stretch of millions of years to truly be considered habitable. Therefore, a planet with a very eccentric orbit may not be habitable, even if the orbit passes through the habitable zone. In addition, perturbations from a larger Jupiter-like planet or another star in the system can destabilize a planet's orbit, possibly causing it to either fall into its host star or be ejected from the system. We define dynamical stability, as the ability for a planet to maintain its orbit, despite outside influences, over timescales reasonable for life to develop.

1.2 Orbital Configurations

There are a number of parameters that affect a planet's behavior in a multiple star system. However, the distance between the component stars in the system, and the distance of the planet from its closest star have the greatest effect. Previous work by Haghighipour, Quintana, and Lissaur [6] has shown that terrestrial planets can indeed form within the habitable zone of binary star environments in two special orbital configurations known as S-Type and P-Type Binaries.

In the S-type configuration, one of the stars acts as host to the planet, while the other star follows it's orbit at a further distance from the planet's host star[2]. This configuration generally occurs in wide binaries, where the separation of the stars is

[1] See Section 1.5.1

[2] See Figure 1.1a

greater than the planet's average distance from its host star. These types of systems likely formed from an early disk fragmentation, in which each star ended up with its own disk. We define an S-Type binary with this criteria: $r_p < R$, where r_p is the radius of the planet's orbit, and R is the separation between the binaries.

The P-type configuration, also known as a circumbinary orbit, occurs when the planet orbits around both stars, as if it was going around one[3]. This configuration would be seen in close binaries with binary separations less than a planet's average distance from it's host star ($R < r_p$). These types of systems likely formed from a single disk around both stars.

Generally, trinary[4] or higher systems, will be a hierarchical structure of binaries. For example, the Alpha Centauri System is a trinary system composed of two Sun-like stars, and one much smaller red dwarf, which orbits in a wide P-type orbit around the two larger stars which have a much tighter orbit. There is also a special configuration, in which a planet could be found in orbit around one of a binary system's Lagrange Points. This possibility is discussed further in section 1.3.

1.3 Lagrange Points

Lagrange Points are a manifestation of a rotating two-body system. They are the points at which the combined gravitational pull of two larger bodies, in our case, the two stars in a binary system, can provide the exact centripetal force needed for a third body of negligible mass, in our case, a planet in a binary system, to rotate with them. In other words, a body at one of the Lagrange Points, would not move relative to the other two bodies in the system. There are a total of five points at which this occurs[5].

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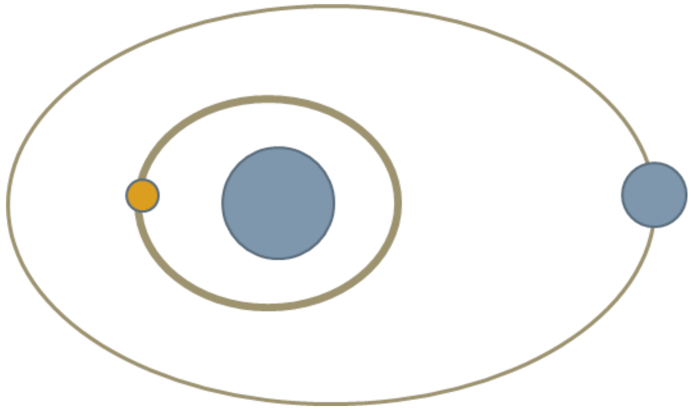
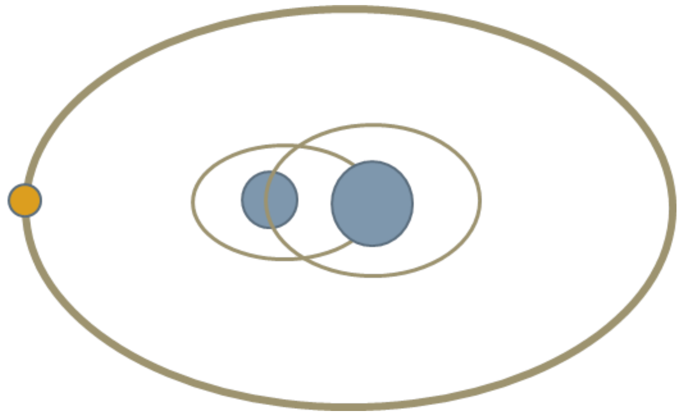
[3] See Figure 1.1b

[4] 3 stars in a system

[5] See Figure 1.2



(a) S-Type Binary



(b) P-Type Binary

Figure 1.1: The two common orbital configurations for binary systems with planets. The heavier lines indicate planetary orbital paths, and lighter lines, the orbital path of the stars.

The first three Lagrange Points, labeled L1-L3, lie on the line connecting the two larger bodies, and are unstable equilibria. Therefore, an object orbiting at this point could be knocked out of its orbit by the slightest perturbation. The L4 and L5 points lie at one of the points of an equilateral triangle whose base is the line between the two larger bodies of the system[6]. Unlike their counterparts, the L4 and L5 points are stable. The L4 and L5 points are maxima in the gravitational potential field in a two body system. Therefore, a small displacement would cause a third object to

[6] See Figure 1.2

move away from the equilibrium. However, provided the mass ratio is large enough, the Coriolis force causes the object to move around the equilibrium point[7]. Therefore, an object near one of these points could potentially maintain its orbit indefinitely. As discussed in Sections 1.4.3 and 5.2.6, this study mainly focus on the habitability of these points. Due to time constraints and other difficulties[8], we leave the stability of these "Lagrange orbits," as the topic of a future study.

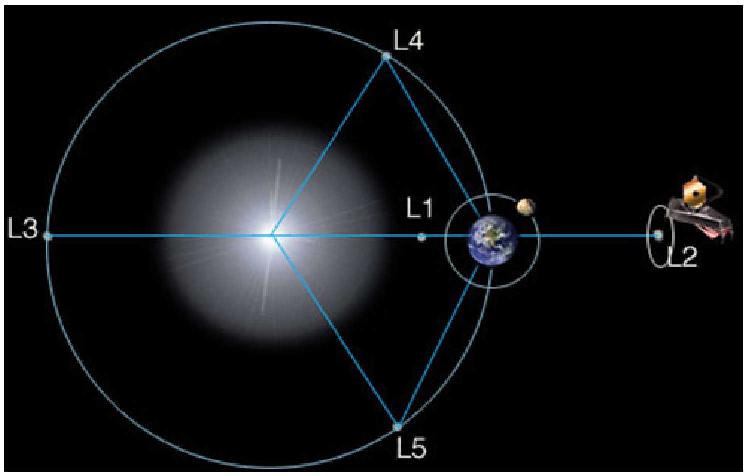


Figure 1.2: The 5 Lagrange Points of the Earth-Sun System with the James Webb Space Telescope orbiting at the L2 point (Not to Scale). Image Credit: NASA

1.4 Overview and Goals

In we aimed to determine the necessary criteria for a planet to sustain long-term habitability in various binary star systems. The notion of day/night and seasonal cycles for planets in our hypothetical systems would be considered and included in the determination of their habitability. Specifics about experiments used to accomplish these goals, as well as predictions regarding these goals, will be discussed further in Chapter 3.

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[7] We see this in our own Solar System where we find numerous asteroids collected around the Sun-Jupiter L4 and L5 Points

[8] See Section 5.2.6

1.4.1 Habitable Zone Geometry and Stability

While the habitable zone of a single star is fairly well understood and Quintana and Lissaur [6] have investigated special cases of habitability of multiple-star planetary systems, we know little about what the habitability zone of a multiple star system may look like in general. Therefore, a primary goal of this study was to fully understand and describe the geometry and long-term stability of the habitable zone of various multiple star configurations. It was the hope of this study to see at what point the habitable zone of two stars in a binary system would begin to merge. Also, we investigate how the separation of the binaries affect the shape of a merged habitable zone.

1.4.2 Determining Critical Separations

We also investigate the stability and habitability threshold of an Earth-like planet in both the S-type and P-type binary configurations with respect to average binary separation and other parameters. The point at a planet can no longer maintain a stable orbit within the system's habitable zone we define as the "critical binary separation," or R_{min} We hope to find a lower limit for these binary separations in S-type system over a reasonable range of binary masses and mass ratios[9]. These values can then be used as rough upper limits for the critical separation in corresponding P-type systems.

1.4.3 Investigation of Lagrange and Transfer Orbits

We extend our investigations beyond the standard S-type and P-type orbits of planets in binary systems. The stability and habitability of planets in both Lagrange Point orbits and transfer orbits were considered as part of this study. A transfer orbit we define as a planet transferring its orbit from its host star to the other star in the binary system. We investigate weather a planet could survive this transfer without losing its orbital stability, and if the habitable zone of the system spans enough of the distance between the two stars to allow the planet to retain its habitability. As

[9] See Section 1.5.1 for more on reasonable stellar masses.

discussed in 1.3, a planet orbiting around a binary system's Lagrange Point would have the unique perspective of never moving relative to the stars in the system. However, only the L4 and L5 points are stable enough to be considered for long-term habitability. Therefore we explore the conditions that would allow for a habitable planet to exist at the L4 or L5 Lagrange Points.

1.5 Restrictions and Scope

It would not be feasible to test every possible orbital configurations available to us in binary systems. Therefore, it is reasonable that we place realistic restrictions on the scope of our study that still allow us to cover the most likely scenarios in which we would find planets. In particular, here we discuss limitations on stellar masses and orbital inclination. Other limitations will be revealed as a part of this study.

1.5.1 Stellar Type

One consideration that had to be made was the type of stars to include in our systems. While it may be possible for planets to exist around a variety of stars, it is main-sequence stars, stars whose primary means of energy generation is core hydrogen fusion, that offer the best opportunities for life-bearing planets. However, higher mass stars stars have considerably shorter lifespans compared to their lower-mass counterparts. We know the main-sequence lifetime of a star scales as $\sim 1/M^2$[7], so with our Sun having a lifetime of 10 Gyr, we know a star 10 times as massive will have a lifetime that is about 100 times shorter, 100 Myr. This is the absolute shortest timescale that most consider to allow for the the formation of planets and simple life[11]. On the other extreme, the mass limit for core hydrogen burning is $\sim 0.08\ M_\odot$. Bodies below this limit are know as Brown Dwarfs, and are unlikely to be suitable for long-term habitability. Therefore we focused our study on stars with masses ranging form 10 to 0.1 $M_\odot$.

1.5.2 Orbital Inclination

Previous work by Haghighipour [5] indicated that a Jupiter-like planet in a binary system would not be stable with an orbital inclination greater than 40° with respect to the orbital plane of the binaries. It is also know that all of the major planetary bodies in our own Solar System have orbital inclinations of less than 10° with respect to the Sun's equator. Therefore, we expect most major habitable bodies to have low orbital inclinations, so it is a valid simplification that, for this study, all bodies orbit in the same plane. It is only for the testing of the stability of the Lagrange orbits that we deviate from this standard.

1.5.3 Orbital Eccentricity

We know that most plants in our solar system do not have eccentricities that exceed 0.1. Particularly, in the case of S-type orbits, a planet with too eccentric of an orbit cannot remain in the star's habitable zone for the entire length of the planet's orbit. Therefore, for our simulations, we initialized all planetary orbits as circular orbits. We discuss the possibility of altering a planet's orbital eccentricity in Section 5.2.3. While it might be conceivable for a habitable planet to survive in a system in which the component stars have relatively eccentric orbits about each other, we limited our selection of binary orbits to circular orbits as well. We discuss the possibility of altering the binary orbital eccentricity in Section 5.2.4.

Chapter 2

GENERAL SIMULATION METHODS

Modeling of hypothetical multiple-star systems was achieved by using the authors' own combination of a dynamic gravitation simulator and a stellar radiative flux calculator, allowing the calculation and observation the surface temperature of a planet over time. This also allowed for the live visual representation of the orbits as well as a graphical representation of the evolving geometry of habitable zone(HZ) superimposed on the orbits.

2.1 Simulator Overview

MATLAB[1] was chosen to run the simulations because of its numerous built-in functions and data visualization tools. If additional longer-term (>1 Myr) dynamic studies are warranted, much of our simulation code could be translated to a faster compiled language, or adapted to use an outside N-body code. A general overview of the simulation process can be seen in Figure 2.1.

2.1.1 Simulation Set-Up

Before a simulation began, a text file containing the initial parameters (mass, radius, luminosity, position, velocity, etc.) of the bodies desired was read in, and then the system of ordinary differential equations (ODEs) representing the equations of motion of the bodies was initialized. One challenge encountered was the proper calculation of initial velocities to meet the desired criteria of the system. This will be discussed further in Chapter 3, where the initialization of parameters for particular simulations are discussed.

[1] MATLAB is a product of The MathWorks, Inc.

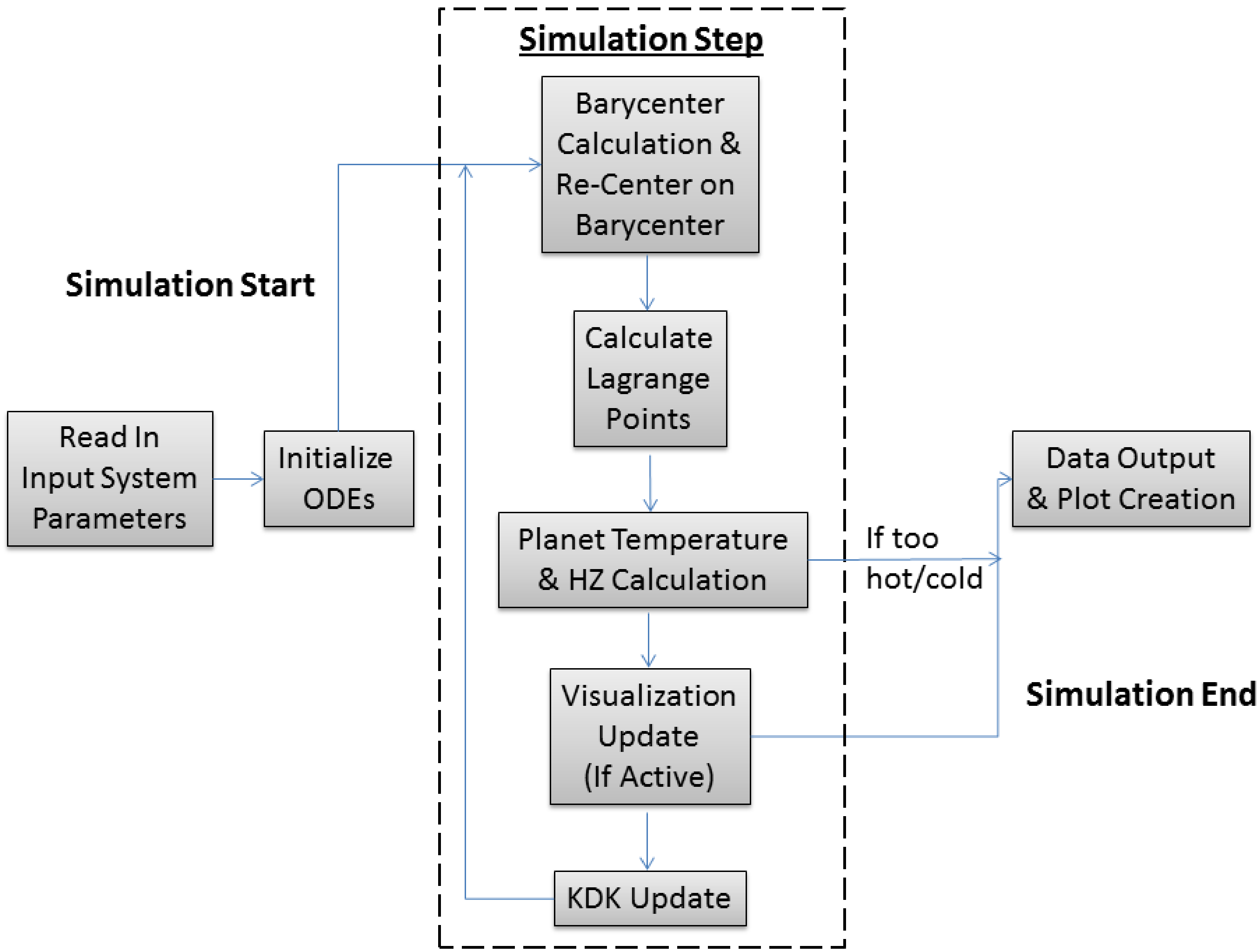

Figure 2.1: Simulation Overview

2.1.2 Simulation Time Steps and Wrap-Up

The process outlined by the dashed box is one simulation time step, which is repeated for the prespecified number of steps, determined by the desired length of the simulation. Each time step is characterized by the following substeps:

1. Centering the coordinate system on the center of mass (barycenter) of the system.

2. Calculation of the positions of the system's Lagrange Points [2].

3. Calculation of the surface temperature of the planets in the system, as well as the determination of the extent of the system's habitable zone [3]. If the planet is outside the temperature tolerance range, end the simulation.

4. Visualization Update: If the live visualization option is active, the display of the habitable zone, and the positions of the bodies and Lagrange Points is updated.

5. KDK Update: Perform the next time step update on the positions and velocities of all the bodies [4].

After the desired number of time steps was achieved, or if the simulations was halted, relevant information was output into text files for later analysis or immediately compiled into plots.

2.1.3 Premature Ends to Simulations

We chose to end simulations if the surface temperature of a planet we were monitoring for habitability went above 450K or below 200 K. Note that this is 75K above and below the range for the boiling and melting point of water because we wished to give a little leeway for instantaneous temperature spikes, which a planet could survive if is has efficient temperature regulating systems. However we believe that regular

[2] See Section 2.2.2

[3] See Section 2.3

[4] See Section 2.2.1

temperature spikes above or below this would be fatal to even most microbial life on the planet, and thus there would little reason to continue considering its habitability, hence the simulation ends. If a simulation reaches its end the average temperature of the planet in question is double checked to see if it remains within the range for liquid water.

2.2 Dynamical Simulation

Our simulations employed N-body gravitational methods to numerically integrate the N discretized ODEs that describe how gravity acts on each object. This allowed for the indefinite calculation of the trajectory of the objects in the field. In addition, the calculation and tracking of the system's Lagrange Points throughout the system's lifetime was achieved.

2.2.1 KDK Method

When choosing a numeric integrator, one must balance speed and accuracy. Ultimately, the second order Leapfrog Integrator, otherwise know as the Kick Drift Kick (KDK) Method, was chosen. To calculate the trajectory of each mass m due to a net force vector $\mathbf{F}(\mathbf{x_n})$, first a half Euler step was performed with time step dt on the mass's momentum vector $\mathbf{p_n}$:

$$\mathbf{p_{n+\frac{1}{2}}} = \mathbf{p_n} + \frac{1}{2}\mathbf{F}(\mathbf{x_n})dt. \tag{2.1}$$

This is the "kick." Then using the half-updated momentum, $\mathbf{p_{n+\frac{1}{2}}}$, a full Euler step was performed, on the position vector of the mass, x_n:

$$\mathbf{x_{n+1}} = \mathbf{x_n} + \frac{\mathbf{p_{n+\frac{1}{2}}}}{m}dt. \tag{2.2}$$

This is the "drift." Finally, using the force vector at the updated position, $\mathbf{F}(\mathbf{x_{n+1}})$, another half step was performed on the momentum to complete the update sequence:

$$\mathbf{p_{n+1}} = \mathbf{p_{n+\frac{1}{2}}} + \frac{1}{2}\mathbf{F}(\mathbf{x_{n+1}})dt. \tag{2.3}$$

This scheme was chosen because of its simplicity and ease to adapt to parallel algorithms. In addition, its speed is comparable to simpler first-order method because gravitational forces only needs to be calculated once per time step, as the previous value, $\mathbf{F}(\mathbf{x_n})$ will be stored by the simulator. It is also known to conserve energy very well. The accuracy of this method is further discussed in Section 2.4.1.

2.2.2 Lagrange Point Calculation

As discussed in 1.3, there are five Lagrange points, only two of which can be stable, the L4 and L5 Lagrange Points. This makes the other three points unsuitable to study for long-term habitability, which is fortunate considering there is no closed form expression for their positions for every mass ratio. However, future study may be warranted to study their short-term affects on orbits.

With the stars initially aligned along the x-axis and the coordinate system centered on the stars' center of mass, the positions of the L4 and L5 Lagrange points are given by:

$$\mathbf{r_{L4/L5}} = \left(\frac{R}{2}\frac{M_1 - M_2}{M_1 + M_2}, \pm\frac{\sqrt{3}}{2}R, 0\right), \tag{2.4}$$

where R is the distance between the two stars, and M_1 and M_2 are the masses of the two bodies[3]. The L4 point is given by the plus sign option. To ensure correct placement in a rotated system, the coordinates are converted to polar coordinates (r, θ), where r is simply the magnitude of $\mathbf{r_{L4/5}}$ given in equation 2.4, and $\theta = \arctan(y/x)$, where x and y are the x and y coordinates of one of the stars in the system.

2.3 Thermal Simulation

Simultaneous to the dynamical calculations, the electromagnetic flux from each star in the simulation could be calculated at any point in the system. From this, the temperature at that point could be calculated, and the habitable zone could be determined. As described in Chapter 1, all stars used in this study were considered to be main-sequence stars. From observations a main-sequence star's mass can be related to its luminosity. For this study the well-accepted main-sequence mass luminosity relation of $L \sim M^{3.5}$ was assumed[5], where M and L are in terms of solar mass and luminosity[7].

2.3.1 Planetary Surface Temperature Calculation

The surface temperature of a planet, T_P, with radius R_P, bond albedo a, and greenhouse effect factor g is determined by balancing the sum of stellar flux absorbed by the planet, $F_{ab} = \pi R_P^2(1-a)\sum F_S$, with the emitted flux of the planet in question: $F_{em} = 4\pi R_P^2 \sigma(1-g)T_P^4$ [10]. Reflected flux does not contribute to the planet's surface temperature.

The calculation is as followed:

$$\pi R_P^2(1-a)\sum F_S \quad = \quad 4\pi R_P^2 \sigma(1-g)T_P^4, \tag{2.5}$$

where σ is the Stephan-Boltzmann Constant, and $\sum F_S$ is the sum of the incoming flux from all of the stars in the system. This is given by:

$$\sum F_S = \sum_{i=1}^{N} \frac{L_i}{4\pi d_i^2}, \tag{2.6}$$

where L_i is the luminosity of the ith star in the system, d_i is the distance between that star and the planet in question, and N is the number of stars in the system. Therefore, the surface temperature of the planet is then given by:

$$T_P \quad = \quad \left(\frac{1}{4\sigma}\frac{1-a}{1-g}\sum_{i=1}^{N}\frac{L_i}{4\pi d_i^2}\right)^{1/4}. \tag{2.7}$$

The bond albedo, a, is the fraction of the total electromagnetic radiation incident on the planet that is scattered back out into space. This particular albedo was chosen as it taken in account all wavelengths of light at all phase angles. Values for a range from zero to one, with 0 representing a perfect blackbody, and 1 representing a perfect reflector. For the course of this study, a value of 0.306 was assumed, as it is the bond

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[5] Stellar mass and luminosity evolve over time, but we are considering relatively short time scales compared to the stellar lifetime, so these are taken to be constant for this study.

albedo of the Earth, although it is quite feasible that a planet with more cloud or surface ice cover could have a significantly higher bond albedo[6].

The value of g represents the greenhouse effect, which is the ability of a planet's atmosphere to trap thermal radiation on the planet in question. We determined this by comparing the known equilibrium temperature of Earth, given by $T_{eq} = \left(\frac{1-a}{4\sigma}F_{\odot}\right)^{1/4} \approx 255K$, with the actual average surface temperature of the Earth, given by $T_S = \left(\frac{1}{4\sigma}\frac{1-a}{1-g}F_{\odot}\right)^{1/4} \approx 288K$, thus giving us an expression for g:

$$g = 1 - \frac{T_{eq}^4}{T_S^4} = 0.385, \tag{2.8}$$

which is the value we assume for this study. To contrast, Venus, with a surface temperature of 735 K, has a greenhouse effect of $g = 0.996$.

2.3.2 Habitable Zone Determination

The habitable zone of a system was determined by applying Equation 2.7 to a square grid, whose size is determined by the body furthest from the system's barycenter, at 0.1 AU intervals. All points whose temperatures laid within the allowable range for liquid water, 273-373 K, were considered to be within the habitable zone. This information was used for the live simulation option where the habitable zone is plotted. When this option is active, the user can watch the habitable zone of the system change as the system evolves. In this view, seen in Figure 2.2 the habitable zone is plotted in green, with temperatures above 373 K displayed in red, and temperatures between 173 and 273 K displayed in blue.

2.4 Verification

One of the dangers of using our own code was the possibility that it would produce results that are not physically accurate. To verify the validity of our results,

---

[6] As reference the bond albedo of Venus, which has significantly more cloud cover than Earth is 0.9, and the icy moon of Enceladus has a bond albedo of 0.99, while our own dry dusty moon only has a bond albedo of 0.123.

the code modeled well-understood systems, and the results were compared to the actual parameters of those systems.

2.4.1 Dynamics

The computational nature of our orbital integrator by nature introduced some error into our calculations. To test the validity of these methods, the orbit of the Earth around the Sun was modeled, and any major discrepancies with the known orbital parameters of the Earth[7] were noted.

The Earth's perihelion, and v_{min} were used as initial parameters, and the Earth's orbit was evolved for half a million years initially using a relatively high time step of $dt = 0.1$ years. As can be seen in Figure 2.3, the system shows no signs of major fluctuations or decay between orbits, so it was concluded that this method does indeed conserved energy. However, it was found that these initial results were qualitatively incorrect. As can be seen in Figure 2.3a, what was intended to be the test planet's perihelion became its aphelion, a clear error resulting from using too large of a time step.

Pertinent Orbital Parameters of Earth	
Perihelion	0.9832 AU
Aphelion	1.0167 AU
v_{max}	30.29 km/s
v_{min}	29.29 km/s

Table 2.1: Orbital parameters of the Earth, acquired from NASA's Planetary Fact Sheets[12], used to verify the correct simulation of orbital dynamics.

Therefore, the logical course of action was to lower the time step. The above procedure was repeated with smaller time steps of 0.05, 0.02, and 0.01 years. We can see in Figure 2.3c the qualitative errors are rectified when the we used $dt = 0.02$ years as a time step. However, the error in the aphelion was too large for accurate dynamic simulations. It is with $dt = 0.01$ years, as seen in Figure 2.3d, that an acceptable level of accuracy was achieved . Unfortunately, systems may contain planets, or stars,

[7] See Table 2.1

with periods shorter than a year, in which $dt = 0.01$ years may be an unacceptable time step. However, as long as the time step remains below $\sim 1/100$ the period of the shortest periodic component, the simulation should be accurate.

2.4.2 Habitable Zone and Surface Temperature Verification

In contrast to the orbital integrator, the methods that calculate the system's habitable zone relies on an analytically derived result, so we can expect an exact result. One obvious check was to map the habitable zone of the Sun and observe the Earth's motion. With the methods that simulate the motion confirmed, we know any deviations of the orbit outside the Habitable Zone will be due to errors in temperature calculations. As can be seen in Figure 2.2, Earth was correctly plotted within the habitable zone (green ring), and we can see that both Earth's perihelion and aphelion are well within the green ring.

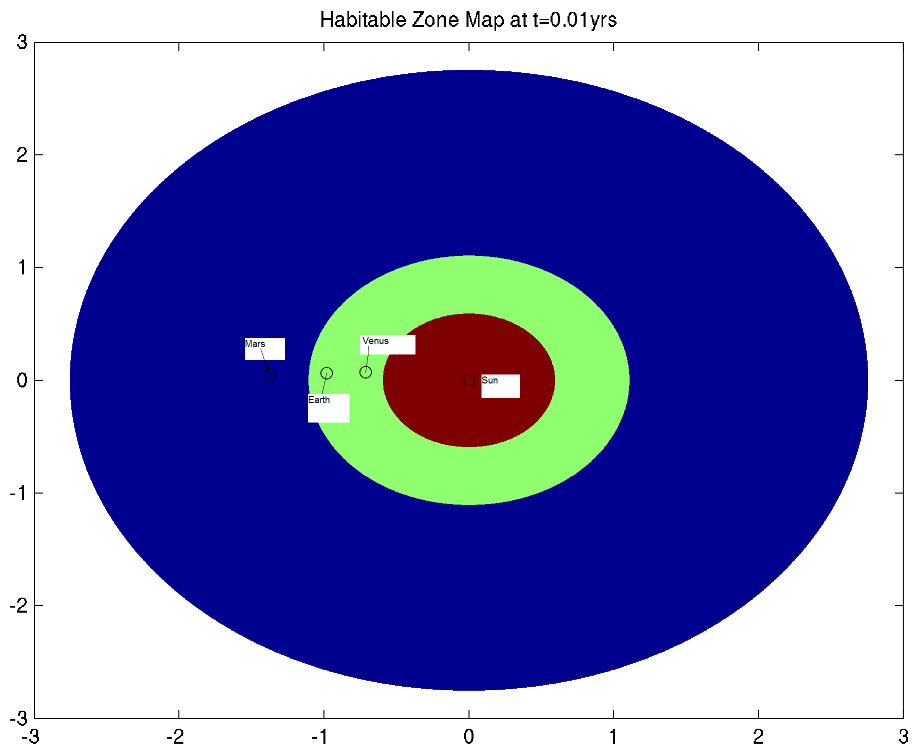
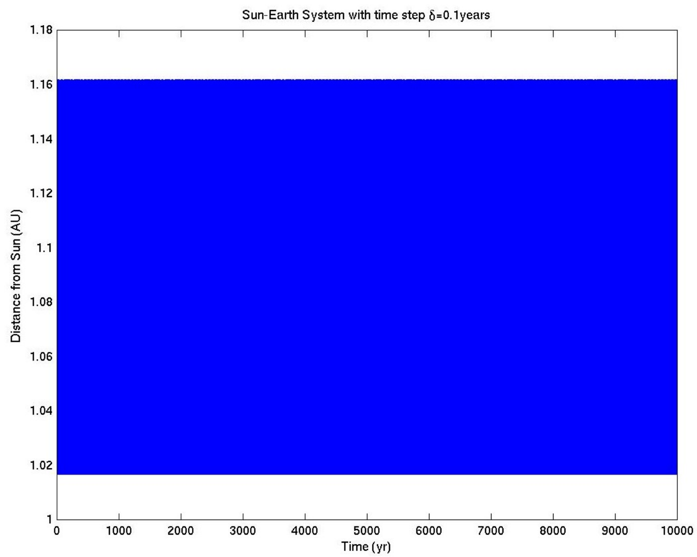
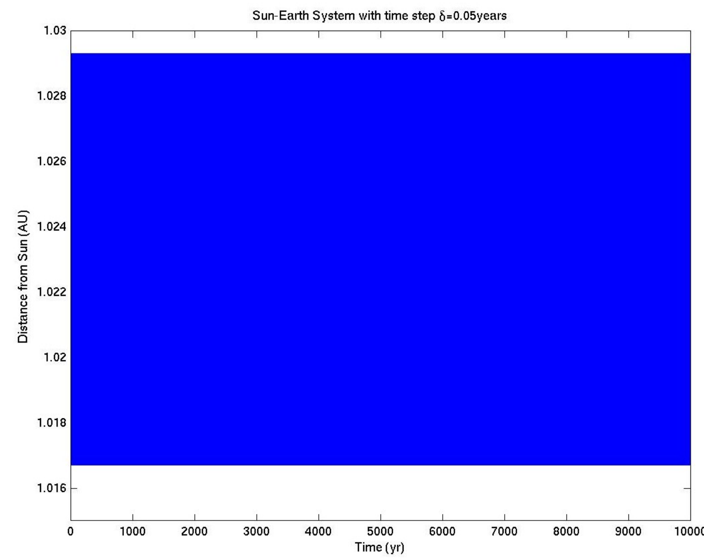


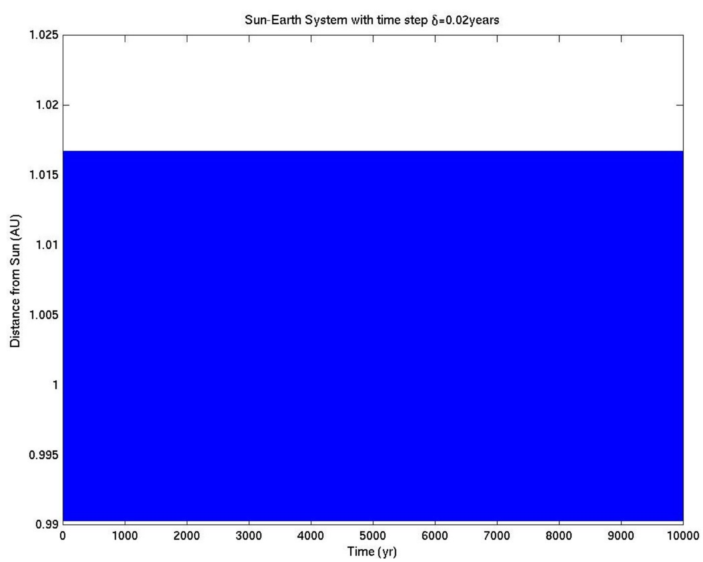
Figure 2.2: Habitable Zone of the Sun, indicated by the green ring, with the position of Venus, Earth, and Mars plotted as small circles. Red indicates the area where the planet would be too warm, and blue, as well as white, indicates regions which are too cold. The Sun would be located at the bullseye of this target shape. All distances are in Astronomical Units (AU).



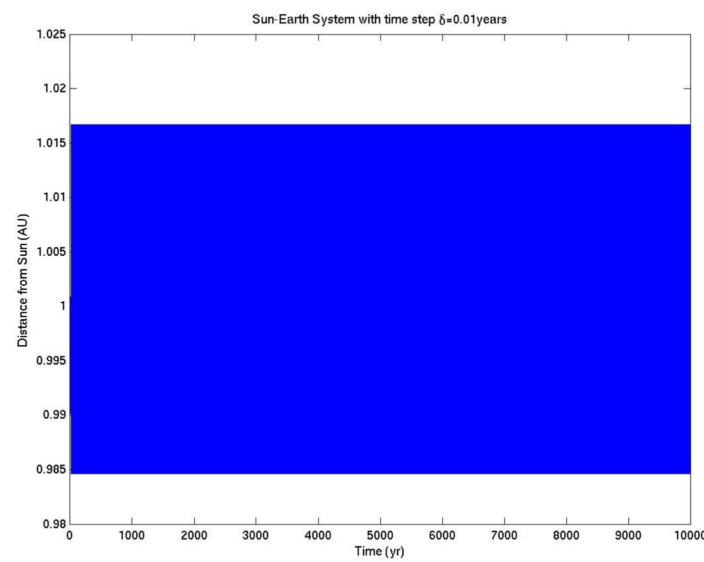
(a) $dt = 0.1$ year



(b) $dt = 0.05$ year



(c) $dt = 0.02$ year



(d) $dt = 0.01$ year

Figure 2.3: Distance between the Earth and the Sun over the evolution of the test system for various time steps dt.

Chapter 3

SIMULATIONS AND HYPOTHESIS

In this chapter we describe the simulations ran to fulfill our goals. Here, we elaborate on the specifics of how our simulations were initialized, and what parameter values were chosen. In addition, we describe the predicted behavior, if any, of the simulations. Our planned rigorous simulations focused mainly on two of our goals: Determining the critical binary separation for planetary stability in an S-type system, and determining the habitability of the L4/L5 Lagrange Points. However, our secondary goals still are addressed with these simulations, as we will discuss in Chapter 4.

3.1 Critical Binary Separation for Planet Habitability in S-Type Systems

The purpose of these simulations was to test the effects of binary separation on a planet's orbit, and determine the minimum safe binary separations to allow for an Earth-like plant to remain habitable. We define this value as the Critical Binary Separation. For these simulations, we will place a planet in circular orbit around one of the stars, which we will call the host star, and put a perturbing star in a wider circular orbit around the host star. We will analyze the case where both stars of equal mass and where the host star's mass, M_2 differs from the mass of the perturbing star, M_1.

In order to focus on the effect of stellar mass, and therefore luminosity, on the critical binary separation, we began with the case of a system with two stars of the same mass. We selected stellar masses based on the guidelines set forth in Section 1.5.1. Therefore we chose masses of 0.1,0.25,0.5,1,2,5, and 10 solar masses to cover the spread of feasible planet-hosting stars. We then proceeded with simulations in which the two stars have different masses. For these simulations, we let the mass of the host star, M_2, always be 1 solar mass, so therefore, our planet had the same orbital radius for all of the mass ratios we tested. We then adjusted the mass of the perturbing star, M_1, which gave us the following mass ratios: $\eta = M_2/M_1 = 10, 5, 2, 1, 0.5, 0.2, 0.1$.

3.1.1 System Parameter Initialization

Three parameters of the simulations were chosen independently: the masses of the two stars in the system, and their separation, R. The others were derived from these three. As stated in Section 2.3.1, stellar luminosity is directly related to the mass of the star by the expression $L \sim M^{3.5}$, where L and M are both in solar units. Therefore, stellar luminosities were determined directly using this formula.

We next set the initial positions of the stars and planets in our system to replicate the conditions we desire. Acting as if the host star was a single star, we place the planet at a distance from its host star to allow for a surface temperature of $T_p = 288K$, the average temperature of the Earth. This is given by a modification of equation 2.7:

$$r_p = \sqrt{\frac{L_2}{16\sigma\pi T_p^4}\frac{1-a}{1-g}}, \tag{3.1}$$

where L_2 is the luminosity of the host star in Watts, and r_p is the initial position of the planet relative to its host star. We place the host star at (0,0), the perturbing star at (R,0), and the planet at ($-r_p$,0), and let the simulators built in re-centering tool adjust the coordinates so that the system's barycenter is at (0,0), and all coordinates are relative to the barycenter.

We initialized velocities using the laws for simple uniform circular motion. For the host star, the initial velocity is given by $v_2 = \sqrt{G\mu/R}$, where $\mu = \frac{M_1 M_2}{M1+M_2}$ is the reduced mass of the two stars, and R is the binary separation. The initial velocity of the planet, v_p, was found by finding the velocity needed for uniform circular motion around the host star, M_2, and adding it to the initial velocity of the host star, v_2: $v_p = \sqrt{GM_2/r_p} + v_2$.

3.1.2 Simulation Time Step

As discussed in Section 2.4.1, the maximum safe time step is $\sim 1/100$ of the smallest period in the system, which for the case of an S-type binary, will always be the period of the planet's orbit. Due to the nature of our mass ratio simulations[1], we were able to use the same time step of $dt = 0.01$ years. A summary of the time steps used in the equal mass case can be found in Table 3.1.

Stellar Masses	r_p (AU)	Time Step
0.10 $M_\odot$	0.018	4×10^{-5} yr
0.25 $M_\odot$	0.088	5×10^{-4} yr
0.50 $M_\odot$	0.295	3×10^{-3} yr
1.00 $M_\odot$	0.993	1×10^{-2} yr
2.00 $M_\odot$	3.342	2×10^{-2} yr
5.00 $M_\odot$	16.61	5×10^{-2} yr
10.0 $M_\odot$	55.87	1×10^{-1} yr

Table 3.1: Sample of time steps used to initialize simulations for the equal mass case.

3.2 Lagrange Point Habitability

For these simulations, we focused on the determination of the habitability of a binary system's Lagrange Points. As discussed in Section 1.3, it has been determined that the L1, L2, and L3 points are all unstable, and therefore unsuitable for consideration for long-term habitability. Our focus was on the L4 and L5 Lagrange points, which can support stable orbits over longer periods of time. We explored the effects of binary separation, mass, and mass ratio, on the surface temperature of an Earth-like planet at the L4 point[2] of a binary system.

3.2.1 L4 Point Temperature Simulations

We began with the testing of equal mass binaries, with seven sample masses of 0.1,0.25,0.5,1,2,5, and 10 Solar Masses. We tested binary separations ranging from 0.1 AU to 10 AU at 0.1 AU intervals. For our simulations, we set up the binaries for circular orbits as it would provide similar temperatures if we averaged over an eccentric orbit. With the circular orbit setup, the position of the L4 point does not change relative

[1] See Section 3.1

[2] The L4 and L5 Lagrange points are the same distances from each star in the system, hence why they can be treated the same when calculating temperatures.

to the two stars. Therefore, we should expect a fairly constant temperature at the L4 point over the evolution of the system, so it was sufficient to simply sample the temperature at the initialization of the simulation. According to equation 2.4, the L4 point is equidistant between the two stars in this case, and therefore, we expect equal thermal contribution from both stars.

We also explore the case of the L4 points in a binary system with unequal mass stars. Using $M_1 = 1$, we chose sample mass ratios of $\eta = M_2/M_1 = 10, 5, 1, 0.5, 0.25, 0.1$, and, as with the equal mass case, use binary separations of 0.1 to 10 AU at 0.1 AU intervals. As explained in the last section, we restrict ourselves to circular binary orbits, and simply sample the temperature of the initial configuration. According to equation 2.4, we find the L4 point closer to the largest star, so we expect the majority of the thermal energy at the L4 point to be from this star.

3.2.2 Expected Behavior

By combining Equation 2.7, from which we can determine the surface temperature of an Earth-like planet at any position in the system, with Equation 2.4, which describes the position of the L4 Lagrange point relative to the system's center of mass, we can derive a relationship to determine the temperature of a planet at the L4 or L5 Lagrange points.

First we begin with equation 2.7:

$$T = \left[C_{pl}\left(\frac{L_1}{d_1^2} + \frac{L_2}{d_2^2}\right)\right]^{1/4}, \tag{3.2}$$

where $C_{pl} = \frac{1}{16\pi\sigma}\frac{1-a}{1-g}$, is a constant consisting of thermal and planetary parameters, and L_1 and d_1 are the Luminosity of the first star and the distance of the first star to the L4 point. We find d_1 by using 2.4:

$$d_1^2 = \left(\frac{R}{2}\frac{M_1 - M_2}{M_1 + M_2} - x_1^2\right)^2 + \frac{3}{4}R^2, \tag{3.3}$$

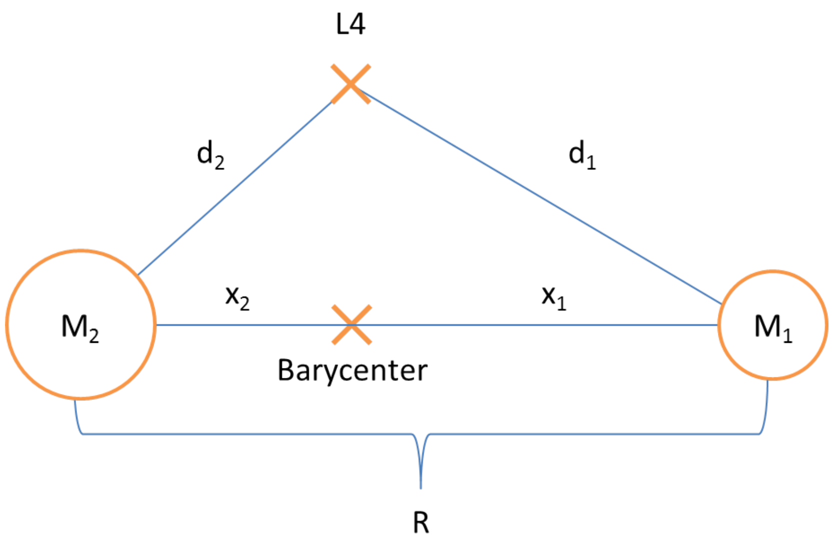


Figure 3.1: Overview of Lagrange Point simulation set-up

where R is the separation between the stars, M_1 and M_2 are the masses of the stars, and $x_1 = \frac{M_2 R}{M_1+M_2}$ is the right star's position relative to the system's barycenter[3]. Substituting the expression for x_1 into equation 3.3 and simplifying yields:

$$d_1^2 = \frac{R^2}{4}\left[\left(\frac{M_1 - 3M_2}{M_1 + M_2}\right)^2 + 3\right]. \tag{3.4}$$

Similarly d_2 can be found using $x_2 = \frac{-M_1 R}{M_1+M_2}$:

$$d_2^2 = \frac{R^2}{4}\left[\left(\frac{M_2 - 3M_1}{M_1 + M_2}\right)^2 + 3\right] \tag{3.5}$$

Substituting 3.4 and 3.5 into 3.2 yields:

$$\begin{aligned} T &= \sqrt{\frac{2}{R}}\left[C_{pl}\left(\frac{L_1}{\left(\frac{M_1-3M_2}{M_1+M_2}\right)^2+3} + \frac{L_2}{\left(\frac{M_2-3M_1}{M_1+M_2}\right)^2+3}\right)\right]^{1/4} && (3.6)\\ &= \sqrt{\frac{M_1+M_2}{R}}\left[C_{pl}\left(\frac{L_1}{M_1^2+3M_2^2} + \frac{L_2}{M_2^2+3M_1^2}\right)\right]^{1/4}. && (3.7)\end{aligned}$$

If we let $\lambda = L_2/L_1$, $\eta = M_2/M_1$, $M = M_1 + M_2$, and rewrite L_1 in terms of solar luminosity ($L_\odot$), then the above becomes:

$$T = \sqrt{\frac{\eta+1}{R}}\left[L_\odot C_{pl}\frac{L_1}{L_\odot}\left(\frac{1}{1+3\eta^2} + \frac{\lambda}{\eta^2+3}\right)\right]^{1/4}. \tag{3.8}$$

[3] See Figure 3.1

Using the standard mass-luminosity relationship, $L \approx M^{3.5}$, we can also conclude that $\lambda = \eta^{3.5}$. With this we find,

$$T = \sqrt{\frac{\eta+1}{R}} \left[L_{\odot} C_{pl} M_1^{3.5} \left(\frac{1}{1+3\eta^2} + \frac{\eta^{3.5}}{\eta^2+3} \right) \right]^{1/4}, \tag{3.9}$$

where M_1 is now in terms of Solar Mass. Finally applying $M_1 = M/(\eta+1)$, we achieve the L4/L5 Temperature function in its final form:

$$T(M,\eta,R) = \sqrt{\frac{1}{R}} \left[\frac{L_{\odot} C_{pl}}{(1\mathrm{AU})^2} \frac{M^{3.5}}{(\eta+1)^{1.5}} \left(\frac{1}{1+3\eta^2} + \frac{\eta^{3.5}}{\eta^2+3} \right) \right]^{1/4} \tag{3.10}$$

$$T(M,\eta,R) = \frac{H(M,\eta)}{\sqrt{R}}, \tag{3.11}$$

where M, the combined mass of the binary stars, is in terms of Solar Mass, R, the separation between the binary stars, is now in AU[4], and $H(M,\eta)$ is given by:

$$H(M,\eta) = \left[\frac{L_{\odot} C_{pl}}{(1\mathrm{AU})^2} \frac{M^{3.5}}{(\eta+1)^{1.5}} \left(\frac{1}{1+3\eta^2} + \frac{\eta^{3.5}}{\eta^2+3} \right) \right]^{1/4}. \tag{3.12}$$

In the next chapter, we use equations 3.11 and 3.12 as the standard of comparison for the Lagrange Point simulations.

[4] 1 AU (Astronomical Unit) = 149 598 000 km, the average distance between the Earth to the Sun.

Chapter 4

RESULTS AND DISCUSSION

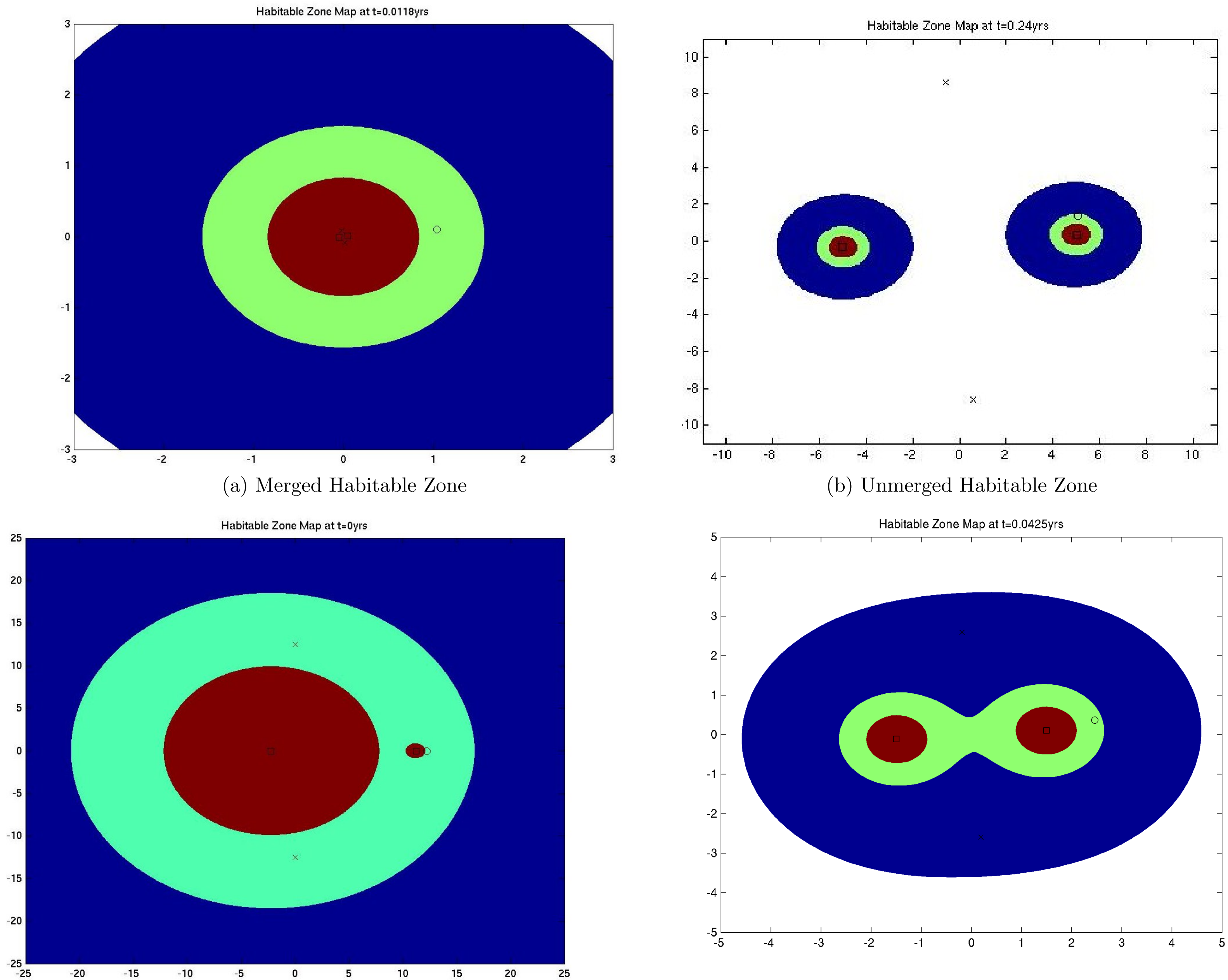
Our simulations produced a number of intended and unintended results. We addressed most of the goals from Section 1.4, as well as discovered some surprising properties of planet-hosting binary systems. In addition, we discuss the climatic and orbital responses of a planet placed in various binary configurations. It is the purpose of this chapter to present and discuss these results.

4.1 Habitable Zone Geometry

We were able to classify the habitable zones of our simulated binary systems into one of two main classes: merged and unmerged. These habitable zone classifications can be used as an analogue to distinguish between habitable P-type and S-type binaries.

In the unmerged case, the habitable zones of the two stars are essentially independent of one another. However, there are some borderline unmerged habitable zones in which we do see some deformation of the standard single-star ring habitable zone as the separation between the stars decreases. In order for an Earth-like planet to survive in this habitable zone class, it would be necessary for it to be in an S-type orbit.

In the merged case, the habitable zones of the stars are essentially indistinguishable from one another. For very close P-type orbits where $R << r_{planet}$, the merged case can be approximated to first order by placing a single star with the sum mass and luminosity of the two stars at the system's barycenter. However, as R becomes comparable to r_{planet}, the gravitational and thermal effects of the stars must be considered individually. Generally, an Earth-like planet would need to be in a P-type orbit to survive.



(a) Merged Habitable Zone

(b) Unmerged Habitable Zone

(c) Engulfed Habitable Zone

(d) Borderline Unmerged

Figure 4.1: Two main classes of Habitable Zone geometries(a and b), with two special cases of each (c and d).

Low mass ratios can result in a special case of the Merged Habitable Zone, in which the Habitable Zone of the larger star engulfs other star's Habitable Zone completely, making it irrelevant to the habitability of the planet. That is, the planet could break free of the orbit of its host star, and likely remain habitable, up to a certain extent. This case only seems to be possible for $\eta < 0.2$, and it is one of the few cases where an Earth-like planet could be in an S-type orbit with a merged habitable zone.

4.2 Transfer Orbits

It is possible for a planet to transfer its orbit, but we have yet to observe a transfer in which the planet survives. In most cases, the planet falls too close to the star, or, more commonly, the planet is ejected from the system. Even if ejection does not occur, the planet does not re-settle into a habitable orbit. All of this implies that there may be only a hairline range of parameters that would allow for a survivable transfer orbit. Continuously habitable transfer orbits are likely to only occur in systems with either a borderline unmerged habitable zone[1] or engulfed habitable zone[2]. More rigorous investigation on this topic is needed, but the probability of a completely habitable and stable transfer orbit actually occurring is negligible.

4.3 S-Type Binary Results

We were able to learn a good deal about the behavior of planets in an S-type binary orbit. As was stated in Section 1.4.2, we hoped to determine the threshold between S-type and P-type orbits. We accomplished this by finding the critical binary separation at which a planet in an Earth-like S-type orbit would destabilize and/or fall out of the habitable zone. The dependence of this value on binary mass and mass ratio are also presented here. We also discuss the effects of the odd day/night cycle these planets would experience.

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[1] See Figure 4.1d

[2] See Figure 4.1c

4.3.1 Sensitivity of Stability and Planetary Eccentricity Perturbation

One surprising discovery was that the stability of these systems was very sensitive to binary separation. Decreasing the separation of the binaries by as little as 0.01 AU below the critical value would not only cause the planet to fall out of the acceptable temperature range for the simulation[3], but it also often completely destabilized the planet's orbit. In most cases, it is the planet falling too close to a star, and getting so hot that ends the simulation. Further investigation is required to verify that planetary ejection or destruction is not an artifact of the constant time step being too small as the planet makes a close approach with a star[4].

In all of the S-type simulations, the planet was perturbed out of its circular orbit into a more eccentric orbit. However, after spending a few periods in this more eccentric orbit, the planet would then return to its circular orbit. We believe that this might be caused by interaction with system's L1 and L2 Lagrange points. As can be seen in Figure 4.2, the shorter period peaks are the individual orbits of the planet around its host star. On top of that we see regular longer period cyclings of the planet's orbital eccentricity, in which the planet's surface temperature varies more widely over a single orbit. In the equal mass case and in most mass ratios, we see that the average temperature of the planet does not increase drastically as R decreases[5]. However, we do see a rise in the temperature peaks as R decreases.

4.3.2 Critical Binary Separations

We first wished to see how the Critical S-type Separation would change with stellar mass, so we began with the equal mass binaries. As described in Section 3.1.1, the initial placement of the planet, r_{planet}, depended on the luminosity, and therefore the mass, of the host star. In Figure 4.3, we see that the Critical Separation scales

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[3] See Section 2.1.3

[4] This is discussed further in Section 5.2.1.

[5] It is only with $\eta < 0.2$, we see the average temperature significantly affected by R.

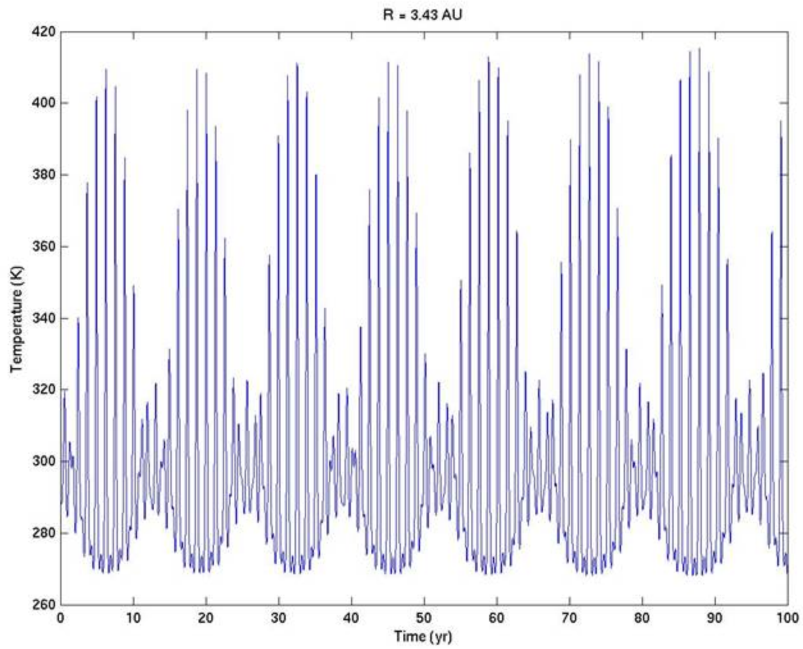

Figure 4.2: Surface Temperature of an Earth-like planet in an S-type orbit near Critical Binary Seperation with $M_1 = 1M_{\odot}$, $\eta = 1$, and $R = 3.43AU$.

with the mass of the perturbing star, and, since this is an equal mass case, scales with the mass of the host star. When comparing these two values we found that on average:

$$R_{min}/r_{planet} \approx 3.43 \pm 0.01, \tag{4.1}$$

where r_{planet} is given by equation 3.1. This expression sets a lower limit for the separation between equal mass binaries for any given planetary orbital radius to allow for a stable S-type binary orbit.

We also wished to explore the effect the binary mass ratio, η, on the critical separation, R_{min}. It was in these simulations that we discovered the special case of the Engulfed Habitable Zone[6]. This made identifying the critical binary separation difficult for $\eta < 0.2$, as the entire host star's habitable zone is within the habitable zone of the primary. Therefore, for these low mass ratios, the lower limit for separation is determined by the inner edge of the habitable zone of the perturbing star. For the others the lower limit is determined by the same manner as the equal mass case. High

[6] See Section 4.1

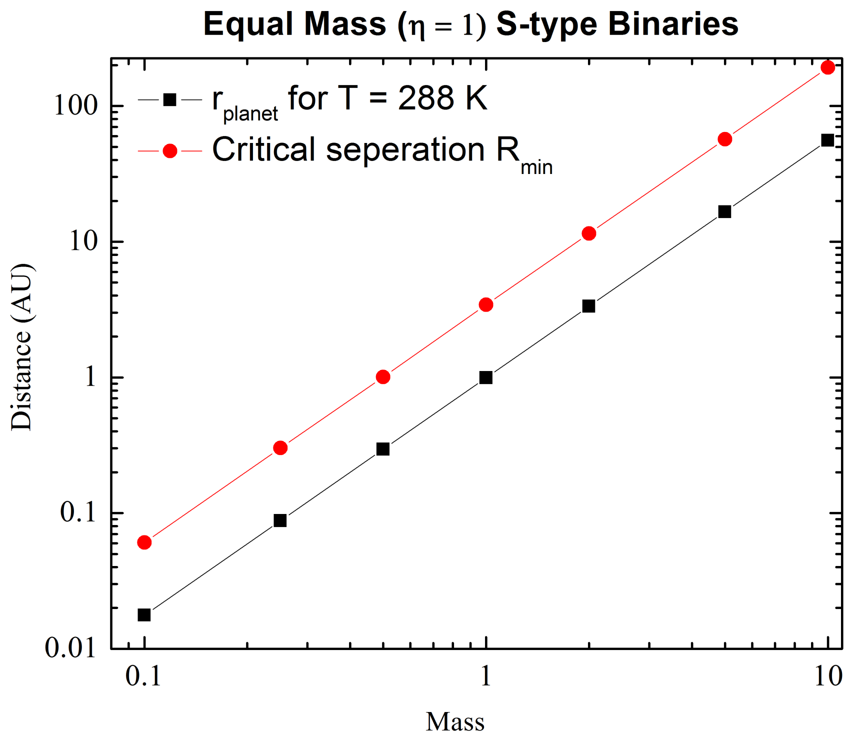

Figure 4.3: Minimum Habitable S-type Binary Separation plotted with the distance the planet needs to be from the host star to have T=288K with $\eta = 1$.

mass ratios reflect the situation of Jupiter's affect on orbits in our own Solar System, just on a larger scale. As can be see from Figure 4.4, the results, as expected, show R_{min} decreasing with η except for the particular case of $\eta = 2$, in which R_{min} was actually lower than the neighboring values of η. We believe this may be caused by some sort of resonance or interaction with the Lagrange points, but further investigation would be required to confirm this. Overall, we found that the critical binary separation followed the relation:

$$R_{min} \sim 3.43\eta^{-1/2}. \tag{4.2}$$

Combining the results from equations 4.1 and 4.2, we arrive at a general result for the critical binary separation:

$$R_{min} = \frac{3.43 r_{planet}}{\sqrt{\eta}} \tag{4.3}$$

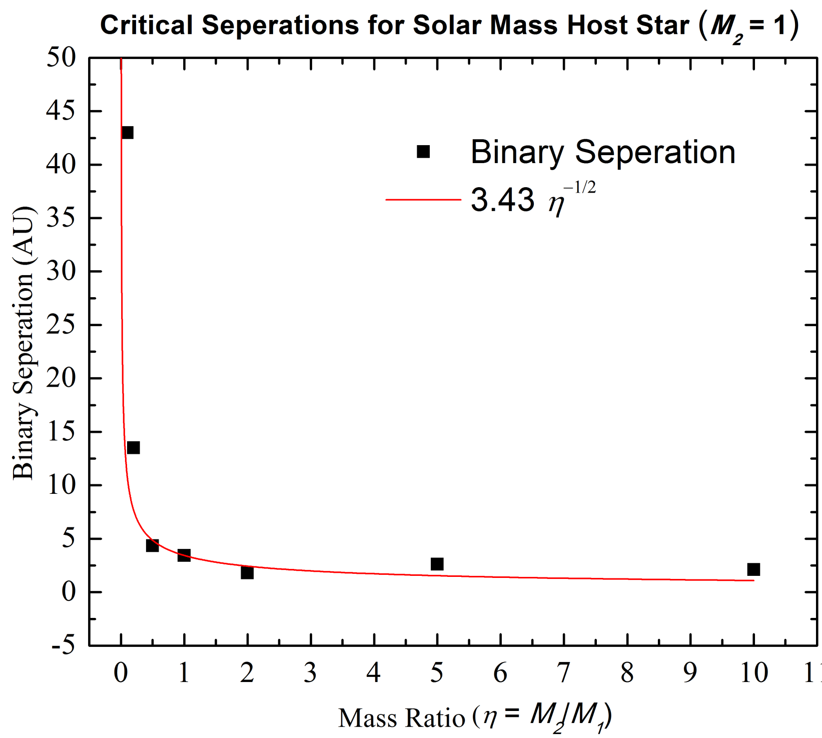

Figure 4.4: Minimum Habitable S-type Binary Separation plotted for varying mass ratios. The line represents the equation of best fit.

4.3.3 Climate Effects of Abnormal Day-Night Cycles in S-type Binaries

A planet in an S-type system would experience a very abnormal day-night cycle where it spends part of the year having no complete night. Looking at Figure 4.5, we follow half of an orbit of a planet in an S-type orbit. In (a) the planet would experience a day/night cycle as we would experience on Earth, except with two Suns in the sky. In (b) the planet would experience two sunrises and two sunsets separated by about a quarter rotation, leaving a quarter rotation for night. In (c), the planet would experience a sunset closely, if not immediately, followed by the sunrise of the other star. Thus the planet would experience no true night, and its surface would be bathed in sunlight for its entire rotation.

There are many consequences that can be considered when a potentially life-bearing planet experiences nearly no night for half of its year. If the two stars are relatively close, i.e. near their critical separation, the secondary star can be as much as 1/10 the brightness of the host star[7]. This is 10000 times brighter than the brightest Full Moon. During the "nightless" season the temperature differential on the two sides of the planet would be much less than normal, if not nearly the same. As the temperature difference on the day and night side of the Earth is one of the driving factors of air currents, and hence, weather on our planet, a planet with a low temperature differential would likely experience an extreme dry season with little weather during the nightless part of the year. On the other extreme, during the other half of the year, the day side of the planet would experience a higher level of radiation than average, and would cause a higher temperature differential on the two sides of the planet. This would likely cause a monsoon-like season with extreme weather on the planet.

Another consideration we must make is how life would respond on a planet with such an odd day/night cycle, especially when it comes to the sleep cycles of animals. The closest example we can find to this type of phenomenon is the Arctic during the summer, where there is 24-hour daylight. Polar Bears are known to be able to sleep almost anywhere at any time, so most animals would either have to have that adaptability, or have dark underground or cave dwellings to which they could retreat for sleep. Nocturnal animals would either be non-existent on a planet such as this or would have to hibernate during the nightless season. Most plants would have to be hardy with excellent water retention mechanisms like trees and desert plants. Much of this is speculative, and the nature of life on these types of planets remains an open question for debate and discussion.

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[7] Potentially more if you consider systems with low stellar mass ratios which would exist in an engulfed habitable zone, as discussed in Section 4.1.

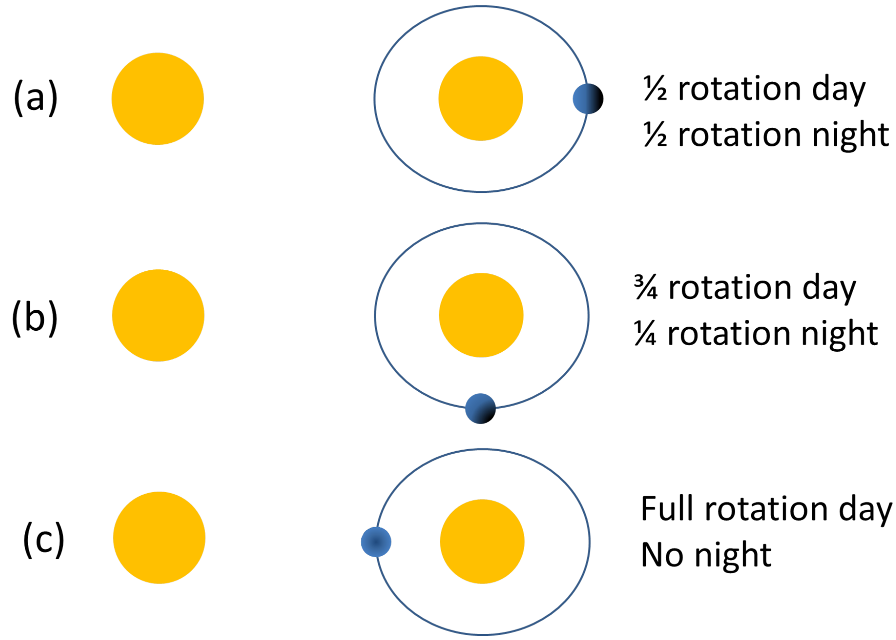

Figure 4.5: Overview of how a planet in an S-type binary orbit would experience day and night.

4.4 Lagrange Point Habitability

One could imagine a forming binary system with material being drawn into an orbit around the stable L4 and L5 Lagrange points created by the two young stars, and that this material could go on to potentially form a planet. Another more likely scenario would be a planet in a standard orbit being perturbed out of its orbit and then captured by the L4 point. Both of these scenarios would have to be tested further[8], but the focus of this study was to determine the habitability of these points once the planet is there.

4.4.1 Comparison to Expected Behavior

As discussed in Section 3.2.2, with exact expressions for planet temperature and location of the L4/L5 points, we derived an analytical expression for the expected temperature of the L4/L5 points. Our first task was to compare the results of our

[8] See Sections 5.2.2 and 5.2.6

simulations to the expected result of Equation 3.11. We were able to fit the lines of both Figure 4.6 and 4.7 to a function of the form

$$T = \frac{C}{\sqrt{R}}, \tag{4.4}$$

where C is a fitting coefficient. This has the exact same form as Equation 3.11, and therefore, we need only to compare the fitting coefficient C with the function $H(M, \eta)$[9]. As we can see from Table 4.1, our analytical result closely matches the simulation results.

$M(M_\odot)$	η	C	$H(\eta, M)$		$M(M_\odot)$	η	C	$H(\eta, M)$
0.20	1.0	45.5	45.5		1.1	0.1	292.5	298.9
0.50	1.0	101.5	101.5		1.25	0.25	296.5	307.7
1.00	1.0	186.0	186.1		1.5	0.5	300.5	309.3
2.00	1.0	341.0	341.4		2.0	1.0	341.0	341.4
4.00	1.0	626.0	626.1		3.0	2.0	551.0	567.2
10.0	1.0	1395	1396		6.0	5.0	1210	1250
20.0	1.0	2560	2560		11	10	2195	2241

Table 4.1: Comparison of the fitting coefficient C from Equation 4.4, and the function $H(M, \eta)$ from Equation 3.11 for various mass ratios, η (right), and combined stellar masses, $M = M_1 + M_2$ (left).

4.4.2 Discussion

As we can see in Figure 4.7, the lines for the lower mass ratios are all grouped together and the higher mass ratios spread. This implies that the mass of largest component of the system has greatest effect on the temperature of the L4 planet. As can be seen in Figure 4.6 and Table 3.1, binary separations which allow for habitable L4 points have the same value as the distance an Earth-like planet needs to be from a host star of mass $M_1 = M/2$ for a surface temperature of T=288K [10]. We also see that the range of acceptable binary separations for a habitable L4 point increases with mass and mass ratio. Overall, we see definite potential for L4 and L5 habitability over the entire range of masses and mass ratios.

[9] See Equation 3.12

[10] See Table 3.1

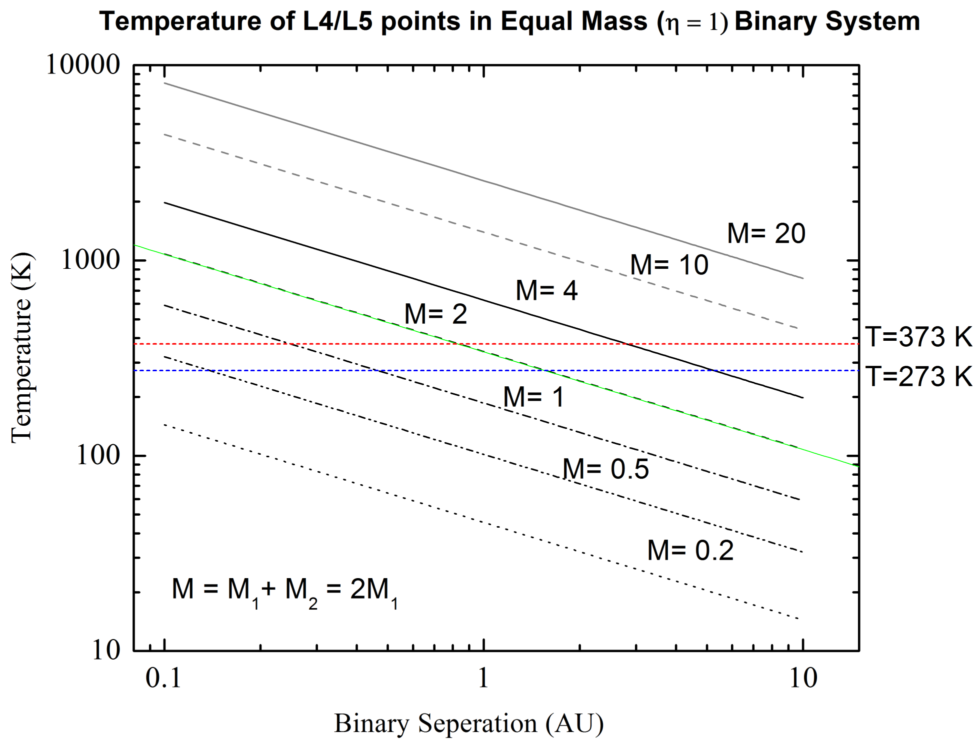

Figure 4.6: Surface Temperature of an Earth-like planet at the L4 point for various equal-mass binaries. The upper and lower horizontal lines represent the boiling point and freezing point of water, our criteria for planet habitability.

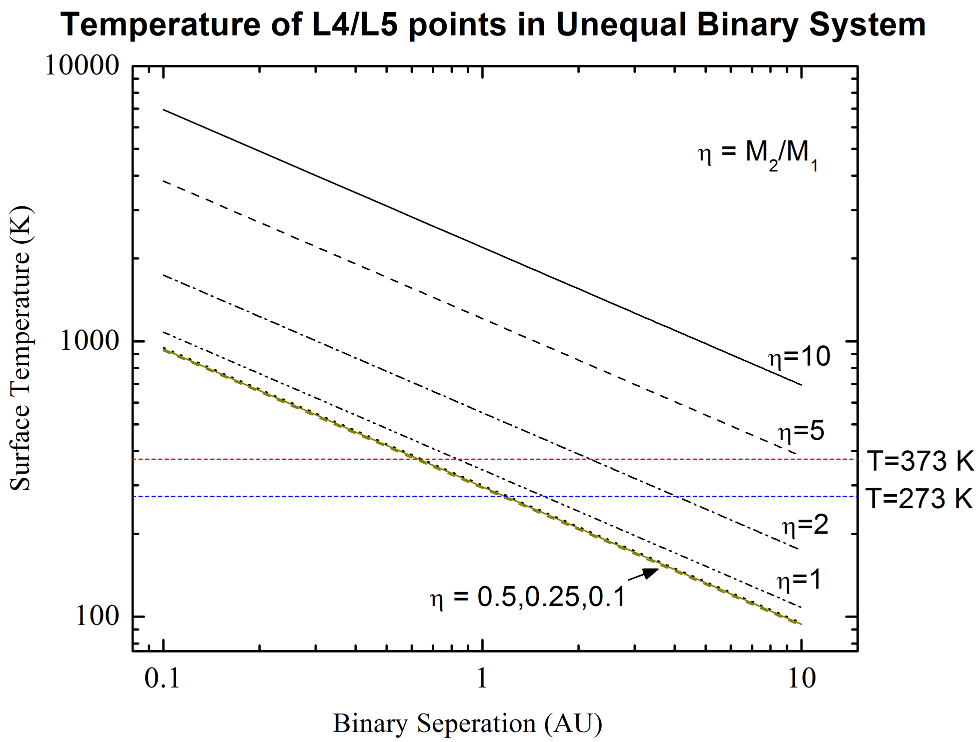

Figure 4.7: Surface Temperature of an Earth-like planet at the L4 point for $M_1 = 1$ and a variety of mass ratios. The upper and lower horizontal lines represent the boiling point and freezing point of water, our criteria for planet habitability.

Chapter 5

CONCLUSIONS AND FUTURE WORK

Using relatively simple computational models, were able to determine a good deal about the possibility of habitable planets in binary star systems. We were able to provide new discussion on the nature and behavior of planets in S-type orbits and at Lagrange Points in binary systems. In this final chapter, we summarize our results from Chapter 4, and discuss the next steps by suggesting future extensions to this study.

5.1 Summary of Results

We were able to classify the habitable zones of our systems into two classes: merged and unmerged. Lower limits for the critical separation for planetary habitability and stability in S-type binaries were determined for all equal mass ranges, and the higher mass ratios. Lower mass ratios, where the host star is much less massive than the perturbing star, resulted in engulfed habitable zones, which simply means the host star needed to stay within the habitable zone of the perturbing star to allow a planet around the host star to be habitable. We have also determined that a planet placed at the L4 or L5 Lagrange point could be habitable for a relatively small range of binary separations for each of the stellar masses and mass ratios used in this study. Also, we have determined that it would be unlikely that a planet would survive a transfer orbit between two stars.

The nuances of a planet's orbital and climatic response in an S-type system were also explored. We found that planets in systems with stars near the critical separation, we see large swings in the eccentricity, which get more dramatic the closer the stars get together and is the source of the eventual destabilization of the system. The spikes in the temperature caused by these sudden increases in eccentricity could end habitability on a planet without a good temperature regulation system. In addition, we showed that a planet in an S-type orbit would undergo a portion of its orbit with no true night, and another portion in which the day side of the planet would receive a heavier dose of radiation than average. We concluded that this abnormal day night cycle would likely cause more dramatic seasonal changes than we see on Earth. The magnitude of the climatic effects of this odd day/night cycle and whether or not it would greatly affect the habitability of the planet remains an open question.

5.2 Future Work

This was a much richer subject than originally anticipated, and there are numerous extensions to this study that could be pursued with additional time and resources. We have the capability of pursuing some of these extension almost immediately, and the only limitation to their completion was time. However, some of these extensions would require either additional research and calculations, or significant reworking of our simulation methods.

5.2.1 Longer-Term and More Efficient Simulations

Unfortunately, due to the large number of simulations run and our limitations on computing power and time, we were not able to evolve systems for longer than a million years. Ultimately, true tests of planetary habitability require simulations on the order of billions of years. There are a number of ways to rectify this situation. The obvious solution is to simply acquire more computing resources and run simulations for longer. However, according to our projections, a billion-year simulation could take on the order of months to complete using our code.

Writing, or obtaining, more efficient code would allow more simulation time to pass per unit real time. While MATLAB is a very powerful programing language, its nature as an interpreted language also makes it slow. Therefore, before pursuing this line of research any further, we would re-write our code in a compiled language such as Java or C. In addition, there are a number of pre-written and well-refined N-body codes that would be more efficient than what we have written. Implementing either of these would improve the speed of our simulations allowing for longer simulations to be run in a shorter amount of time.

The method we used for solving the N-body problem used a constant time step. However, using a constant time step is only efficient when your bodies stay on average the same distance away from each other, and in the most general cases, these distances would change. Having an small time step when bodies are far apart is inefficient and a waste of computing time, and on the other hand, when bodies have close encounters, a smaller time step is needed to ensure accurate results. Luckily, there are methods such as Bulirsch-Stoer Integration [2], that can do just that. For the next revision of our simulator, we would like to incorporate such a method.

5.2.2 Testing the Odds of Planet Formation

All of the systems we used in this study were preformed. This was sufficient for this study, but we also must answer the question of: Could these systems even form in the first place? Therefore, we would like to extend our simulations to model the formation of these binary systems to allow us to explore this question. There are a number of factors that affect the formation of an Earth-like planet, one of which is the proximity of a Jupiter-sized planet. The asteroid belt is a testament of how Jupiter-sized planets can disrupt planet formation [11]. This problem is compounded when you add a second star into the system, which has at least 10 times the mass of Jupiter. In addition, it would be prudent to investigate if there would be enough material from the original nebula to allow for planet formation after the formation of two stars.

5.2.3 Exploring Planetary Parameters

All of our simulated systems only contained planets with Earth-like conditions. Adjusting the mass or radius of our planets would not have had a large effect on the results of our simulations. However, we know both the greenhouse effect and the albedo of the planet have huge effects on the planet's surface temperature without changing its relative position. One could easily imagine a world with more ice or water on the surface, greatly increasing the albedo of the planet, or a planet like Venus with significantly more cloud cover causing both increased albedo and an increased greenhouse effect. After exhausting our experimental options with Earth-like worlds, we would like to explore how these worlds might fare in various binary systems.

5.2.4 Orbital Eccentricities

In order to reduce the number of parameters we were testing for this study, all of the orbits used in our simulations were initiated to be circular orbits. However we do have the capability to create any sort of eccentric orbit we wish, in which an orbit becomes more elliptical creating times where bodies are further and closer from each other rather than the constant separation a circular orbit provides[1]. Therefore, given the time and resources, we could indeed explore the effects of orbital eccentricities.

While the average overall surface temperature of a planet might not be affected by the eccentricity of its orbit, it could cause dangerous spikes in its surface temperature, as we have already seen in Section 4.3.1. A planet with an initialized eccentric orbit would be in a more dangerous position to be perturbed out of the habitable zone than its circular orbit counterpart. The first parameter explored should be the maximum allowable eccentricity for a planet to remain in the habitable zone, as it would give us a guideline for what kind of eccentricities should be tested for a given system's habitable zone.

Another parameter that remained unexplored was the effect eccentricity of the orbits of the stars about the system's barycenter has on the stability of the planetary orbits. Of particular interest would be the effect on the planet as the stars pass through their closest approach. Another problem to consider is the dynamic effect of binary eccentricity on the geometry of the habitable zone of a P-type binary. Unlike a in a circular orbit, an eccentric binary orbit would create a situation where the habitable

[1] See Figure 5.1

zone's geometry would change over time, causing issues for both the habitability as well as the stability of a planet in a P-type Orbit.

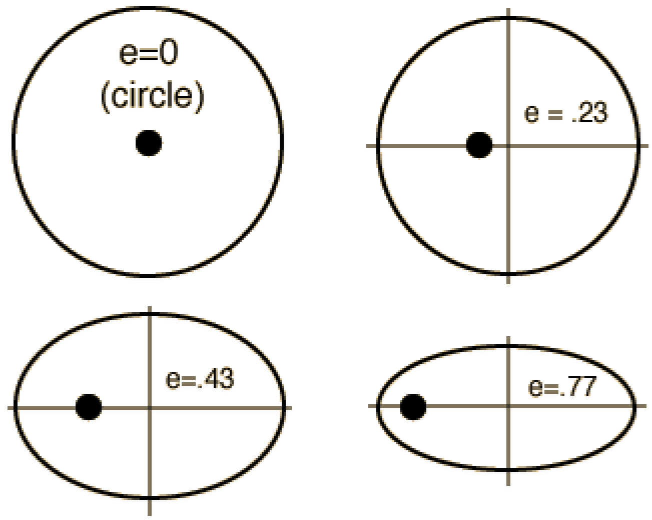

Figure 5.1: Examples of eccentricity of orbits, with $e = 0$ representing a perfectly circular orbit, and larger e representing increasingly elliptical orbits. Image Credit: HyperPhysics, Georgia State University.

5.2.5 P-type Habitability

For closely orbiting binaries, with $R << r_p$, we can estimate the habitable zone and planetary orbits of the system by replacing the binaries with a single star with the sum of the mass and luminosity of the two stars at the system's barycenter. However, wide-set P-type binaries provide a challenge for calculating habitable planetary orbits. In these merged habitable zones, we no longer always get the nice ring-shaped habitable zones we are used too, so putting a planet in a circular orbit about the system's barycenter would not lead to a habitable configuration. An eccentric orbit of some kind would be required for the planet to stay in the habitable zone of the system. Our study of P-type orbits would primarily focus finding the correlation between binary separation and the needed orbital eccentricity of the planet's orbit.

5.2.6 L4/L5 Orbit Stability

One difficulty we encountered was how to calculate suitable initial orbital velocities of a planet to place it in orbit around the system's L4 point. Once this problem is solved, we would be able to run long-term stability simulations for planets orbiting about a binary system's L4 point. This study has provided necessary constraints on the parameters of the binary systems for L4 habitability as a starting point for these simulations. However, as stated in Section 1.3, the stability of the L4 and L5 points depend on the mass ratios of the two main bodies. For the Coriolis force to make a planet orbit the L4 or L5 point, the stars need a mass ratio of at least 25. While not unrealistic, it is certainly not a common binary mass ratio, but luckily it was with high mass ratios that we saw more freedom for L4 habitability. Therefore, even with the mass ratio limitation, this would still be a worthwhile study to pursue.

5.3 Conclusion

Ultimately, we hope that the results of this study, as well as any future extensions of it, can be applied to future observations of multiple star systems and allow for immediate confirmation of an observed planet's or stellar system's potential habitability. This study has put definitive constraints on the habitability of planets in binary star systems. We have shown it would take very special circumstances for a habitable planet to survive in a binary system, yet the author of this study is more optimistic than others and believes that it should be possible to find habitable planets in binary systems. This study is just the beginning of this endeavor. We hope to continue to discover configurations and constraints that would lead to a habitable binary system, and further investigate the possibility of complex life arising in a binary system.

REFERENCES

[1] Beutler, G. Methods of Celestial Mechanics. Berlin: Springer, 2010. Print.

[2] Bodenheimer, Peter. Numerical Methods in Astrophysics: an Introduction. New York: Taylor and Francis, 2007. Print.

[3] Cornish, Neil J.. "The Lagrangian Points". Montana State University - Department of Physics. Retrieved 29 July 2011. < http://wmap.gsfc.nasa.gov/media/ContentMedia/lagrange.pdf >

[4] Giordano, Nicholas J., and Hisao Nakanishi. Computational Physics. Upper Saddle River, NJ: Pearson/Prentice Hall, 2006. Print.

[5] Haghighipour, Nader. Dynamical Stability and Habitability of Gamma Cephei Binary-Planetary System. ApJ 25, 2005.

[6] Haghighipour, Nader. Planets in Binary Star Systems. Dordrecht [etc.: Springer, 2010. Print.

[7] Kippenhahn, Rudolf, and A. Weigert. Stellar Structure and Evolution. Berlin: Springer-Verlag, 1994. Print.

[8] Mason, John. Exoplanets: Detection, Formation, Properties, Habitability. Berlin: Springer, in Association with Praxis Pub., Chichester, UK, 2008. Print.

[9] Seager, Sara, and Renee Dotson. Exoplanets. Tucson: University of Arizona, 2010. Print.

[10] Vazquez, M., Manuel., Enric Palle, and Rodriguez Pilar. Montanes. The Earth as a Distant Planet. New York: Springer, 2010. Print.

[11] Ward, Peter D. and Brownlee, Donald. Rare Earth. Copernicus Books: New York, 2000. Print.

[12] Williams, David R. "Planetary Fact Sheets." National Space Science Data Center (NSSDC), 06 Jan. 2005. Web. 18 Jan. 2012. <http://nssdc.gsfc.nasa.gov/planetary/planetfact.html>