

**ANALOG JOINT SOURCE-CHANNEL CODING FOR
NON-STANDARD SCENARIOS**

by

Bo Lu

A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical and Computer Engineering

Spring 2015

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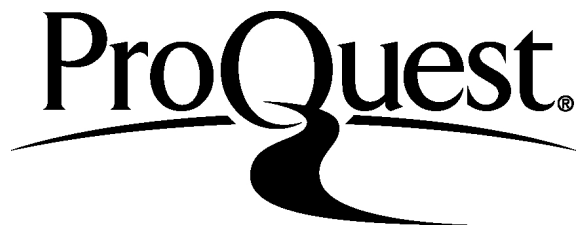
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by

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ABSTRACT

Recently, analog joint source-channel coding (JSCC) systems based on mappings have become one of the most promising schemes for transmitting discrete-time, continuous-amplitude sources (e.g., audio and video samples) over time-varying channels (e.g., wireless channels) under complexity and delay constraints. In contrast to traditional digital communication systems based on Shannon's separation principle, analog JSCC schemes are robust to changes in the channel quality, and do not require near infinite block lengths to approach the theoretical limits. As a result, the encoding/decoding complexity and delay can be greatly reduced compared with digital schemes.

Direct source-channel mappings take K discrete-time, continuous-amplitude symbols (a K dimensional vector in the source space) and map them directly into L discrete-time, continuous-amplitude channel symbols (an L dimensional vector in the channel space), achieving either bandwidth reduction ($K > L$) or bandwidth expansion ($K < L$). Most of the work on these schemes has dealt with the transmission of memoryless sources over noisy channels for point-to-point communications. In this dissertation, we focus on distributed scenarios and non-i.i.d. sources. Specifically, we first study the problem of transmitting multivariate correlated Gaussian samples over AWGN channels. A direct source-channel mapping designed utilizing power constrained channel optimized vector quantization (PCCOVQ) is proposed, taking into account the channel noise and power constraints. Simulation results show that, for bandwidth reduction, direct source-channel mappings achieve the theoretical limits for low channel signal to noise ratios (CSNR) when the samples are correlated. The performance is also quite close to the theoretical bound for higher CSNR.

Second, we work on the design of direct source-channel mappings using space-filling curves for the transmission of memoryless Gaussian samples over AWGN channels when side information is available at the receiver (Wyner-Ziv scenario). We first propose a 1:1 mapping for $K = 1$ and $L = 1$ using a periodic piece-wise linear curve. Then, we propose a 1: M bandwidth expansion mapping based on the use of existing space-filling curves in a periodic fashion. A simplified decoding algorithm is also proposed to reduce the cost of decoding without losing much performance compared to MMSE decoding. Then, we propose a flexible rate $K : L$ bandwidth expansion mapping for the Wyner-Ziv scenario by combining the proposed 1 : 1 and 1 : M schemes with the optimum energy allocation. These schemes are shown to outperform existing systems for a wide range of CSNRs, especially for high CSNRs and highly correlated side information.

We also study analog JSCC for the transmission of correlated Gaussian senders transmitted over separated AWGN channels. An asymmetric distributed coding scheme is proposed. One of the senders is encoded using standard mappings and the reconstructed symbol at the common receiver is used as distorted “side information” for the other sender, which is encoded by a periodic mapping designed for the point-to-point Wyner-Ziv scenario. A power allocation strategy is proposed to minimize the distortion while still satisfying the power constraint. Simulation results shows that the proposed schemes perform very close to the theoretical limits, outperform existing analog zero-delay mapping schemes, and are robust against CSNR mismatch and correlation mismatch.

Finally, we extend our research on analog JSCC to multiple access channel (MAC) scenarios. The proposed scheme contains a CDMA-like access scheme which converts the MAC into orthogonal channels. We showed that the proposed CDMA-like access scheme achieves capacity if optimum power allocation is used. Simulations show that by pairing with periodic mappings, the proposed scheme performs very close to the theoretical limits irrespective of the rate of each user when independent Gaussian sources are transmitted over a Gaussian MAC with side information at the receiver.

When transmitting correlated Gaussian sources over a Gaussian MAC, the performance of the proposed scheme is comparable to that of the best existing zero-delay schemes, but with the benefits of easiness of adaption to different rates.

Chapter 1

INTRODUCTION

1.1 Motivation

Analog signals (audio, image, video, etc.) are usually processed and transmitted using digital communications systems. A typical digital communications system for point-to-point transmission of analog sources is shown in Fig. 1.1. The analog source of interest produces a realization of a stochastic process, which is sampled and quantized into bits. Redundancy among these bits is eliminated by the source encoder, while the channel encoder introduces controlled redundancy to protect the information against the channel noise. Finally, the modulator specifies the constellation symbol that will be sent through the channel. At the receiver, the demodulator sends the distorted constellation symbol to the channel decoder, where the most likely codeword is generated. Then, the source decoder reconstructs the quantization index assuming that all the channel errors have been corrected by the channel decoder.

One important characteristic of standard digital communications systems based on separation is that source coding and channel coding are completely separated. This approach is indeed optimum for point-to-point communications, as long as source coding and channel coding themselves are optimum as proven in [77] (lossless source coding case) and in [7] (lossy source coding case). Therefore, the source code is designed assuming that the channel is error-free and the channel code is designed assuming that input bits do not have any redundancy. However, the analog source has to be source encoded up to the desired rate/distortion pair (e.g. using very powerful vector quantization) [51, 75, 88], and then capacity approaching channel codes such as turbo codes [8] or LDPC [32] codes should be applied. This means that the price to pay to achieve

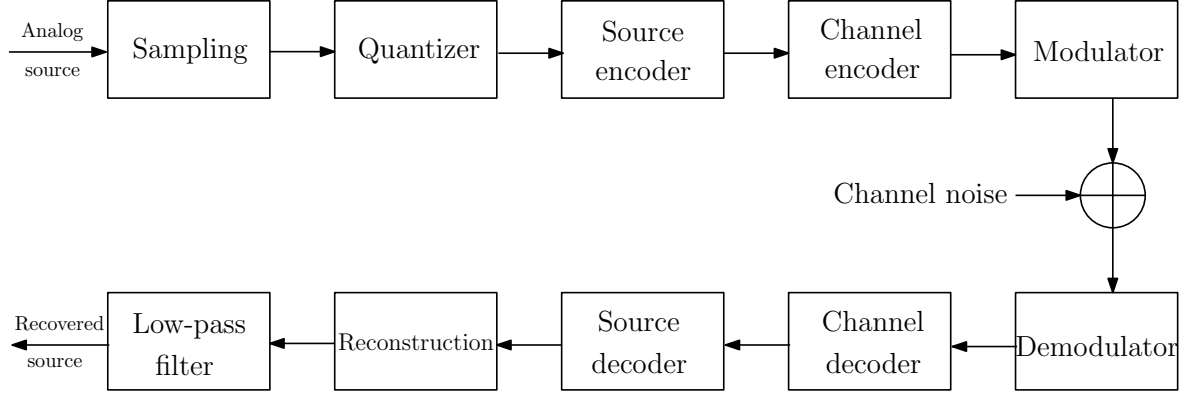


Figure 1.1: Digital communications system based on separation between source and channel coding for the transmission of analog sources.

near-optimum performance is very high encoding/decoding complexity and significant delays, since any capacity approaching channel code (and any quasi-optimal vector quantizer) requires long block lengths.

The aforementioned requirements have some drawbacks. For applications that require real time communications, the communication quality has to be sacrificed due to the delay constraint. Moreover, these systems lack robustness and adaptivity when transmitting analog sources. Capacity approaching codes such as LDPC codes have steep BER curves, meaning that above a certain channel signal to noise ratio (CSNR), the BER drops abruptly towards zero. This means that such codes behave well provided that the actual CSNR is at least as high as the design CSNR for the code, but the system will quickly break down below the design CSNR because source coding is very sensitive to the errors from the channel, and thus it requires channel codes to correct all the channel errors. Moreover, if the actual CSNR increases above the design CSNR, the source is over-protected meaning that bits that could have been used in the source coder are wasted on unnecessary error-correction and the reconstructed quality will not improve, but will be limited by the irreversible quantization error. Thus, digital communications system have to be redesigned if channel conditions change. For time-varying channel conditions and imperfect channel state information (CSI), which is quite common in wireless channels and in underwater communication channels, this is

an important issue.

These limitation in digital communications systems based on separation between source and channel coding can be alleviated by employing joint source-channel coding (JSCC). This can be done in many different ways, with the degree of optimization varying from a simple rate-allocator added to the separated source and channel encoder to a fully combined encoder where there is no distinction between the reduction of redundancy in the source encoder and the addition of error protection in the channel encoder. The idea is that in some situations (e.g., when the encoder/decoder complexity is limited) digital systems based on JSCC can present some advantage over systems based on separation. Fig. 1.2 shows a digital JSCC system where, instead of traditional separated source and channel coding, the source is just encoded using a single code, which, depending on the desired information rate, introduces redundancy or performs compression [10, 55, 57]. In this approach, the statistics of the source are utilized in the encoding process and/or exploited (and sometimes also estimated) in the decoding process, resulting in a performance close to the theoretical Shannon limits [10, 53, 54, 55, 57, 87]. Even in the case of practical systems based on separation, JSCC is relevant. Notice that in practical situations, the source encoder is not able to eliminate all redundancy existing in the source, and thus exploiting this residual redundancy in the decoding process will lead to performance improvements [2].

Although digital JSCC systems can improve spectral efficiency upon digital systems based on separation, in practical situations where finite block lengths are used, the overall complexity is still quite high and the system still suffers from lack of robustness to fast changing channel conditions and when accurate CSI is not available. In [37], the author shows that for the case of i.i.d Gaussian sources and AWGN channels, both of the same bandwidth, it is optimum to just send the source directly through the channel (using optimal scaling factors at the transmitter and receiver). In that sense, it is said that Gaussian sources are perfectly matched to Gaussian channels [33, 34, 35, 36, 62]. Fig. 1.3 shows the linearity of the direct transmission. This scheme is elegant, not only because it is optimal in terms of spectral efficiency, but

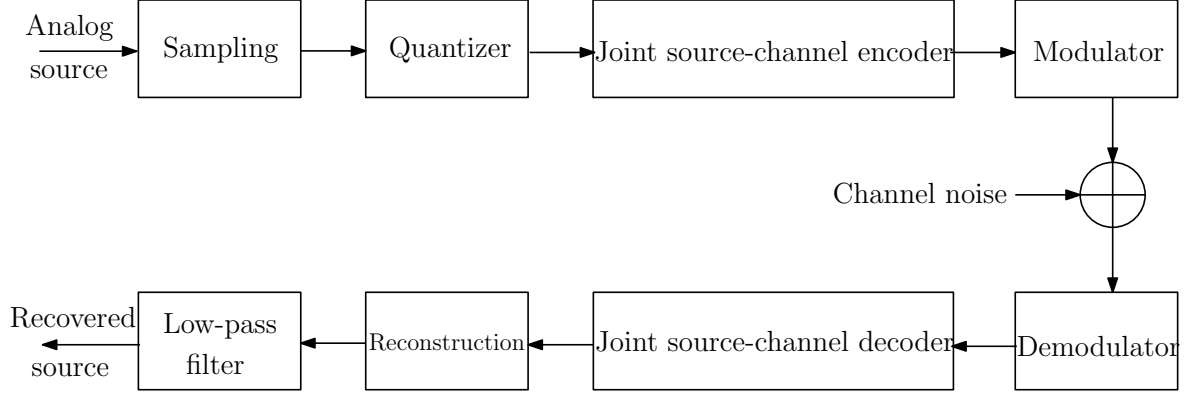


Figure 1.2: Digital communications system based joint source-channel coding for the transmission of analog sources.

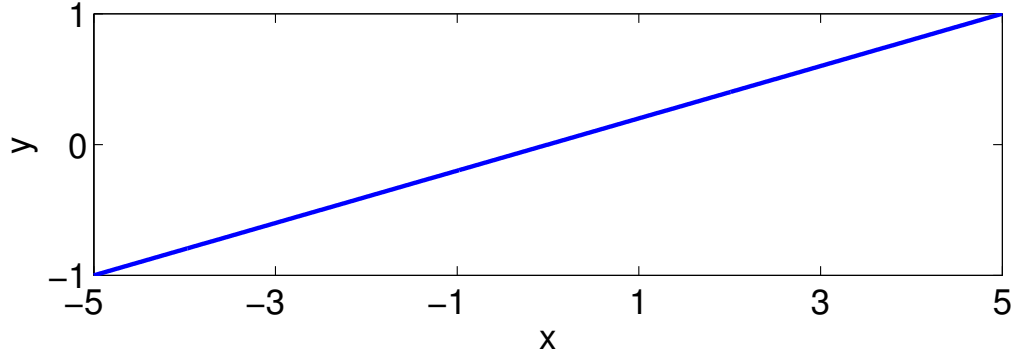


Figure 1.3: Direct transmission scheme: source x is linearly scaled (y) according to the power constraint before transmission through the channel.

also because it requires low complexity (simple encoding/decoding), zero delay (very short block lengths) and has high robustness. Fig. 1.4 compares the robustness of the direct transmission scheme using binary pulse amplitude modulation (PAM) with that of an optimal digital system, and also provides the curve of the optimal performance theoretically attainable, OPTA [6]. Although both systems achieve the theoretical limit at 20 dB, which is the CSNR used for the design, for CSNRs below 20 dB the digital system breaks down completely, while direct transmission shows graceful degradation. When the CSNR is above 20 dB, the digital system does not improve while direct transmission shows graceful improvement.

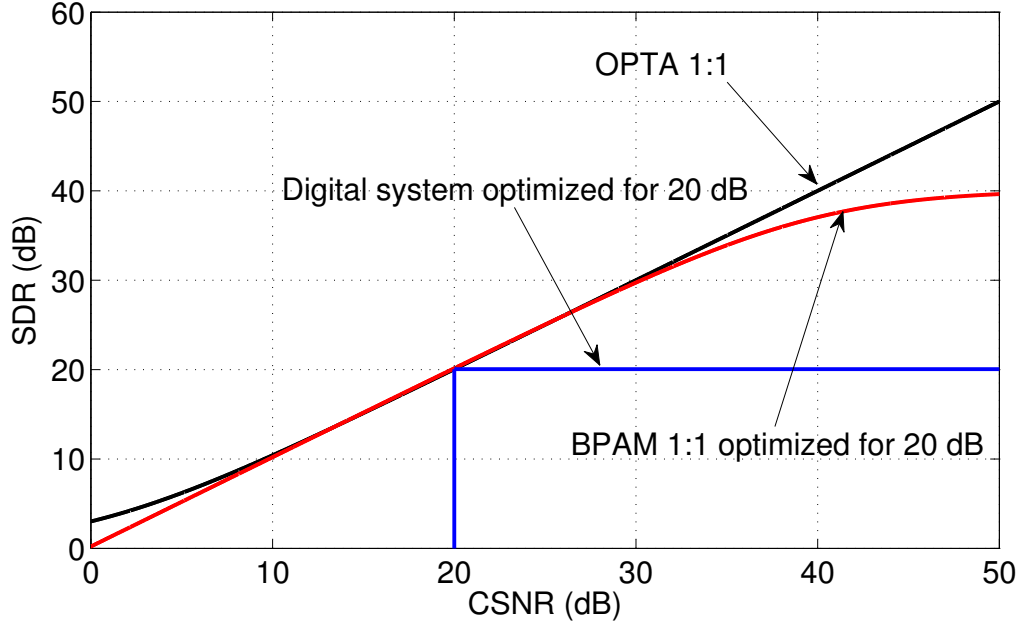


Figure 1.4: An i.i.d Gaussian source is transmitted directly over an AWGN channel. The performance of the direct transmission scheme (BPAM), and that of a digital system based on the separation principle are shown and compared with the theoretical limit. Both direct transmission and the digital system are optimized for 20 dB. SDR stands for signal to distortion ratio.

1.2 Non-Linear Mappings

As we mentioned earlier, if i.i.d. Gaussian samples are transmitted directly over an AWGN channel, then a simple uncoded system can achieve Shannon's theoretical limit for any given value of CSNR. The idea of direct source-channel mappings is to consider both the discrete time analog source and the channel signal as points in vector spaces of dimensions K and L , respectively, so that the source space is mapped directly onto the channel space. This is illustrated in Fig. 1.5, where a source point X is projected directly onto the channel space using a given function. The channel symbol Y is sent and the received observation $Z = Y + N$ is decoded to obtain an estimate $\hat{X}$ of the original source signal. If we look at Fig. 1.1 again, we notice that standard digital communication systems based on the separation principle can be actually considered as a special case of the direct source-channel mapping if we consider quantizer, source

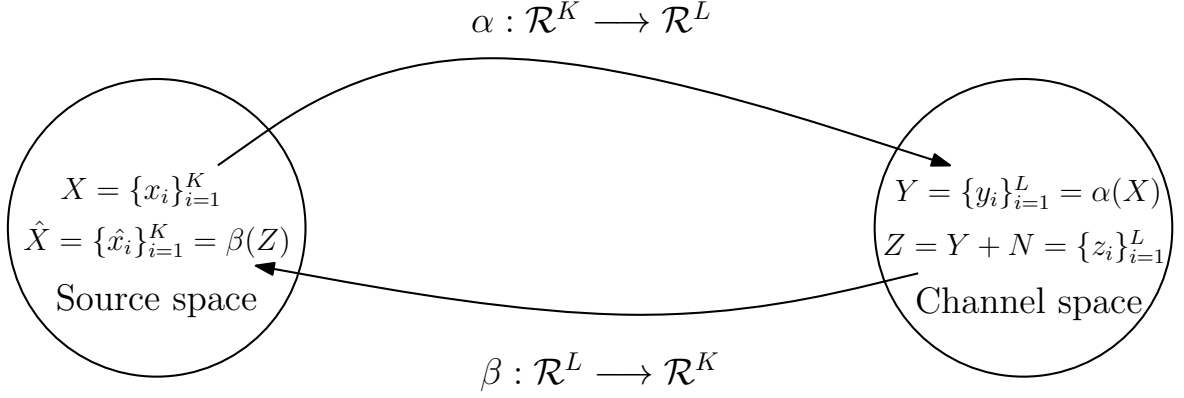


Figure 1.5: Direct source channel mapping between the source and channel spaces. The function α performs a projection from the source space of dimension K to the channel space of dimension L . The function β brings the channel signal back into the source space and provides an estimate of the transmitted source signal.

coding, channel coding and modulation as a single transformation, although this mapping is very complex. If $K < L$, the source dimension is expanded onto the channel, meaning that redundant dimensions are available, and can be used for noise reduction (error control). If $K > L$, the dimension of the source is reduced, meaning that the information content of the source must be reduced before transmission, and so lossy compression is necessary. In the following, an operation where a source of dimension K is mapped onto a channel of dimension L is called a $K : L$ mapping with rate $\frac{K}{L}$.

The theoretical limit for the transmission of discrete-time continuous-amplitude sources over noisy channels can be derived by evaluating the distortion-rate function [79] of the source at the rate equal to the channel capacity [77]. This limit is called optimal performance theoretically attainable (OPTA), and it is defined as the maximum achievable signal to distortion ratio (SDR) for a given CSNR and a given rate. The OPTA is the theoretical upper bound of any communication system, digital or analog, which transmits K source samples using the channel L times.

An example of a nonlinear direct source-channel mapping that approaches optimally is the mapping designed using Power Constrained Channel Optimized Vector Quantization (PCCOVQ) [26, 30, 31]. The idea of direct source-channel mapping using

PCCOVQ is that the discrete-time continuous-amplitude source is quantized and the index is directly mapped to a channel symbol in the channel space. PCCOVQ uses a modified generalized Lloyd algorithm that takes channel and power constraint into account to train the codebook, and has been shown to be able to find good mappings for bandwidth reduction. Moreover, the trained codebooks of PCCOVQ have regular structures that can be approximated by parametric curves and surfaces. This is of interest, since the mathematical equation of the curves makes it very simple to adapt them to varying channel conditions by merely changing their coefficients. In [30], a direct source-channel mapping using PCCOVQ for transmitting i.i.d Gaussian sources over AWGN channels when $K = 2$ and $L = 1$ was proposed. The resulting performance is only about 1 dB away from the theoretical limit. PCCOVQ works best for the bandwidth reduction case. This is probably because in the case when $K < L$, the same points in the signal space will have multiple points in the channel space. This implies that decoding of a noise contaminated channel symbol will not always give the lowest possible reconstruction error.

When the number of centroids provided by PCCOVQ is large, the points in the source and channel space can be considered as an approximation of a curve. Thus, if the source points are projected onto the space-filling curve that resembles the centroids of PCCOVQ, we should get similar results. Therefore, instead of using PCCOVQ, a mathematical function describing the space-filling curve can be directly utilized for the mapping. The idea of using space-filling curves dates back to [64, 78]. This type of mapping is also called Shannon-Kotel'nikov mapping, and it has been shown to perform quite well for memoryless sources in point-to-point communications systems with zero delay and low complexity, and to be quite robust against varying channel conditions [9, 11, 16, 17, 27, 45, 46, 47, 48, 50, 52, 65, 66, 74, 81]. Analog JSCC based on Shannon-Kotel'nikov mappings are preferred to PCCOVQ because the whole Shannon-Kotel'nikov mapping is represented by a mathematical equation with few parameters, and, therefore, it is easier to adapt to varying channel conditions by merely

changing a few coefficients in the corresponding equation, and it is also easier to optimize and analyze than the mapping based on PCCOVQ. They are also more practical to be used for high CSNR and high dimensional cases, as the training complexity of PCCOVQ increases very fast when CSNR and the dimension increase. Recently, Shannon-Kotel'nikov mappings have been successfully applied to applications such as compressive sensing (CS) [5, 12, 13, 14, 19] in [49, 38, 39] and underwater communications in [41].

Spiral-like curves are a particular case of Shannon-Kotel'nikov mappings, used for the transmission of Gaussian sources over AWGN channels. Specifically, when $K = 2$ and $L = 1$ or $K = 1$ and $L = 2$, the curve is shown in Fig. 1.6 and can be defined in the following parametric form [45, 47]

$$\begin{cases} x_1 = \frac{\Delta}{\pi}\theta \sin \theta \\ x_2 = \frac{\Delta}{\pi}\theta \cos \theta \end{cases} \quad \text{for } \theta \geq 0, \quad (1.1)$$

$$\begin{cases} x_1 = -\frac{\Delta}{\pi}\theta \sin \theta \\ x_2 = \frac{\Delta}{\pi}\theta \cos \theta \end{cases} \quad \text{for } \theta < 0,$$

where Δ is the distance between two neighboring spiral arms and θ is the angle from the origin to the point (x_1, x_2) on the curve.

Notice that in the curve described above there is a one-to-one correspondence between parameter θ and the points (x_1, x_2) on the curve, so that the curve gradually fills in the whole two-dimensional space as the absolute value of θ grows.

1.2.1 2 : 1 mapping

1.2.1.1 Encoder

When $K = 2$ and $L = 1$, the 2 : 1 bandwidth reduction mapping using spiral-like curves is based on projecting a pair of source samples $X = (x_1, x_2)$ onto the curve

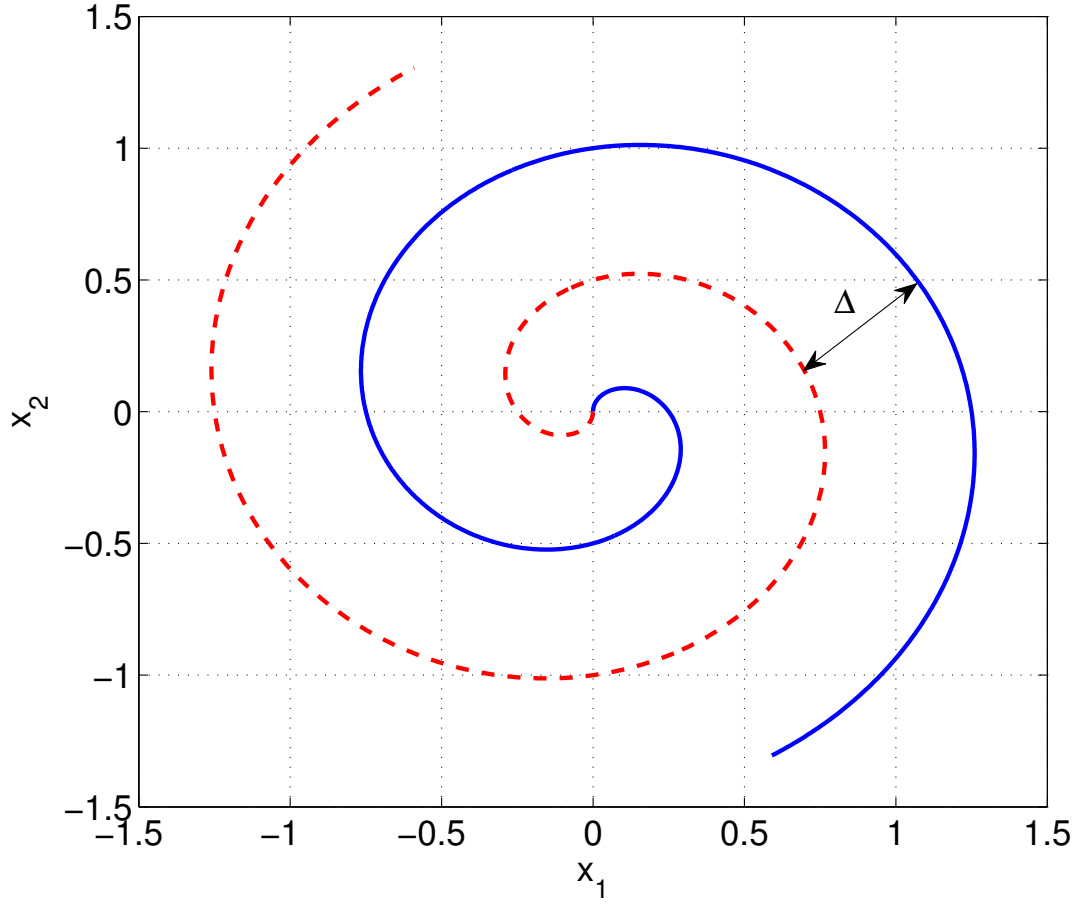


Figure 1.6: Spiral-like curve performing a 2 : 1 mapping.

by finding the closest point on the curve, and representing the projection using the parameter θ . Specifically, the mapping function $M_{\Delta}()$ is defined as

$$\begin{aligned}\hat{\theta} &= M_{\Delta}(X) \\ &= \arg \min_{\theta} \left(x_1 \pm \frac{\Delta}{\pi} \theta \sin \theta \right)^2 + \left(x_2 - \frac{\Delta}{\pi} \theta \cos \theta \right)^2.\end{aligned}\tag{1.2}$$

Then, an invertible function of $\hat{\theta}$ is normalized in order to satisfy the power constraint and transmitted through the AWGN channel. The invertible function is

$$T_{\alpha}(x) = x^{\alpha},\tag{1.3}$$

where $\alpha \in (0, 2]$ is a variable controls the distribution of channel symbol. In general α needs to be optimized for different CSNRs in order to make the distribution of channel symbol as close to Gaussian as possible, but we will use $\alpha = 2$ in this dissertation. In this case, the transmitted symbol $y = \hat{\theta}^2$ is proportional to the length of the curve.

1.2.1.2 Decoder

The received observation at the decoder can be expressed as

$$r = M_{\Delta}(X)^2 + \sqrt{\gamma}n, \quad (1.4)$$

where γ is the normalization factor.

ML decoding

Given the received symbol r , the estimate $\hat{X}_{ML}$ is obtained as the source pair $X = (x_1, x_2)$ belonging to the curve and satisfying

$$\begin{aligned} \hat{X}_{ML} &= \arg \max_{X \in \text{curve}} p(r|X) \\ &= X | X \in \text{curve and } M_{\Delta}(X)^2 = r. \end{aligned} \quad (1.5)$$

ML decoding is equivalent to first get $\hat{\theta}' = \text{sign}(r)\sqrt{|r|}$, and then performing inverse mapping on $\hat{\theta}'$ according to (1.1).

MMSE decoding

Although ML decoding is very simple and performs very well for high CSNRs, it is in general not optimal for the mean square error distortion criterion (MSE). The estimate that minimizes the MMSE criterion can be expressed as

$$\begin{aligned} \hat{X}_{MMSE} &= E[X|r] = \int X p(X|r) dX \\ &= \frac{1}{p(r)} \int X p(r|X) p(X) dX \\ &= \frac{\int X p(r|X) p(X) dX}{\int p(r|X) p(X) dX}. \end{aligned} \quad (1.6)$$

Since the conditional probability, $p(r|X)$, involves the mapping function $M_\Delta(X)$ which is discontinuous and highly non-linear, (1.6) can only be calculated numerically. The algorithm that calculates (1.6) numerically is as follows: First, we discretize X using a uniform step, and calculate the mapping value for each discretized point according to the encoding process described in Section 1.2.1. By doing so, we obtain a discretized version of $p(r|X)$. We also need to apply the same discretization procedure to X and $p(X)$ so that the calculation of the above integral consists of multiplication and addition operations. The only downside of the MMSE decoding is increased complexity.

1.2.2 1 : 2 mapping

1.2.2.1 Encoder

When $K = 1$ and $L = 2$, we first use the stretching function [47], which maps the source x to θ

$$\theta(cx) = \begin{cases} \sqrt{\frac{cx}{0.16\Delta}} & x \geq 0 \\ -\sqrt{-\frac{cx}{0.16\Delta}} & x < 0. \end{cases} \quad (1.7)$$

where c is the normalization factor. Then $Y = (y_1, y_2)$ is mapped from θ by (1.1) with x_1, x_2 replaced by y_1, y_2 , respectively.

1.2.2.2 Decoder

The received observation at the decoder can be expressed as

$$R(r_1, r_2) = Y + N, \quad (1.8)$$

ML decoding

Given the received symbols R , the estimate $\hat{x}_{ML}$ is obtained by projecting R on to the curve to get $\hat{\theta}$

$$\begin{aligned} \hat{\theta} &= M_\Delta(R) \\ &= \arg \min_{\theta} (r_1 \pm \frac{\Delta}{\pi} \theta \sin \theta)^2 + (r_2 - \frac{\Delta}{\pi} \theta \cos \theta)^2. \end{aligned} \quad (1.9)$$

Then the ML estimate is

$$\hat{x}_{ML} = \text{sign}(\hat{\theta})0.16\hat{\theta}^2\Delta/c \quad (1.10)$$

MMSE decoding

The MMSE decoding is expressed as

$$\begin{aligned} \hat{x}_{MMSE} &= E[x|R] = \int xp(x|R)dx \\ &= \frac{1}{p(R)} \int xp(R|x)p(x)dx \\ &= \frac{\int xp(R|x)p(x)dx}{\int p(R|x)p(x)dx}. \end{aligned} \quad (1.11)$$

The same numerical algorithm as in 2 : 1 mappings is applied here. However, since the integral is now in 1-D as opposed to 2-D in the 2 : 1 case, the complexity is reduced exponentially.

Notice that the encoding in the 2 : 1 system is equivalent to ML decoding in the 1 : 2 system, while the encoding in the 1 : 2 system is equivalent to ML decoding in the 2 : 1 system.

1.2.3 Shannon-Kotel'nikov mapping design guidelines

Although there is no systematic way to construct good Shannon-Kotel'nikov mappings, some design rules have been discussed in the literature [70]:

1. Cover the source space: In order to minimize the projection error incurred when mapping a source space of high dimension to a channel space of lower dimension, the mapping must cover the entire source space, so that every source vector has a projection point on the curve as close as possible.
2. Minimize the channel signal power: In order to minimize the average channel power, source vectors with high probability should be mapped to channel vectors with low amplitude.
3. Increase robustness: In order to avoid large error for some channel noise, vectors that are close in the channel space should correspond to vectors close in the source space. The opposite, however, is not necessary.

1.3 Scope of the Dissertation

Shannon-Kotel'nikov mappings have so far been mostly applied in point-to-point communications. However, they have the potential to be specially relevant in scenarios where the application of standard digital coding techniques may complicate the decoding process. For instance, in a sensor network the correlation among sensors may be defined in a very simple way in the analog domain (at the sample level), so that its exploitation using direct source-channel mapping would be straightforward. On the other hand, in order to optimize performance using a digital system, it would be necessary to apply Slepian-Wolf coding techniques [80] over the digitalized versions of the signals of interest, which requires the use of the proper correlation models at the bit/symbol level [56, 58, 59, 84, 85, 86, 87]. This is an important constraint in practical systems: even if the correlations are very simple at the sample level, they may be very complicated (and thus difficult to exploit) at the bit/symbol level of the digitalized versions of the waveforms.

Most of the previous work on the design of source-channel mappings has focused on point-to-point communications of memoryless discrete-time continuous-amplitude sources. In this dissertation, we move one step further, focusing mainly on the design of mappings for the transmission of multivariate correlated Gaussian sources over AWGN channels and for the transmission of memoryless Gaussian sources with side information available at the receiver, as well as on more general distributed JSCC schemes.

1.4 Organization of the Dissertation

The remainder of the proposal is organized as follows. In Chapter 2, we propose an analog JSCC bandwidth reduction scheme using PCCOVQ for the transmission of multivariate Gaussian sources over AWGN channels. Next, we study the problem of designing bandwidth expansion JSCC schemes for the transmission of memoryless Gaussian samples over Additive White Gaussian Noise (AWGN) channels when side information is present at the receiver (Wyner-Ziv scenario) in Chapter 3. We first propose $1 : 1$ and $1 : M$ Shannon-Kotel'nikov mappings for the Wyner-Ziv scenario,

then we combine both mappings to construct the analog JSSC schemes for flexible rate transmission of Gaussian sources with side information.

In Chapter 4, we investigate the problem of transmitting correlated Gaussian sources over separated Gaussian channels using analog JSSC schemes. An asymmetric analog mapping scheme is proposed for the transmission of quadratic Gaussian sources, where one of the sources is transmitted using a standard mapping designed for point-to-point communications and the other source is transmitted using a periodic mapping. We also proposed a simplified decoding algorithm to reduce the computational complexity, especially for bandwidth reduction and high correlation.

Finally, in Chapter 5 we focus on the design of analog JSSC schemes for the Gaussian multiple access channel under different scenarios. In the proposed scheme, each user is first encoded using a space-filling curve and then a CDMA-like access scheme converts the multiple access channel into several orthogonal channels. We also proposed a simplified decoding algorithm for the proposed scheme based on the simplified algorithm proposed in Chapter 4.

Chapter 2

NON-LINEAR MAPPINGS FOR TRANSMISSION OF MULTIVARIATE GAUSSIAN SOURCES OVER NOISY CHANNELS

In this chapter, we investigate the design of zero-delay non-linear bandwidth reduction JSCC schemes for transmission of multivariate correlated Gaussian sources over average power constrained AWGN channels. The non-linear direct source-channel mappings are designed by utilizing PCCOVQ. Practical examples for $2 : 1$, $3 : 1$ and $4 : 2$ systems are provided, and simulation results are compared with the respective OPTA.

2.1 Introduction

Previous work in the literature of analog JSCC design focuses on sources that generate independent and identically distributed (i.i.d.) samples. However, in some applications, such as wireless sensor networks (WSN) and 3D cameras, it is likely that the samples are correlated. In this chapter, we focus on the analog transmission of discrete-time continuous-amplitude multivariate correlated Gaussian sources over average power constrained AWGN channels.

Since, there is no systematic way of constructing good space-filling curves for different source/channel statistics, and specifically when the source samples are correlated, we will utilize PCCOVQ, mentioned in the previous chapter and described in detail later, to derive well performing analog mappings for the transmission of multivariate correlated Gaussian sources. The resulting mappings are very different from the ones optimized for i.i.d. sources. We present the results for $2 : 1$, $3 : 1$ and $4 : 2$ bandwidth reduction systems and compare them with the corresponding OPTA.

The remainder of this chapter is organized as follows. In next section we describe the system model, including the source model and the channel model. Section 2.3 provides the derivation of the OPTA for multivariate Gaussian sources over AWGN channels. In Section 2.4, we review the PCCOVQ algorithm. Finally, simulation results and the conclusion are given in Section 2.5 and 2.6, respectively.

2.2 System Model

2.2.1 Source model

In this chapter, we consider the transmission of K correlated Gaussian samples, proceeding from a multivariate Gaussian source $X = \{x_i\}_{i=1}^K$ in $\mathcal{R}^K$. Without loss of generality each sample x_i has zero mean and a variance σ_x^2 . Specifically, the samples are generated following a Markov process defined as

$$x_k = \begin{cases} \rho \times x_{k-1} + \sqrt{1 - \rho^2} \times i_k & \text{if } 2 \leq k \leq K \\ i_k & \text{if } k = 1, \end{cases} \quad (2.1)$$

where $i_k \sim \mathcal{N}(0, \sigma_x^2)$, independent of x_{k-1} , and ρ is the correlation coefficient.

2.2.2 Encoder/Decoder

In analog JSCC communications systems based on direct source-channel mappings, K discrete-time continuous-amplitude source symbols X are mapped directly into L discrete-time continuous-amplitude channel symbols $Y = \{y_i\}_{i=1}^L$ in $\mathcal{R}^L$ by an encoder function $\alpha : \mathcal{R}^K \rightarrow \mathcal{R}^L$ (see Fig. 2.1). The coding rate is defined as $R_c = K/L$ ($K : L$ mapping). In this chapter, we only consider the bandwidth reduction case ($R_c > 1$).

When transmitted over the channel, Y is corrupted by additive white Gaussian noise, $N = \{n_i\}_{i=1}^L$ with each symbol $n_i \sim \mathcal{N}(0, \sigma_n^2)$. The received vector

$$\hat{Y} = Y + N \quad (2.2)$$

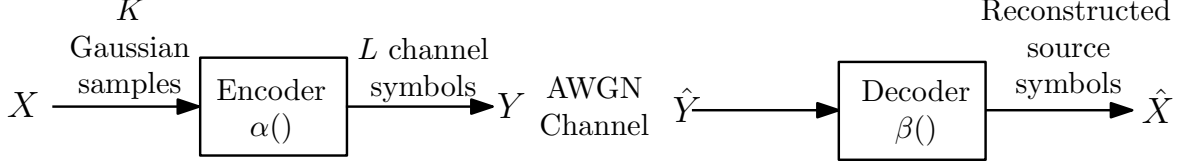


Figure 2.1: Analog JSCC communications system.

is processed by the decoder function $\beta : \mathcal{R}^L \rightarrow \mathcal{R}^K$ (see Fig. 2.1). The decoding distortion is defined as

$$D = E[\|X - \hat{X}\|^2]/K \quad (2.3)$$

and the power constraint is defined as

$$P = E[\|Y\|^2]/L \leq P_{max}. \quad (2.4)$$

Because there is no bit representation in this analog communication system, bit error rate (BER) is not applicable. Rather, we measure the system performance in terms of signal to distortion ratio (SDR) defined as

$$SDR = 10\log_{10}(\sigma_x^2/D). \quad (2.5)$$

The channel signal to noise ratio (CSNR) is defined as

$$CSNR = 10\log_{10}(P/\sigma_n^2). \quad (2.6)$$

2.3 OPTA for Multivariate Gaussian Sources in AWGN Channel

It is well known that the theoretical limit is specified by

$$R_c R(D) < C, \quad (2.7)$$

where R_c is the code rate, $R(D)$ is the rate distortion function and C is the channel capacity. When an AWGN channel is considered, the channel capacity C is given by [18]

$$C = \frac{1}{2} \log(1 + \frac{P}{\sigma_n^2}), \quad (2.8)$$

where σ_n^2 is the power of the channel noise and P is the power of the channel symbols.

For a multivariate Gaussian source $N(\mu_{l_x}, \Sigma_{l_x l_x})$ of length l_x , its rate distortion function can be represented parametrically as [63]

$$\begin{cases} R(\theta) = \frac{1}{l_x} \sum_{i=0}^{l_x-1} \max(0, \frac{1}{2} \log \frac{\lambda_{l_x,i}}{\theta}) \\ D(\theta) = \frac{1}{l_x} \sum_{i=0}^{l_x-1} \min(\theta, \lambda_{l_x,i}), \end{cases} \quad (2.9)$$

where R is the rate of the source, D is the mean squared error defined before and $\lambda_{l_x,i}$ are the eigenvalues of covariance matrix $\Sigma_{l_x l_x}$.

Next, we will provide the derivation of the OPTA for 2 : 1, 3 : 1 and 4 : 2 bandwidth reduction systems. Notice that in the derivations below the OPTA is obtained as an expression for CSNR dependent on the distortion, D , while the range defining each case is provided in terms of SDR. From the expressions below, it is straightforward to obtain the SDR vs CSNR curves defining OPTA.

2.3.1 2 : 1 bandwidth reduction

In this case, $l_x = 2$, and the covariance matrix is

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x^2 \\ \rho\sigma_x^2 & \sigma_x^2 \end{bmatrix}, \quad (2.10)$$

whose eigenvalues are $\lambda_{2,0} = \sigma_x^2(1 + \rho)$ and $\lambda_{2,1} = \sigma_x^2(1 - \rho)$. According to (2.9), the rate distortion function can be calculated by reverse water-filling on the eigenvalues.

We have two cases since there are two different ordered eigenvalues:

Case 1

When $0 < \theta < \lambda_{2,1}$, which corresponds to

$$SDR > 10 \log_{10} \left(\frac{1}{1 - \rho} \right), \quad (2.11)$$

(2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{2} \sum_{i=0}^1 \frac{1}{2} \log \frac{\lambda_{2,i}}{\theta} \\ D(\theta) = \frac{1}{2} \sum_{i=0}^1 \theta. \end{cases} \quad (2.12)$$

From (2.7), we get

$$CSNR > 10 \log_{10} \left(\left(\frac{\sigma_x^2}{D} \right)^2 (1 - \rho^2) - 1 \right). \quad (2.13)$$

Case 2

When $\lambda_{2,1} < \theta < \lambda_{2,0}$, which corresponds to

$$0 < SDR < 10 \log_{10} \left(\frac{1}{1 - \rho} \right), \quad (2.14)$$

(2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{2} \times \frac{1}{2} \log \frac{\lambda_{2,0}}{\theta} \\ D(\theta) = \frac{1}{2} \times (\theta + \lambda_{2,1}). \end{cases} \quad (2.15)$$

From (2.7), we get

$$CSNR > 10 \log_{10} \left(\frac{2 - 2 \frac{\sigma_x^2}{D}}{(1 - \rho) \frac{\sigma_x^2}{D} - 2} \right). \quad (2.16)$$

2.3.2 3 : 1 bandwidth reduction

In this case, $l_x = 3$, and the covariance matrix is

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho \sigma_x^2 & \rho^2 \sigma_x^2 \\ \rho \sigma_x^2 & \sigma_x^2 & \rho \sigma_x^2 \\ \rho^2 \sigma_x^2 & \rho \sigma_x^2 & \sigma_x^2 \end{bmatrix}, \quad (2.17)$$

whose eigenvalues are $\lambda_{3,0} = \sigma_x^2(1 - \rho^2)$, $\lambda_{3,1} = \frac{1}{2} \sigma_x^2(\rho^2 + 2 + \rho\sqrt{\rho^2 + 8})$ and $\lambda_{3,2} = \frac{1}{2} \sigma_x^2(\rho^2 + 2 - \rho\sqrt{\rho^2 + 8})$. We have three cases since there are three different ordered eigenvalues:

Case 1

When $0 < \theta < \lambda_{3,2}$, which corresponds to

$$SDR > 10\log_{10}\left(\frac{2}{\rho^2 + 2 - \rho\sqrt{\rho^2 + 8}}\right), \quad (2.18)$$

(2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{3} \sum_{i=0}^2 \frac{1}{2} \log \frac{\lambda_{3,i}}{\theta} \\ D(\theta) = \frac{1}{3} \sum_{i=0}^2 \theta. \end{cases} \quad (2.19)$$

From (2.7), we get

$$CSNR > 10\log_{10}\left(\left(\frac{\sigma_x^2}{D}\right)^3(1 - \rho^2)^2 - 1\right). \quad (2.20)$$

Case 2

When $\lambda_{3,2} < \theta < \lambda_{3,0}$, which corresponds to

$$\begin{cases} SDR > 10\log_{10}\left(\frac{6}{-3\rho^2 + 6 - \rho\sqrt{\rho^2 + 8}}\right) \\ SDR < 10\log_{10}\left(\frac{2}{\rho^2 + 2 - \rho\sqrt{\rho^2 + 8}}\right), \end{cases} \quad (2.21)$$

(2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{3} \left(\frac{1}{2} \log \frac{\lambda_{3,0}}{\theta} + \frac{1}{2} \log \frac{\lambda_{3,1}}{\theta} \right) \\ D(\theta) = \frac{1}{3} (\theta + \theta + \lambda_{3,2}). \end{cases} \quad (2.22)$$

From (2.7), we get

$$CSNR > 10\log_{10}\left(\frac{2\left(\frac{\sigma_x^2}{D}\right)^2(1 - \rho^2)(\rho^2 + 2 + \rho\sqrt{\rho^2 + 8})}{3 - \frac{1}{2}\frac{\sigma_x^2}{D}(\rho^2 + 2 - \rho\sqrt{\rho^2 + 8})^2} - 1\right). \quad (2.23)$$

Case 3

When $\lambda_{3,0} < \theta < \lambda_{3,1}$, which corresponds to

$$0 < SDR < 10\log_{10}\left(\frac{6}{-3\rho^2 + 6 - \rho\sqrt{\rho^2 + 8}}\right), \quad (2.24)$$

(2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{3} \left(\frac{1}{2} \log \frac{\lambda_{3,1}}{\theta} \right) \\ D(\theta) = \frac{1}{3} (\theta + \lambda_{3,0} + \lambda_{3,2}). \end{cases} \quad (2.25)$$

From (2.7), we get

$$CSNR > 10 \log_{10} \left(\frac{a}{b} - 1 \right), \quad (2.26)$$

where $a = \frac{1}{2} \frac{\sigma_x^2}{D} (\rho^2 + 2 + \rho \sqrt{\rho^2 + 8})$ and $b = 3 - \frac{\sigma_x^2}{D} (1 - \rho^2) - \frac{1}{2} \frac{\sigma_x^2}{D} (\rho^2 + 2 - \rho \sqrt{\rho^2 + 8})$.

2.3.3 4 : 2 bandwidth reduction

In this case, $l_x = 4$, and the covariance matrix is

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho \sigma_x^2 & \rho^2 \sigma_x^2 & \rho^3 \sigma_x^2 \\ \rho \sigma_x^2 & \sigma_x^2 & \rho \sigma_x^2 & \rho^2 \sigma_x^2 \\ \rho^2 \sigma_x^2 & \rho \sigma_x^2 & \sigma_x^2 & \rho \sigma_x^2 \\ \rho^3 \sigma_x^2 & \rho^2 \sigma_x^2 & \rho \sigma_x^2 & \sigma_x^2 \end{bmatrix}, \quad (2.27)$$

whose eigenvalues are $\lambda_{4,0} = \frac{1}{2} \sigma_x^2 (\rho^3 - (\rho^2 + \rho) \sqrt{\rho^2 - 2\rho + 5} + \rho + 2)$, $\lambda_{4,1} = \frac{1}{2} \sigma_x^2 (\rho^3 + (\rho^2 + \rho) \sqrt{\rho^2 - 2\rho + 5} + \rho + 2)$, $\lambda_{4,2} = \frac{1}{2} \sigma_x^2 (-\rho^3 - (\rho^2 - \rho) \sqrt{\rho^2 + 2\rho + 5} - \rho + 2)$ and $\lambda_{4,3} = \frac{1}{2} \sigma_x^2 (-\rho^3 + (\rho^2 - \rho) \sqrt{\rho^2 + 2\rho + 5} - \rho + 2)$. We have four cases since there are four different ordered eigenvalues:

Case 1

When $0 < \theta < \lambda_{4,3}$, which corresponds to

$$SDR > 10 \log_{10} \left(\frac{\sigma_x^2}{\lambda_{4,3}} \right), \quad (2.28)$$

(2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{4} \sum_{i=0}^3 \frac{1}{2} \log \frac{\lambda_{4,i}}{\theta} \\ D(\theta) = \frac{1}{4} \sum_{i=0}^3 \theta. \end{cases} \quad (2.29)$$

From (2.7), we get

$$CSNR > 10\log_{10}(\sqrt{(\frac{\sigma_x^2}{D})^4(1-\rho^2)^3} - 1). \quad (2.30)$$

Case 2

When $\lambda_{4,3} < \theta < \lambda_{4,0}$, which corresponds to

$$\begin{cases} SDR > 10\log_{10}(\frac{8}{t}) \\ SDR < 10\log_{10}(\frac{\sigma_x^2}{\lambda_{4,3}}), \end{cases} \quad (2.31)$$

where $t = 2\rho^3 - 3(\rho^2 + \rho)\sqrt{\rho^2 - 2\rho + 5} + (\rho^2 - \rho)\sqrt{\rho^2 + 2\rho + 5} + 2\rho + 8$, (2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{4} \sum_{i=0}^2 \frac{1}{2} \log_{10}(\frac{\lambda_{4,i}}{\theta}) \\ D(\theta) = \frac{1}{4}(3\theta + \lambda_{4,3}). \end{cases} \quad (2.32)$$

From (2.7), we get

$$CSNR > 10\log_{10}(\sqrt{\frac{b}{c}} - 1), \quad (2.33)$$

where $b = (3\frac{\sigma_x^2}{D})^3 \lambda_{4,0} \lambda_{4,1} \lambda_{4,2}$ and $c = (4\sigma_x^2 - \frac{\sigma_x^2}{D} \lambda_{4,3})^3$.

Case 3

When $\lambda_{4,0} < \theta < \lambda_{4,2}$, which corresponds to

$$\begin{cases} SDR > 10\log_{10}(\frac{4}{m}) \\ SDR < 10\log_{10}(\frac{8}{t}), \end{cases} \quad (2.34)$$

where $t = 2\rho^3 - 3(\rho^2 + \rho)\sqrt{\rho^2 - 2\rho + 5} + (\rho^2 - \rho)\sqrt{\rho^2 + 2\rho + 5} + 2\rho + 8$ and $m = -\frac{1}{2}(\rho^2 + \rho)\sqrt{\rho^2 - 2\rho + 5} + 4 - \rho^3 - \frac{1}{2}(\rho^2 - \rho)\sqrt{\rho^2 + 2\rho + 5} - \rho$, (2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{4}(\frac{1}{2}\log_{10}(\frac{\lambda_{4,1}}{\theta}) + \frac{1}{2}\log_{10}(\frac{\lambda_{4,2}}{\theta})) \\ D(\theta) = \frac{1}{4}(2\theta + \lambda_{4,3} + \lambda_{4,0}). \end{cases} \quad (2.35)$$

From (2.7), we get

$$CSNR > 10\log_{10}(\sqrt{\frac{b}{c}} - 1), \quad (2.36)$$

where $b = (\frac{\sigma_x^2}{D})^2 \lambda_{4,1} \lambda_{4,2}$ and $c = (2\sigma_x^2 - \frac{1}{2}\frac{\sigma_x^2}{D} \lambda_{4,0} - \frac{1}{2}\frac{\sigma_x^2}{D} \lambda_{4,3})^2$.

Case 4

When $\lambda_{4,2} < \theta < \lambda_{4,1}$, which corresponds to

$$\begin{cases} SDR > 0 \\ SDR < 10\log_{10}(\frac{4}{m}), \end{cases} \quad (2.37)$$

where $m = -\frac{1}{2}(\rho^2 + \rho)\sqrt{\rho^2 - 2\rho + 5} + 4 - \rho^3 - \frac{1}{2}(\rho^2 - \rho)\sqrt{\rho^2 + 2\rho + 5} - \rho$, (2.9) becomes

$$\begin{cases} R(\theta) = \frac{1}{4}(\frac{1}{2}\log_{10}(\frac{\lambda_{4,1}}{\theta})) \\ D(\theta) = \frac{1}{4}(\theta + \lambda_{4,3} + \lambda_{4,0} + \lambda_{4,2}). \end{cases} \quad (2.38)$$

From (2.7), we get

$$CSNR > 10\log_{10}(\sqrt{\frac{b}{c}} - 1), \quad (2.39)$$

where $b = (\frac{\sigma_x^2}{D})^2 \lambda_{4,1}$ and $c = 4\sigma_x^2 - \frac{\sigma_x^2}{D} \lambda_{4,0} - \frac{\sigma_x^2}{D} \lambda_{4,3} - \frac{\sigma_x^2}{D} \lambda_{4,2}$.

2.4 Review of the PCCOVQ Algorithm

Power constrained channel optimized vector quantization has been successfully applied to the design of bandwidth reduction source-channel mappings [30] and bandwidth expansion source-channel mappings [26] for i.i.d sources. In the case of i.i.d. sources, the algorithm produces a mapping whose performance is close to the OPTA. Our aim in this chapter is to extend the algorithm to the general case of multivariate sources, where the samples are correlated.

2.4.1 Encoding process

Each channel symbol is restricted to belong to the Cartesian product of uniform m -ary PAM, giving rise to $Q = m^L$ points in the channel space and in the source space. The actual encoding process consists of 2 stages: vector quantization followed by a signal selection module.

The vector quantizer first maps X into an index $i \in \mathcal{I} = \{0, 1, \dots, Q - 1\}$ whenever $X \in \Omega_i$, where Ω_i is one of the partitions $\mathcal{P} = \{\Omega_i\}_{i=0}^{Q-1}$ of the source space

$(\mathcal{R}^K)$. Next, the signal selection module maps the index i into the corresponding channel symbol $Y = \Delta \cdot s_i$, where s_i is a unit distance PAM signal in the channel space, and Δ is a scale factor.

2.4.2 Recovery process

At the receiver, the decoder chooses index j to minimize $\|\hat{Y} - \Delta \cdot s_j\|^2$. Then, $\hat{X} = c_j$, where $c_j \in C = \{c_i\}_{i=0}^{Q-1}$, and C is called the codebook of the source space.

2.4.3 Optimization process

The distortion per source sample is

$$D(\mathcal{P}, \Delta, C) = \frac{1}{K} E[\|X - \hat{X}\|^2] = \sum_{i=0}^{Q-1} \int_{\Omega_i} d_i(X) f_x(X) dX, \quad (2.40)$$

where $f_x(X)$ is the source pdf, and

$$d_i(X) = \frac{1}{K} \sum_{j=0}^{Q-1} p(j|i) \|X - c_j\|^2 \quad (2.41)$$

is the distortion associated with partition i , where $p(j|i)$ is the probability of receiving index j given that index i was transmitted. The power per channel symbol is given by

$$P = \frac{\Delta^2}{L} \sum_{i=0}^{Q-1} \|s_i\|^2 \int_{\Omega_i} f_x(X) dX. \quad (2.42)$$

The optimization problem can then be expressed as

$$\min_{\{\mathcal{P}, \Delta, C\}} [D(\mathcal{P}, C, \Delta) + \lambda P], \quad (2.43)$$

where λ is a Lagrange multiplier.

The minimization of (2.43) is performed by a variation of the generalized Lloyd algorithm. For a given Δ , codebook C , and λ , whose value is related to the CSNR after optimization, the partition $\mathcal{P}$ is updated according to

$$\Omega_i = \{X | g_i(X) \leq g_j(X), \forall j \in \mathcal{I}, i \in \mathcal{I}, \quad (2.44)$$

where

$$g_i(X) = d_i(X) + \frac{\lambda \Delta^2}{L} \sum_{i=0}^{Q-1} \|s_i\|^2. \quad (2.45)$$

The codebook is updated by using the new partition and Δ , and can be expressed as

$$c_j = \frac{\sum_{i=0}^{Q-1} p(j|i) \int_{\Omega_i} X f_x(X) dX}{\sum_{i=0}^{Q-1} p(j|i) \int_{\Omega_i} f_x(X) dX}, j \in \mathcal{I}. \quad (2.46)$$

Δ is updated by using an iterative search method to minimize (2.43). The process will not stop until the cost function in (2.43) converges.

As any other iterative optimization method, different initial values may lead to different optimization results, some of which may not be global optima. When implementing PCCOVQ, in order to avoid local optima, we make λ large enough so that the CSNR is low at the very beginning (σ_n^2 can be fixed as 1), and we use a linear codebook given as

$$c_i(m) = \begin{cases} ks_i(m) & \text{if } 0 \leq m \leq L-1 \\ ks_i(L-1) & \text{if } L \leq m \leq K-1, \end{cases} \quad (2.47)$$

where $c_i(m)$ and $s_i(m)$ are the m 'th components of the vectors c_i and s_i , respectively, and k is a scale factor. The reason for using a linear codebook initially is that for very noisy channels (low CSNR), linear mappings work well [22]. Once we have the optimized codebook for the current CSNR, we will use it as the initial codebook for a slightly higher CSNR. The optimization process continues until the optimized codebook of the target CSNR is obtained.

2.5 Simulation Results

In this section, we show simulation results for 2 : 1, 3 : 1 and 4 : 2 mappings of multivariate Gaussian sources generated using the PCCOVQ algorithm. For the 2 : 1 bandwidth reduction case, we use training vectors of length 400000, which are long enough for $Q = 256$ and low CSNR. This configuration is a good trade-off between simulation duration and performance, and also good enough to see the shape of the mapping. Our simulation covers a wide range of values for the correlation coefficient ρ , from 0.2 to 0.99. Fig. 2.2 shows the output SDR versus CSNR for the resulting mappings. From this figure, we see that the simulation results obtained using optimized PCCOVQ matches OPTA when the CSNR falls in the region corresponding to the

case 2 defined by (2.14). The explanation is that in this case it is possible to design a scheme that theoretically matches the OPTA by using a linear transformation that drops the second dimension of the source symbol and only transmits the first dimension through the channel (the receiver applies MMSE estimation to the received channel symbols to recover both symbols). On the other hand, when the CSNR falls in the region corresponding to case 1 defined by (2.11), similar to the result in [30] for i.i.d. Gaussian sources, there exists a gap from the OPTA, but the performance is still quite close to the upper bound. It is interesting to remark that the higher the value of ρ , the higher CSNR case 2 can reach, which means we can achieve OPTA for wider range of CSNRs. The fact that the gap to OPTA becomes larger when $\text{CSNR} > 30$ dB is due to the length of the training vector and the fact that Q is not big enough for this high CSNR.

We also compare our simulations with the results obtained when using a space filling curve optimized for an i.i.d. Gaussian source (which basically agrees with the mapping obtained by PCCOVQ for an i.i.d. Gaussian source). Fig. 2.2 also shows that mappings designed for i.i.d. sources [50] are no longer appropriate for correlated samples, especially when ρ is high. This is corroborated in Fig. 2.3, 2.4, 2.5, which show the codebooks in the source signal space (i.e., mapping shape) for $\rho = 0.2, 0.5$ and 0.9 for four different CSNRs. The shape for case 2 (low CSNR) is basically a straight line irrespective of the correlation level, which agrees with our explanation above. For case 1 (high CSNR), the shape when $\rho = 0.2$ (Fig. 2.3) is similar to the Archimede's spiral, which is the mapping determined by PCCOVQ for i.i.d Gaussian sources ($\rho = 0$). However, for higher ρ (Fig. 2.5), we obtain wave-shaped curves. If we compare the codebook shapes with the contours of the respective 2-D Gaussian pdfs, both are quite similar, which is a good sign for a good mapping [74], since this means that it covers the target source space in an efficient way.

For the 3 : 1 case, we use training vectors of length 600000, and choose Q equal to 256. Fig. 2.6 shows the output SDR versus CSNR. Similar to the results obtained for the 2 : 1 mapping, when the CSNR falls in the regions corresponding to case 3, the

OPTA is reached, while in all other cases a gap begins to appear but the results are still quite reasonable.

We also derive the mappings for the 4 : 2 case, because we want to investigate how the performance changes as K increases when R_c stays fixed (in this case $R_c = 2$). In the simulation of the 4 : 2 case, we use training vectors of length 120000, and choose Q to be 32^2 . Fig. 2.7 compares the performance of optimized 4 : 2 and 2 : 1 mappings in terms of SDR versus CSNR when $\rho = 0.8$ and $\rho = 0.99$. The OPTA for 4 : 2 and 2 : 1 are shown in the same figure as well. The results clearly show that for a fixed R_c the performance is better when K increases. This is expected, since the decoding of 4 : 2 mappings exploits the correlations among all four source samples while 2 : 1 only exploits the correlation between two. The performance loss when CSNR is above 20 dB is due to Q and it is also explained because the length of the training vector is not sufficiently large for that CSNR. This is indeed the main drawback of the PCCOVQ method: the training complexity increases very fast as the number of dimensions and the CSNR increase. We can expect better performance for high CSNRs by using larger Q and longer training vectors.

2.6 Conclusion

We have investigated the performance of bandwidth reduction analog mappings for the transmission of multivariate Gaussian sources over AWGN channels by using the PCCOVQ algorithm for the mapping design, comparing the results with theoretical bounds. We have shown that when the correlation is high, linear mappings can reach OPTA up to a pretty high CSNR. Beyond that CSNR, the performance is still close to the theoretical limits if we use optimized non-linear mappings. Furthermore, the optimal signal space structure is not a spiral-like space filling curve (as for i.i.d. sources) any more. Although we did not provide a parameterized space-filling curve, we did show the possibility to construct good analog mappings for multivariate Gaussian sources.

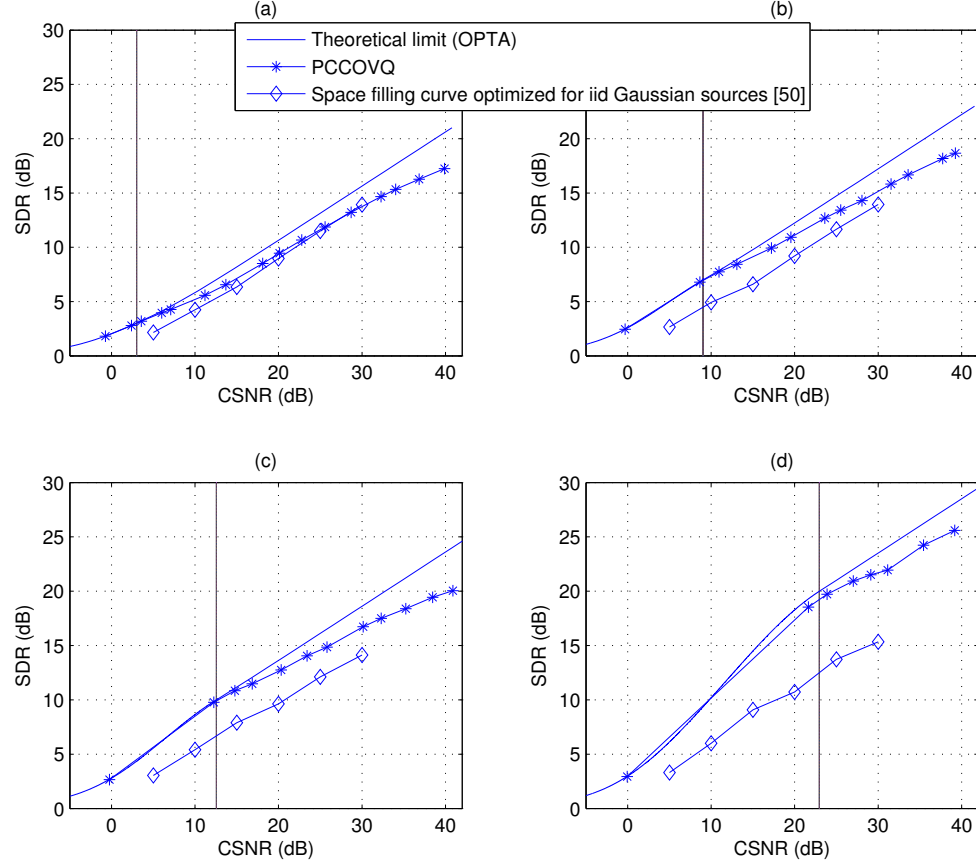


Figure 2.2: Performance of a 2 : 1 system obtained as explained in the text for the transmission of multivariate Gaussian sources over AWGN channels (a) $\rho = 0.5$, (b) $\rho = 0.8$, (c) $\rho = 0.9$, (d) $\rho = 0.99$. The vertical line in each of the sub-figures separates the regions corresponding to case 2 (left) and case 1 (right)

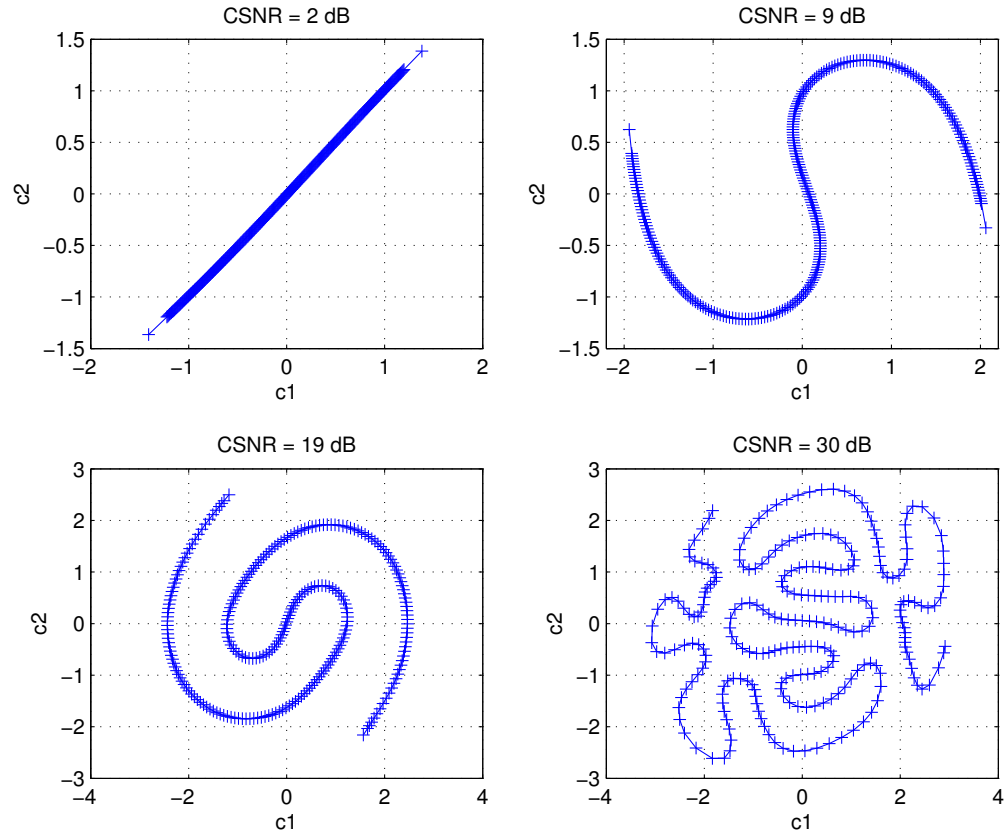


Figure 2.3: Signal space after optimization by the PCCOVQ algorithm when $\rho = 0.2$.

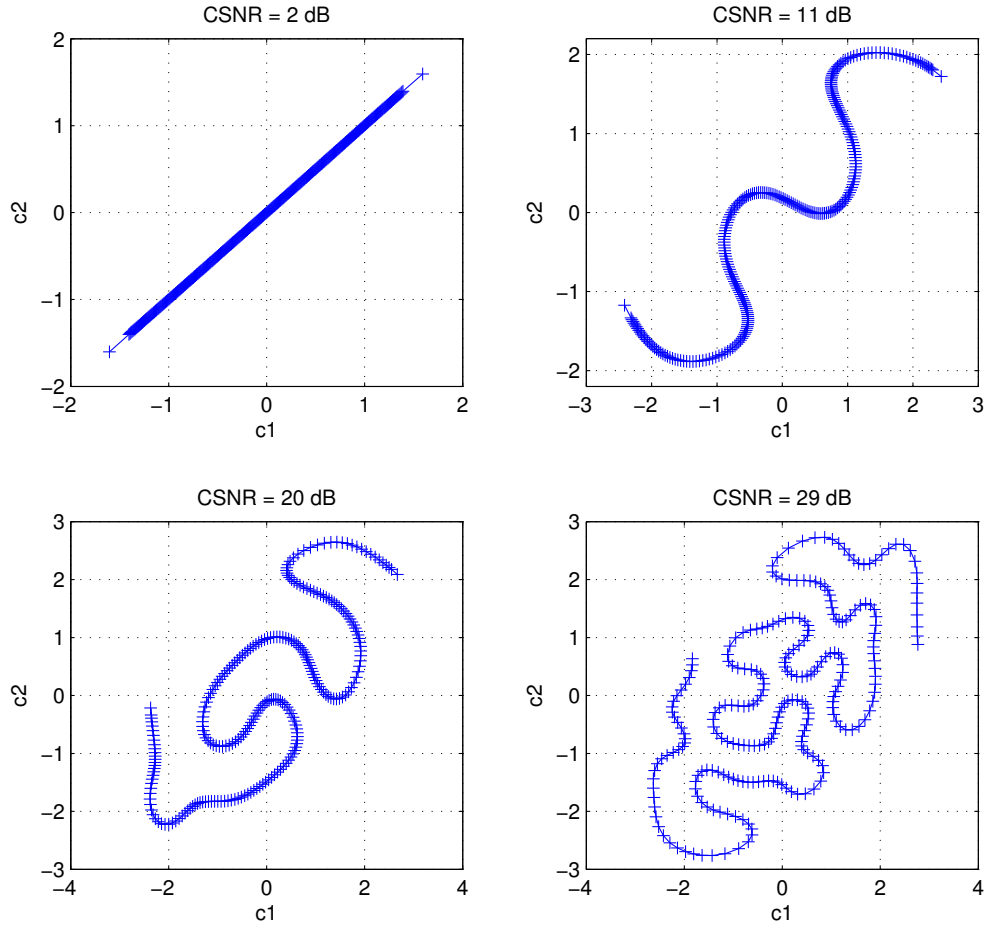


Figure 2.4: Signal space after optimization by the PCCOVQ algorithm when $\rho = 0.5$.

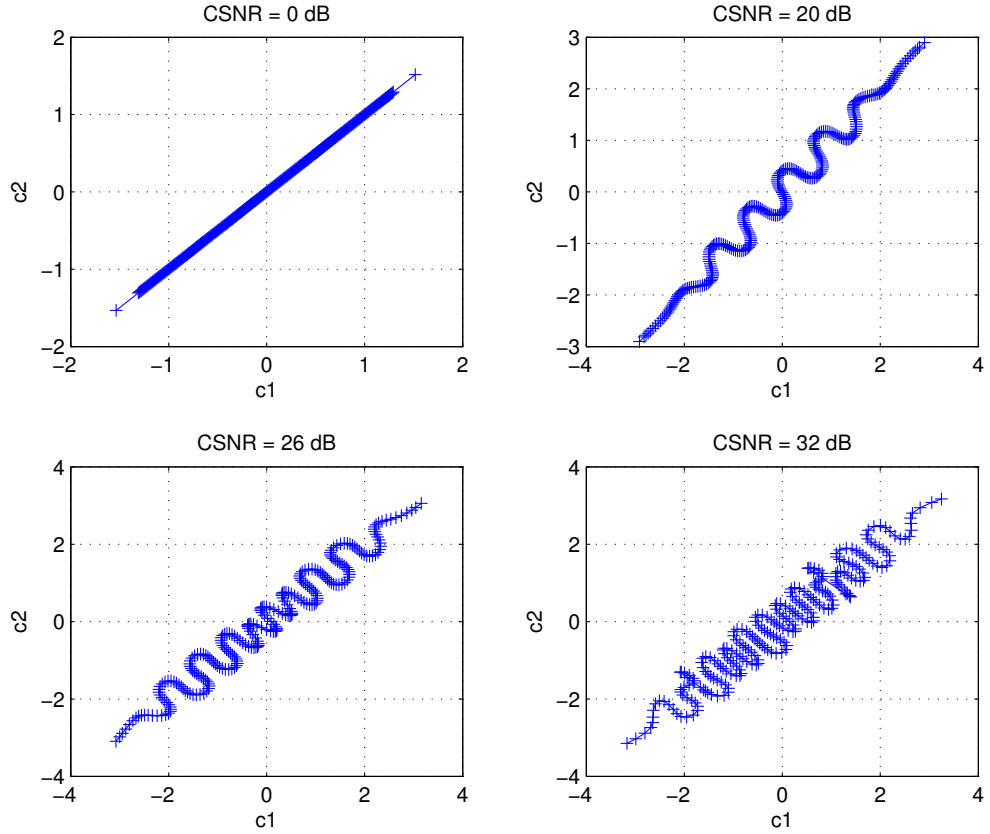


Figure 2.5: Signal space after optimization by the PCCOVQ algorithm when $\rho = 0.9$.

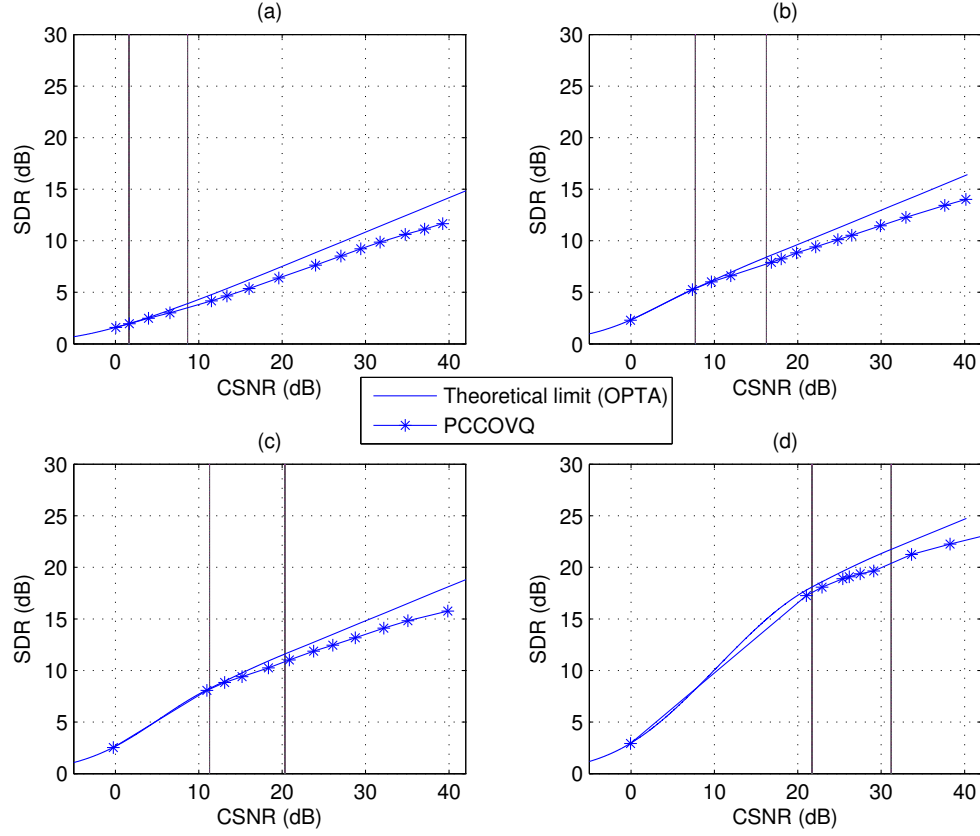


Figure 2.6: Performance of a 3 : 1 system for multivariate Gaussian sources over AWGN channels when (a) $\rho = 0.5$, (b) $\rho = 0.8$, (c) $\rho = 0.9$, (d) $\rho = 0.99$. The vertical lines in each of the sub-figures separate the regions corresponding to case 3 (left), case 2 (center), and case 1 (right)

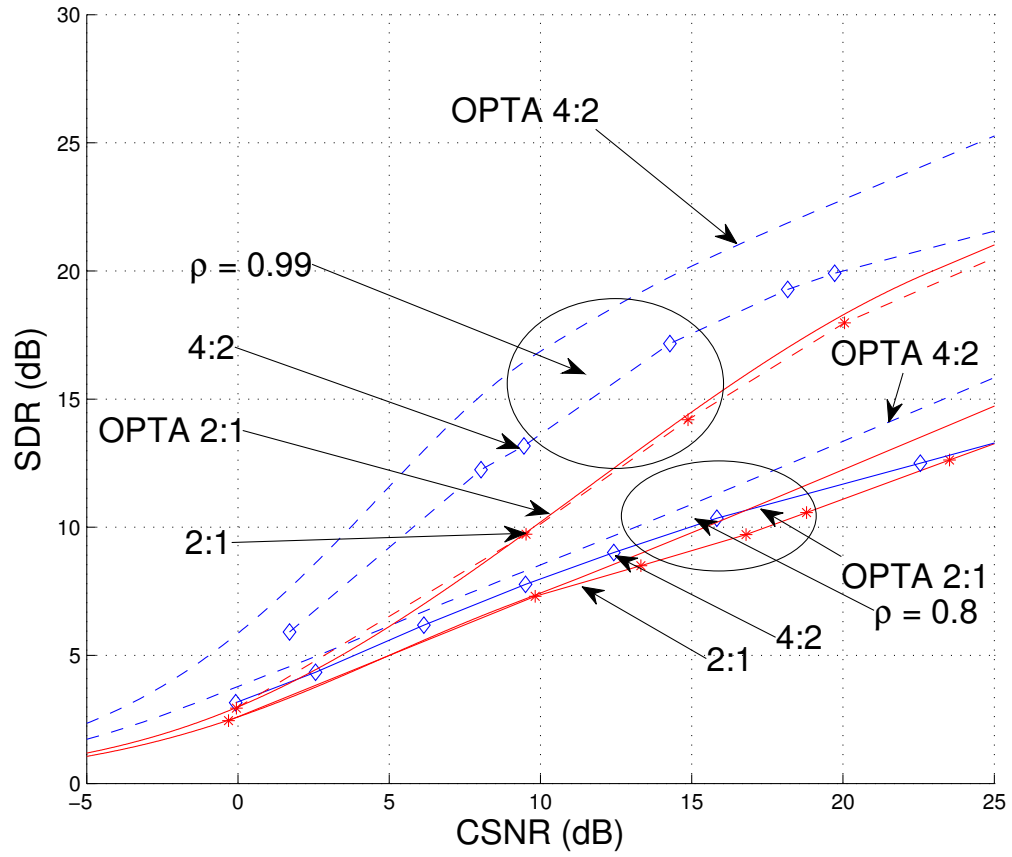


Figure 2.7: Comparison between 4 : 2 and 2 : 1 system for multivariate Gaussian sources over AWGN channels when $\rho = 0.8$ and $\rho = 0.99$.

Chapter 3

BANDWIDTH EXPANSION MAPPINGS FOR TRANSMISSION OF GAUSSIAN SOURCES WITH SIDE INFORMATION

In this chapter, we consider the problem of designing non-linear zero-delay bandwidth expansion analog joint source-channel coding (JSCC) schemes for the transmission of memoryless Gaussian samples over an additive white Gaussian noise (AWGN) channels when side information is present at the receiver (Wyner-Ziv scenario). We first propose a $1 : 1$ scheme and a $1 : M$ scheme based on the use of Shannon-Kotel'nikov mappings in a periodic fashion. To reduce the complexity of the numerical MMSE decoding, we propose a simplified decoding algorithm for the proposed schemes. Then, we combine the two proposed mappings to construct a flexible rate bandwidth scheme whose rate can be anywhere from $1 : 1$ to $1 : M$. Simulation results show that the performance of the proposed $1 : 2$ scheme is better than that of existing zero-delay systems for a wide range of signal to noise ratios, but especially for high signal to noise ratios and highly correlated side information. Simulation also shows that the simplified decoding performs very similar to the numerical MMSE for a wide range of CSNRs, but with a greatly reduced complexity. The proposed flexible rate $K : L$ bandwidth expansion system also offers satisfactory performance, especially when considering its flexibleness in terms of rate.

3.1 Introduction

The problem of transmitting analog source samples (audio, video, etc.) over a noisy channel with side information at the decoder (Wyner-Ziv scenario) can be solved by serially concatenating efficient source coding schemes and capacity approaching

channel codes such as Turbo and LDPC codes. The reason is that digital communication systems based on separation between source and channel coding are optimal for the transmission of samples over noisy channels when side information is available at the decoder [34]. However, just as in point-to-point communication without side information (see Chapter 1), digital communication systems based on separation for the Wyner-Ziv scenario have some drawbacks, such as high delay and high complexity encoder/decoder and poor robustness. In this chapter, we study Shannon-Kotel'nikov mappings for the Wyner-Ziv scenario. Specifically, we design bandwidth expansion analog mappings based on the use of space-filling curves for transmitting memoryless Gaussian sources over AWGN channels when side information is available at the receiver. There is not much work in the literature about analog mappings for the Wyner-Ziv scenario. A hybrid scheme is proposed in [15]. Bandwidth-expansion analog mapping using sinusoids, as proposed in [61], performs quite well for the low-CSNR regime. In [3], a mapping utilizing spiral like curves for bandwidth-reduction was proposed, but for high CSNRs the results are far from the theoretical limits.

In this chapter, we first propose $1 : 1$ and $1 : M$ schemes that use periodic space-filling curves: a piece-wise periodic linear mapping for the $1 : 1$ case and the concatenation of a piece-wise linear mapping with a standard space-filling curve (Archimedes' spiral) for the $1 : 2$ case. The proposed schemes benefit from reusing the output symbols and the correlation model of the source, as well as from the side information, to recover the source samples while maintaining high spectral efficiency. A simplified decoding is also proposed to reduce the complexity of the numerical MMSE decoding without losing much performance. Next, we propose a flexible rate $K : L$ bandwidth expansion scheme that consists of the parallel combination of the proposed $1 : 1$ and $1 : M$ schemes with optimized power allocation. This scheme greatly increases the practicality of the proposed mappings.

The remainder of this chapter is organized as follows. In Section 3.2, we formulate the problem and we also provide the derivation of the theoretical limits for the transmission of Gaussian samples over AWGN channels in the Wyner-Ziv scenario. In

Section 3.2.3, we analyze the distortion for standard bandwidth expansion mappings using space-filling curves. Section 3.3 discusses the proposed 1 : 1 and 1 : M schemes for the Wyner-Ziv problem, and Section 3.4 introduces the proposed K : L system. Simulation results are presented in Section VI, and Section VII concludes the chapter.

3.2 Problem Formulation and OPTA

3.2.1 Problem Formulation

The point-to-point communication system for the Wyner-Ziv scenario using analog JSCC is depicted in Fig. 3.1. In general, we consider the transmission of K independent and identically distributed (i.i.d.) Gaussian samples $X = \{x_i\}_{i=1}^K$ in $\mathcal{R}^K$. Without loss of generality, we assume that each sample x_i has zero mean and variance σ_x^2 . The K side information samples at the receiver $Y = \{y_i\}_{i=1}^K$ in $\mathcal{R}^K$ are modeled as

$$Y = X + U, \quad (3.1)$$

where $U = \{u_i\}_{i=1}^K$ in $\mathcal{R}^K$ are i.i.d. Gaussian random variables, $u_i \sim \mathcal{N}(0, \sigma_u^2)$, independent of X .

In analog JSCC based on direct source-channel mapping, each K source samples X are mapped to L channel samples $Z = \{z_i\}_{i=1}^L$ in $\mathcal{R}^L$ by an encoder function $\alpha() : \mathcal{R}^K \rightarrow \mathcal{R}^L$. In this chapter, we consider the bandwidth expansion case ($R_c = K/L < 1$) in which channel bandwidth (the number of channel samples) is greater than source bandwidth (the number of source samples) and the bandwidth matching case ($R_c = K/L = 1$) in which channel bandwidth equals source bandwidth.

When transmitted over the channel, Z is corrupted by additive Gaussian noise samples, N , with zero mean and noise variance σ_n^2 . At the receiver, the vector $Z' = Z + N$ and the side information vector Y are processed jointly by the decoder function $\beta() : (\mathcal{R}^L, \mathcal{R}^K) \rightarrow \mathcal{R}^K$. The distortion per sample is defined as

$$D = \frac{E[\|X - \hat{X}\|^2]}{K}, \quad (3.2)$$

and the power constraint is defined as

$$P = \frac{E[\|Z\|^2]}{L} \leq P_{max}. \quad (3.3)$$

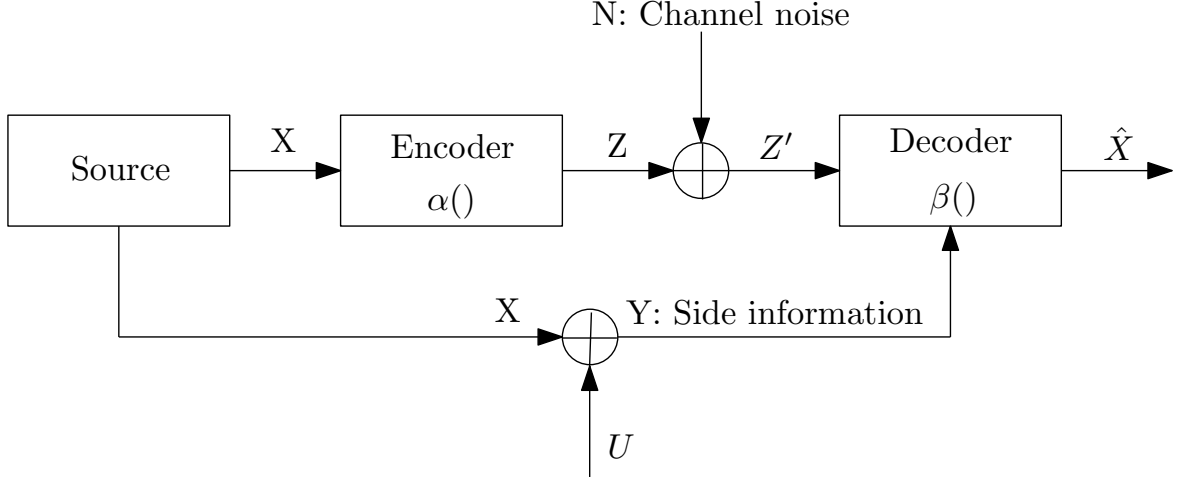


Figure 3.1: Analog communication system based on JSCC with side information available at the decoder.

Therefore, the goal is to find the $\alpha()$ and the $\beta()$ that minimize (3.2) under the power constraint defined in (3.3). As in the previous chapter, the system performance is measured in terms of SDR versus CSNR.

3.2.2 Optimal Performance Theoretically Attainable (OPTA)

When AWGN channels are considered the capacity is given by [18]

$$C = \frac{1}{2} \log \left(1 + \frac{P}{\sigma_n^2} \right), \quad (3.4)$$

where σ_n^2 is the power of the channel noise and P is the average power of the channel samples.

The rate distortion function of memoryless Gaussian sources for the case where perfect side information is available at the receiver is [82]

$$R(D) = \begin{cases} \frac{1}{2} \log \frac{\sigma_x^2 \sigma_u^2}{(\sigma_x^2 + \sigma_u^2) D}, & 0 < D < \frac{\sigma_x^2 \sigma_u^2}{\sigma_x^2 + \sigma_u^2} \\ 0, & D \geq \frac{\sigma_x^2 \sigma_u^2}{\sigma_x^2 + \sigma_u^2}, \end{cases} \quad (3.5)$$

where D is the mean squared error as defined before and σ_x^2 and σ_u^2 are the variance of the source, X , and of U , respectively.

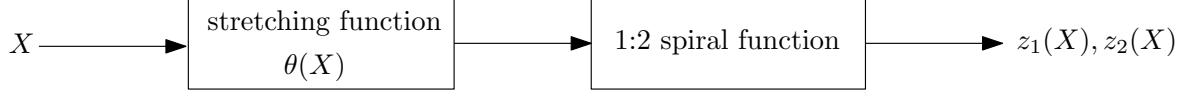


Figure 3.2: Encoding process of 1 : 2 analog JSCC using a spiral-like mapping. The stretching function is followed by the Archimedes’ spiral function.

Defining ρ as the correlation coefficient between X and Y , it is easy to obtain

$$\rho = \frac{E[xy]}{\sqrt{E[x^2]E[y^2]}} = \frac{\sigma_x}{\sqrt{\sigma_x^2 + \sigma_u^2}}, \quad (3.6)$$

so that for the case of $R_c = \frac{K}{L}$, from the condition $R(D)R_c < C$, the theoretical limit can be obtained as

$$\frac{\sigma_x^2}{D} = \frac{1}{1 - \rho^2} \left(1 + \frac{P}{\sigma_n^2}\right)^{\frac{L}{K}}, \quad (3.7)$$

where ρ is the correlation coefficient between X and Y . We can rewrite (3.7) in terms of CSNR and SDR as

$$\text{SDR} = 10\log_{10}\left(\frac{\sigma_x^2}{D}\right) = \frac{L}{K} \times 10\log_{10}\left(1 + 10^{\frac{\text{CSNR}}{10}}\right) - 10\log_{10}(1 - \rho^2). \quad (3.8)$$

3.2.3 Distortion analysis for standard bandwidth expansion mappings

As explained in Chapter 1, the encoder of the 1 : 2 mapping system, depicted in Fig. 3.2, consists of two steps: stretching function and a mapping using a spiral-like curve. Fig. 3.3 and Fig. 3.4 show the spiral-like curve in 3D and 2D, respectively.

The study of bandwidth expansion direct source-channel mapping using space-filling curves was initiated by V. A. Kotel’nikov [64]. He mentioned in his dissertation that for a fixed source space (e.g. $[-1, 1]$), the space-filling curve should be made longer in order to reduce the noise influence. This should be done without leaving a certain hyper-sphere in order to satisfy the power constraint, so that the curve has to be twisted inside the hyper-sphere defined by the power constraint if we want to make it as long as possible. However, the length of the curve cannot be increased beyond a certain length without introducing what Kotel’nikov called anomalous errors or threshold distortion. The occurrence of these anomalous errors depends on the standard deviation of the

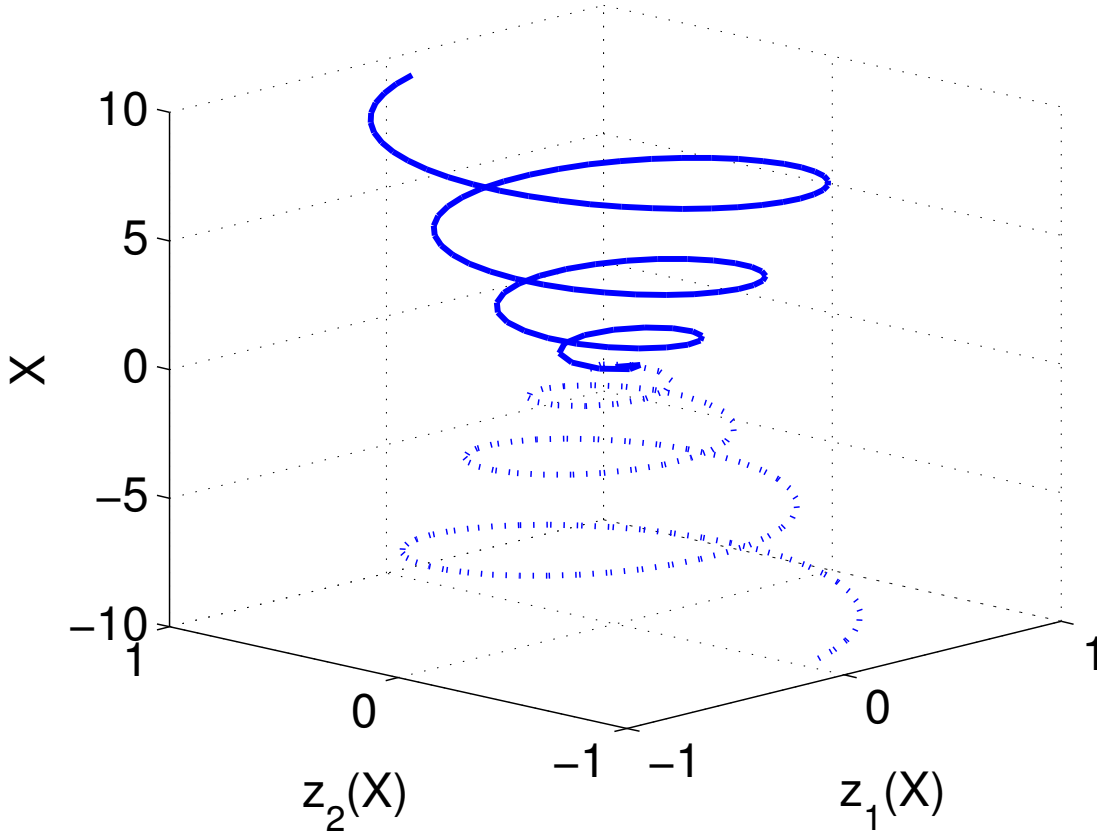


Figure 3.3: 1 : 2 spiral like mapping in 3D: The dotted curve represents the negative branch ($X < 0$) while the positive ($X \geq 0$) branch is shown by the solid curve.

channel noise and the density of the curve. Therefore, the overall distortion for any bandwidth expansion mapping using space-filling curves can be categorized into two: threshold distortion and weak noise distortion. Threshold distortion refers to the case when distortion is not continuous with respect to the received symbol Z' , for example when the decoder decodes source samples to the opposite branch ($\text{sign}(X) = -\text{sign}(\hat{X})$) of the spiral even when Z' is very different from zero, which introduces large distortion and quickly degrades the system's performance. Weak noise distortion refers to the distortion incurred in all the other cases and it is much smaller than the threshold distortion. Both types of distortion for the 1 : 2 mapping are discussed in [47], where

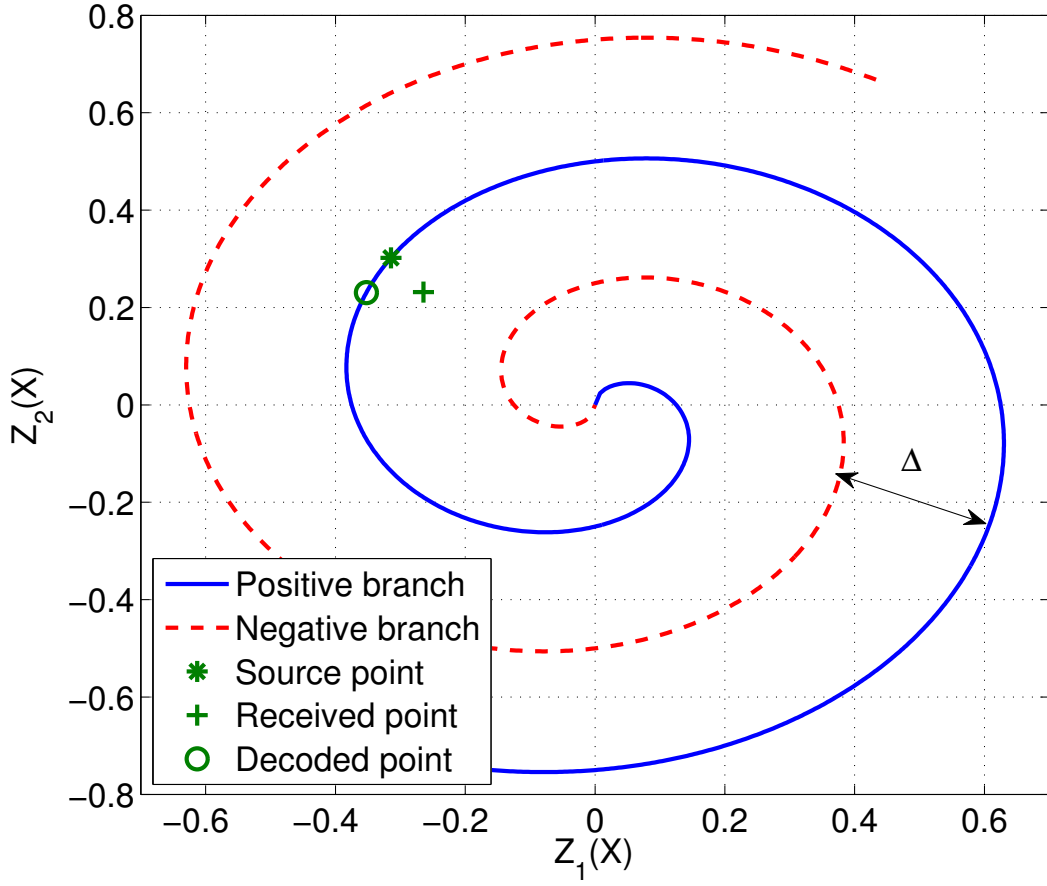


Figure 3.4: 1 : 2 spiral like mapping in 2D: The dash curve represents the negative branch ($X < 0$) while the positive ($X \geq 0$) branch is shown by the solid curve.

it is shown that there is a trade-off between threshold and weak noise distortion. The weak noise distortion decreases as Δ decreases, but the threshold distortion increases as Δ decreases. Therefore, the optimized value of Δ is somewhere in the middle to minimize the overall distortion for each CSNR. Fig. 3.5 shows two 1 : 2 mappings with two different values of Δ under the same power constraint. The mapping in Fig. 3.5(a) uses shorter curves to cover the source space and thus for the same noise level it will have larger weak noise distortion than the mapping in Fig. 3.5(b). The curve in Fig. 3.5(b) is more dense and thus the mapping has a smaller threshold for the anomalous

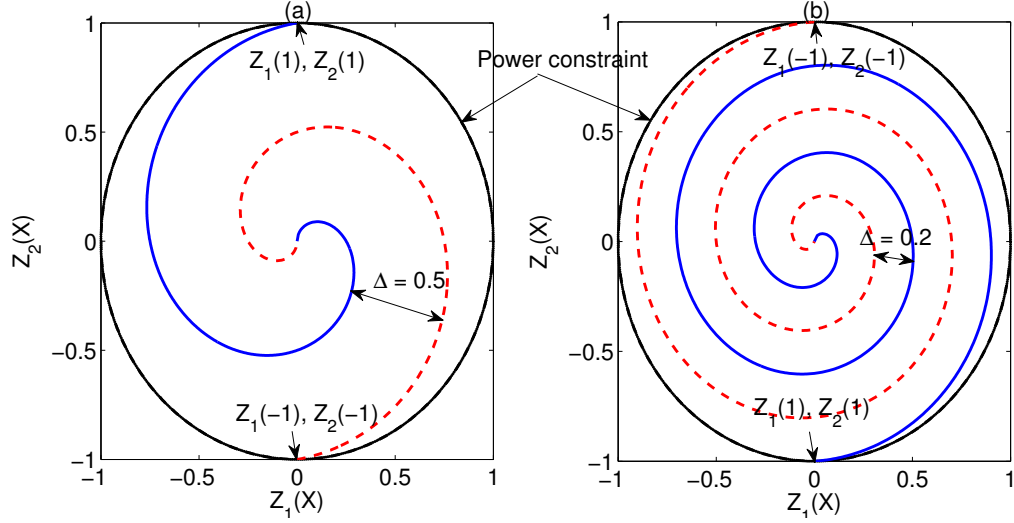


Figure 3.5: Two 1 : 2 mappings using a spiral-like curve with different values of Δ . Both mappings have the same power constraint: $P_{max} = 1$ and $X \subseteq [-1, 1]$: (a) $\Delta = 0.5$, (b) $\Delta = 0.2$.

errors to occur than the mapping in Fig. 3.5(a). As a result, the mapping in Fig. 3.5(b) is more likely to have threshold distortion than the mapping in Fig. 3.5(a).

3.3 Proposed 1 : 1 and 1 : M Schemes for the Wyner-Ziv Problem

In this section, we introduce our 1 : 1 scheme and 1 : M bandwidth expansion non-linear mappings using space-filling curves designed for the transmission of memoryless Gaussian sources over AWGN channels under the Wyner-Ziv scenario.

As we mentioned in section 3.2.3, the curve used for bandwidth expansion mappings needs to be long in order to reduce the weak noise distortion, but this will increase the threshold distortion because the curve becomes dense. The question is: by taking advantage of the side information available at the decoder, can we further reduce the weak noise distortion without introducing more threshold distortion?

The key idea to achieve this goal is to make the space-filling curve periodic so that the mapping produces the same output values for input values that are separated by a period. In this way, we can make the curve longer, which reduces the weak noise distortion but controls the value of the period so that the threshold distortion is not

increased. Notice that this approach does not work if side information is not available at the decoder. The reason is that multiple source samples (the ones separated by a period) are mapped into the same output samples, which makes it impossible to decode the source sample by only looking at the received sample. The side information will help the decoder decide which period of the curve the source sample comes from, and has to be combined with the received sample to decode the source sample. Although we focus on the case when $M = 2$, the idea can be extended to the design of general $1 : M$ bandwidth expansion mapping for the Wyner-Ziv scenario.

3.3.1 $1 : 1$ mapping

As we mentioned earlier, when $K = L$, linear mapping (direct transmission) is optimal for point-to-point transmission of memoryless Gaussian sources over AWGN channel with no side information. However, for the Wyner-Ziv scenario the linear mapping performs far from OPTA, especially for high correlation and high CSNRs. In this work, we improve upon the linear mapping by employing a piece-wise linear mapping with period T .

Specifically, the mapping is defined as

$$Z = ((X - \frac{T}{2}) \pmod{T} - \frac{T}{2})/c, \quad (3.9)$$

where T is the fundamental period of the piece-wise linear function, c is a constant used to satisfy the power constraint and $a \pmod{T}$ is the modulo operation with a as the dividend and T as the divisor. An example of the mapping when $T = 2$ is shown in Fig. 3.6. Although in [4], the author uses an iterative algorithm and derives a $1 : 1$ mapping for the Wyner-Ziv scenario that is similar to our proposed mapping, ours is more applicable due to the fact that it is represented by parametric equations, which makes it easier to adapt to different CSNRs.

3.3.2 $1 : M$ mapping

When $M = 2$, we serially concatenate the $1 : 1$ piece-wise linear mapping to the standard $1 : 2$ mapping that we introduced in Section 3.2.3. As a result, the proposed

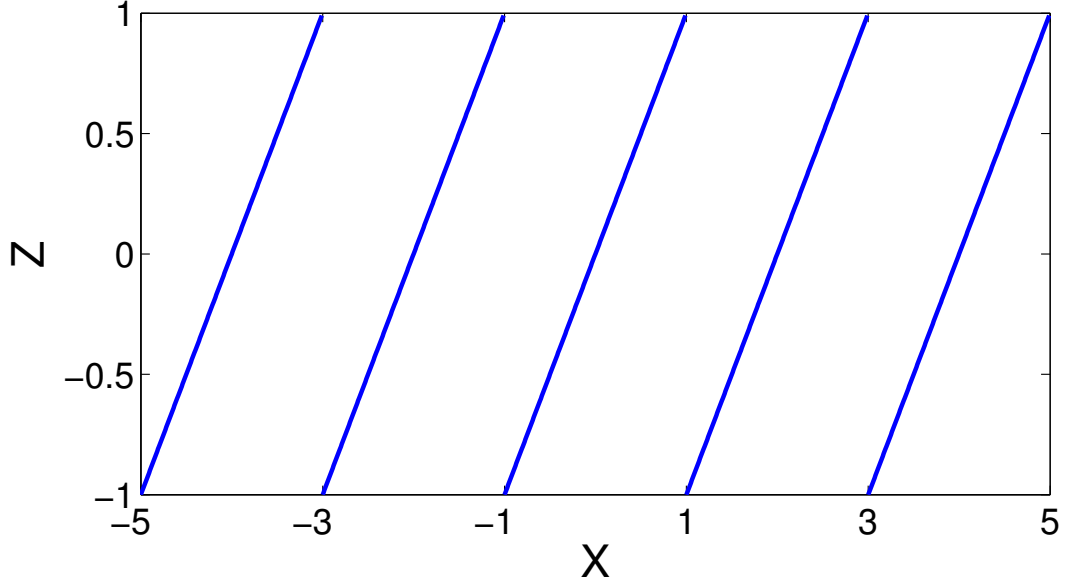


Figure 3.6: 1 : 1 periodic piece-wise linear mapping with $T = 2$.

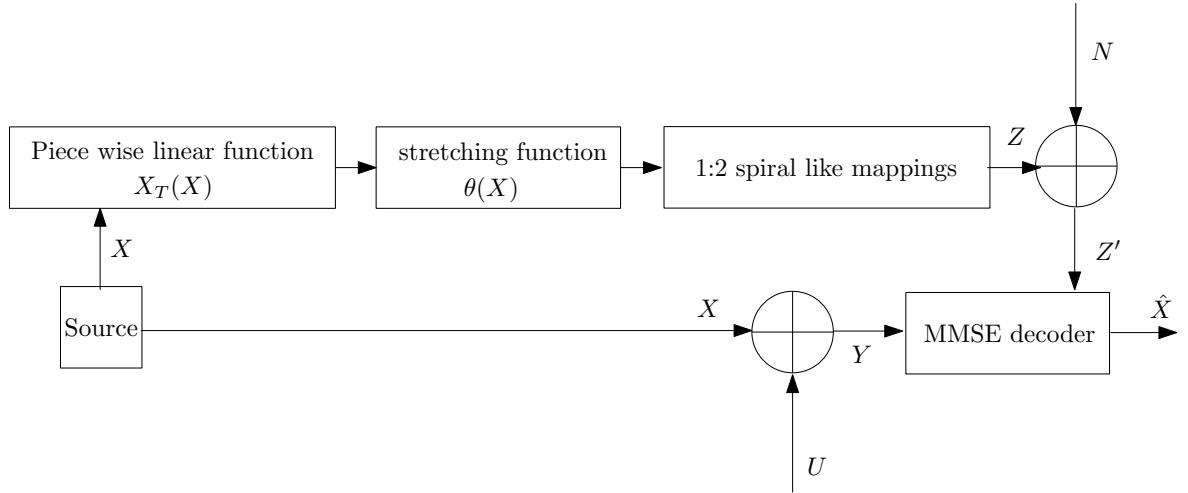


Figure 3.7: 1 : 2 proposed scheme.

1 : 2 mapping is a periodic mapping where each period is the 1 : 2 mapping using the Archimedes' spiral. Fig. 3.7 presents the complete system block and Fig. 3.8 shows the proposed mapping in 3D.

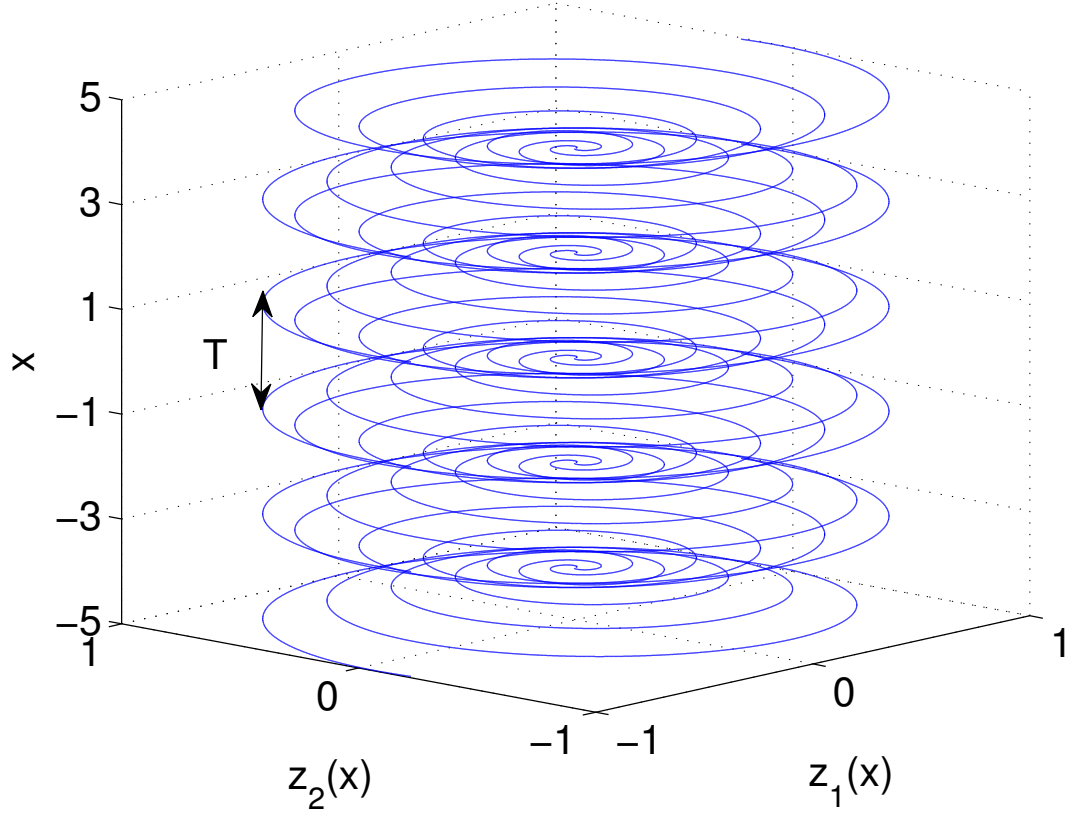


Figure 3.8: Proposed 1 : 2 mapping for $T = 2$ in 3D.

3.3.3 Decoding

MMSE decoding is optimum when mean square error is considered. However, due to the non-linearity of the proposed mappings, computationally intensive numerical methods need to be used. In this work, we propose a simplified decoding algorithm which massively reduces the cost of the decoding without losing much performance.

3.3.3.1 MMSE decoding

The MMSE decoding is described as

$$\begin{aligned}
\hat{X} &= E[X|Z', Y] = \int X p(X|Z', Y) dX \\
&= \int X \frac{p(Z', Y|X) p(X)}{p(Z', Y)} dX = \int X \frac{p(Y|X) p(Z'|X) p(X)}{p(Z', Y)} dX \\
&= \int X \frac{p(Y, X) p(Z'|X)}{p(Z', Y)} dX = \int X \frac{p(Y, X) p(Z'|X)}{\int p(Y, X) p(Z'|X) dX} dX.
\end{aligned} \tag{3.10}$$

We need to calculate the integral numerically since it is not possible to get a closed form expression for the conditional probability $p(Z'|X)$ due to the non-linearity of the mappings. The numerical method is similar to the one we mentioned in Chapter 1 for 2 : 1 standard mapping.

3.3.3.2 Simplified decoding

To illustrate the simplified decoding algorithm, we denote the index of the period as k , which is defined as

$$k = \lfloor X/T \rfloor, \tag{3.11}$$

where $\lfloor a \rfloor$ rounds a to the closest integer. The simplified decoding algorithm divides the decoding process into two stages. The first stage of the simplified decoding is to estimate X' from Z' , where X' corresponds to the estimate of X assuming X belongs to the period containing 0 (i.e., assuming $k = 0$). The second stage is to estimate the value of k , the index of the period where X comes from, using Y . Finally, $\hat{X}$ is described as

$$\hat{X} = X' + kT. \tag{3.12}$$

To estimate X' , we first estimate $\hat{Z}$, the estimate of Z , from Z' using MMSE estimation, which can be described as

$$\hat{Z} = E[Z|Z'] = Z' \frac{P}{P + \sigma_n^2}. \tag{3.13}$$

X' is then estimated using ML decoding from $\hat{Z}$. When $M = 2$, we have already mentioned the ML decoding in Chapter 1. When $M = 1$, $X' = \hat{Z}$.

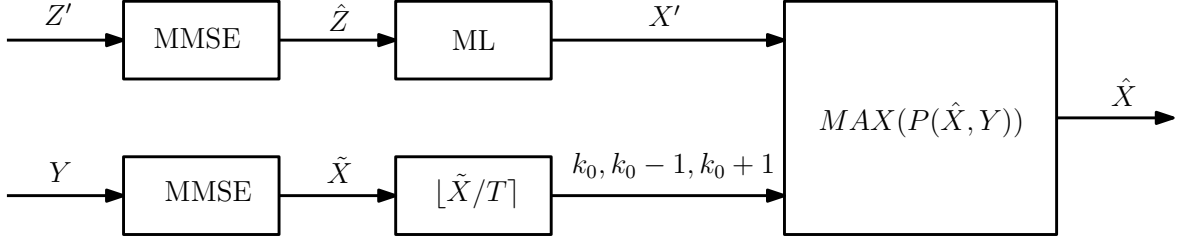


Figure 3.9: Simplified decoding algorithm.

To estimate k , we first get $\tilde{X}$ from Y using MMSE estimation, which is given by

$$\tilde{X} = E[X|Y] = \rho^2 Y. \quad (3.14)$$

Then, the initial guess of k , k_0 , is derived from $\tilde{X}$ as

$$k_0 = \lfloor \tilde{X}/T \rfloor, \quad (3.15)$$

To reduce the distortion, we consider a bigger pool of candidate indexes from k_0 's neighbor period: $k_0 - 1$ and $k_0 + 1$. Once we get k_0 , $k_0 - 1$, $k_0 + 1$ and X' , we can apply (3.12) to get $\hat{X}_{k_0}$, $\hat{X}_{k_0-1}$ and $\hat{X}_{k_0+1}$, and choose $\hat{X}_k$ which maximizes $P(\hat{X}, Y)$. The whole decoding process is depicted in Fig. 3.9.

3.3.4 Parameter optimization and distortion analysis

In this section, we discuss the parameter optimization and distortion analysis for our proposed mappings.

3.3.4.1 1 : 1 mapping

The only parameter in the proposed 1 : 1 mapping is the fundamental period T . When T is smaller, the length of the curve is longer, so the weak noise distortion is smaller. If we consider the aperiodic linear mapping used in the case when no side information is available as the limiting case of the proposed mapping when T goes to infinite, we can see that the proposed mapping with a finite T has smaller weak noise distortion. Fig. 3.10 illustrates the effect of weak noise distortion. Notice, however,

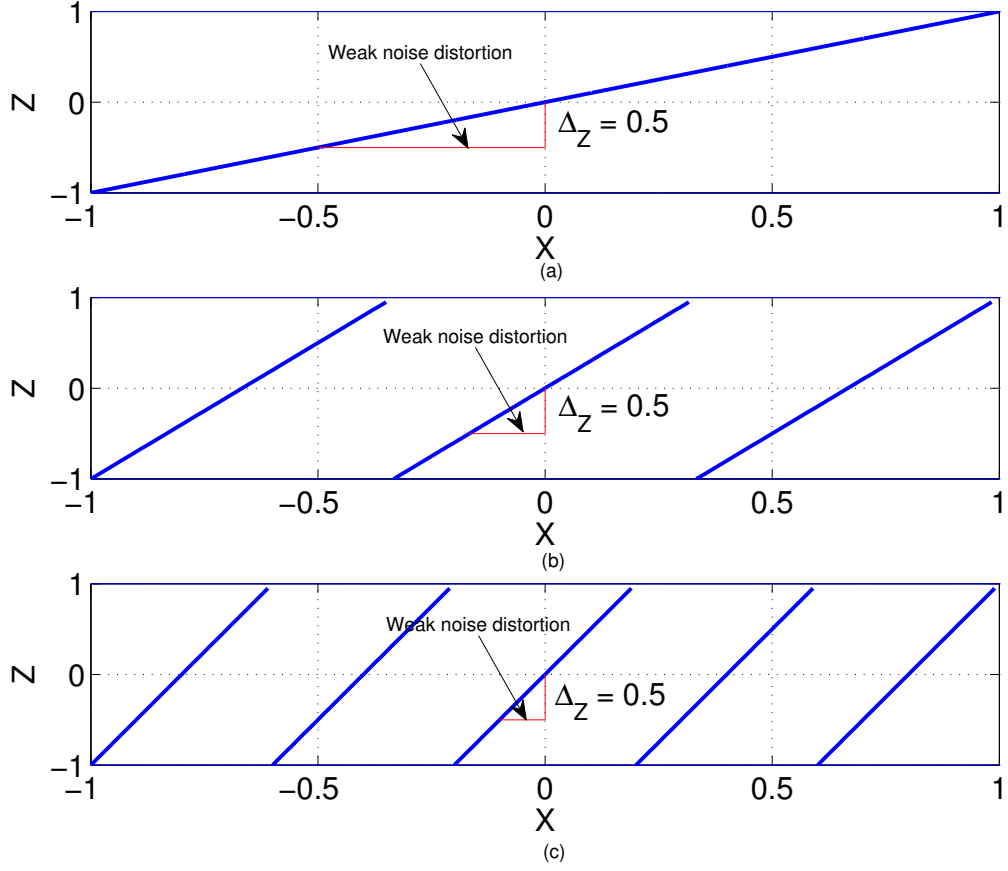


Figure 3.10: Weak noise distortion comparison: (a) linear mapping, (b) proposed mapping with $T = 2/3$, (c) proposed mapping with $T = 0.4$. For all three mappings, we assume $Z = 0$ and $Z' = -0.5$ ($\Delta_Z = Z - Z' = 0.5$). The corresponding weak noise distortion (Δ_X) decreases from (a) to (c).

that the value of T cannot be too small: when T is smaller than a threshold, the side information will not be powerful enough to distinguish source samples coming from different periods, and, as a result, threshold distortion occurs. The optimization process needs to consider the trade-off between the weak noise distortion and the threshold distortion. The optimal value of T is a function of the correlation ρ and CSNR.

3.3.4.2 1 : M mapping

In the proposed 1 : M scheme, both Δ and T need to be optimized. Due to the non-linearity of the mapping, we do this using numerical methods. Notice that in addition to the standard weak noise distortion and Δ -threshold distortion, we have a new T -threshold distortion due to the piece-wise linear mapping. In order to reduce the weak noise distortion, we should decrease both Δ and T , since for a given T reducing Δ increases the length of the curve, and for a given Δ reducing the value of T also increases the length of the curve. However, when Δ is reduced beyond a threshold for a given CSNR, the standard Δ -threshold distortion will occur. On the other hand, for a given value of ρ , the value of T cannot be reduced beyond a certain threshold because the side information will not be powerful enough to distinguish source samples coming from different periods, as in the 1 : 1 scheme. As a result, the decoder may confuse the recovered sample as coming from the wrong period. We name this distortion as T -threshold distortion in order to differentiate it from the Δ -threshold distortion. Therefore, the trade-off now is between weak noise distortion and the two threshold distortions. From our simulations, the rule of thumb for the optimized value is: the larger the CSNR the smaller the Δ and the larger the ρ the smaller the T .

3.4 Proposed $K : L$ Scheme for the Wyner-Ziv Problem

In this section, we propose a system that can produce bandwidth expansion mappings of any rate between 1 : 1 and 1 : M by combining the 1 : 1 and 1 : M schemes proposed in section 3.3. The proposed scheme is more flexible, and applicable in practice. Notice that designing a space-filling curve for a specific rate is far more difficult than just choosing the number of samples transmitted in both 1 : 1 and 1 : M subsystems, which is what our proposed system does.

The proposed system is shown in Fig. 3.11. s_1 out of $s_1 + s_2$ source samples are transmitted through the 1 : 1 system and the rest are transmitted using the 1 : M system, so that the total number of samples transmitted through the channel is $s_1 + Ms_2$ and the rate is $(s_1 + s_2) : (s_2 + Ms_2)$. By properly choosing the values of s_1 and s_2 ,

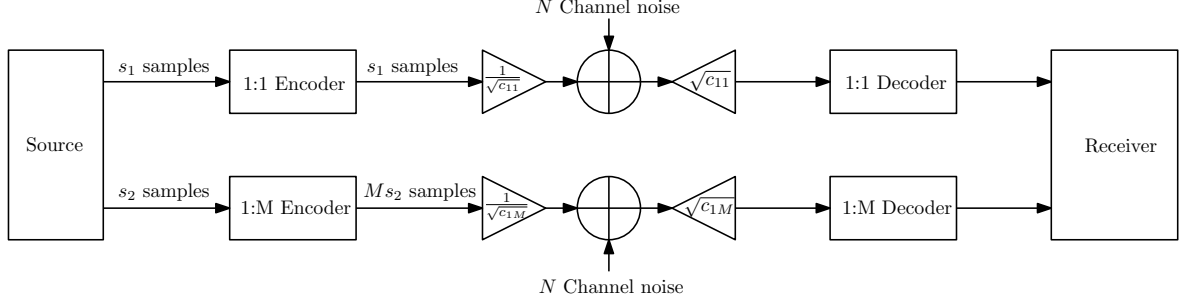


Figure 3.11: Proposed system of rate $K : L$. The rate is changed by choosing the number of samples s_1 and s_2 .

any rate between $1 : 1$ and $1 : M$ can be obtained. Notice that the average power assigned to the $1 : 1$ sub-system and to the $1 : M$ sub-system can change as long as the power constraint for the whole system is satisfied. Therefore, we will optimize the fraction of the total power allocated to sub-system $1 : 1$ which we will denote as r_{11} ($1 - r_{11}$ is thus the fraction of the total power allocated to sub-system $1 : 2$). For both subsystems, MMSE decoding is used and evaluated numerically. Δ , T and r_{11} should be optimized according to the values of each CSNR and ρ .

3.4.1 $1 : 1$ subsystem

We use the $1 : 1$ scheme introduced in section 3.3 to obtain

$$Z = (\text{mod}(X - \frac{T}{2}, T) - \frac{T}{2})/\sqrt{c_{11}}, \quad (3.16)$$

where T is the fundamental period. c_{11} is the coefficient used to control the total power assigned to the $1 : 1$ subsystem and can be expressed as:

$$c_{11} = \frac{P_{11}s_1}{P(s_1 + Ms_2)r_{11}}, \quad (3.17)$$

where P_{11} is the average power of the $1 : 1$ system before power allocation.

Table 3.1: Optimized Parameter (T) for the 1 : 1 scheme

T	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	3.7	3.4	3.3	3.5
$\rho = 0.99$	1.1	1.0	1.0	1.1

Table 3.2: Optimized Parameter (T) for the 1 : 2 scheme

T	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	4.1	4.2	3.6	3.9
$\rho = 0.99$	1.4	1.1	1.1	1.2

3.4.2 1 : M subsystem

We utilize the 1 : M non-linear system described in section 3.3, except that the coefficient used in power allocation, denoted as c_{1M} , is expressed as:

$$c_{1M} = \frac{P_{1M} M s_2}{P(s_1 + M s_2)(1 - r_{11})}, \quad (3.18)$$

where P_{1M} is the average power of the 1 : M system before power allocation.

3.5 Simulation Results

In this section, we present simulation results for the proposed periodic JSCC schemes and compare them with the theoretical limit (OPTA as defined in (3.7)) and with other existing schemes. We first evaluate the performance of the 1 : 1 and 1 : 2 systems proposed in section 3.3 when numerical MMSE decoding is used. In Fig. 3.12, we plot the performance of the proposed 1 : 1 system for $\rho = 0.99$ and $\rho = 0.9$. For comparison, we also plot the performance of direct transmission when optimal MMSE decoding is applied. Fig. 3.13, Fig. 3.14 and Fig. 3.15 show the SDRs achieved with the proposed 1 : 2 scheme, the sinusoidal scheme proposed in [61], and the standard 1 : 2 mapping introduced in Section 3.2.3 for $\rho = 0.99$, $\rho = 0.95$ and $\rho = 0.9$, respectively. The optimal values for parameter T for the 1 : 1 and 1 : 2 schemes are presented in Tables 3.1 and 3.2, respectively.

From the figures, we clearly see that the proposed system performs better than the other zero-delay systems, especially for large CSNR and for highly correlated side information. Interestingly, the proposed scheme shows an almost-constant gap to the OPTA, while other schemes present an increasing gap as the CSNR increases. Notice that OPTA is actually the theoretical limit for the case when K and L go to infinite, so the real gap between our scheme and the theoretical limit when $L = 2$ is even smaller. From Fig. 3.15, we notice that for $\rho = 0.9$ the performance of the proposed scheme is close to that of the standard spiral-like mapping designed for the case when there is no side information available at the receiver. The reason is that the optimized value of T when $\rho = 0.9$ is relatively large, and, as a result, there is not much practical difference between the proposed mapping and the standard spiral-like expansion mapping. From Table 3.1 and Table 3.2, we notice that the optimum T only depends on the value of ρ and decreases as ρ increases.

Next, we simulate the proposed $K : L$ system introduced in Section 3.4. Specifically, we consider a 6:10 system and a 9 : 10 system based on the combination of 1 : 2 and 1 : 1 systems to assess how the proposed system performs when the overall rate is close to 1 : 1 and 1 : 2. Fig. 3.16 shows the performance of the 6:10 system for both $\rho = 0.99$, and $\rho = 0.9$ when r_{11} is optimized, and Fig. 3.17 shows the results of the 9 : 10 system for both $\rho = 0.99$ and $\rho = 0.9$, also when r_{11} is optimized. In these figures, we also plot the OPTA and the simulated performance of the system which combines the uncoded 1 : 1 mapping and the 1 : 2 system proposed in section 3.2.3, which can be considered as a lower bound since these mappings are not designed for the Wyner-Ziv scenario.

Notice that in both cases (6 : 10 and 9 : 10), for $\rho = 0.99$ and high CSNR the proposed system performs much better than the system designed for the case without side information. When comparing with the OPTA, the performance is not as remarkable as in Fig. 3.13, Fig. 3.14 and Fig. 3.15, but it is still quite reasonable when considering the great flexibleness of the scheme in terms of rate. Similar to the 1 : 2 case (see Fig. 3.15) when $\rho = 0.9$, the proposed system does not present substantial

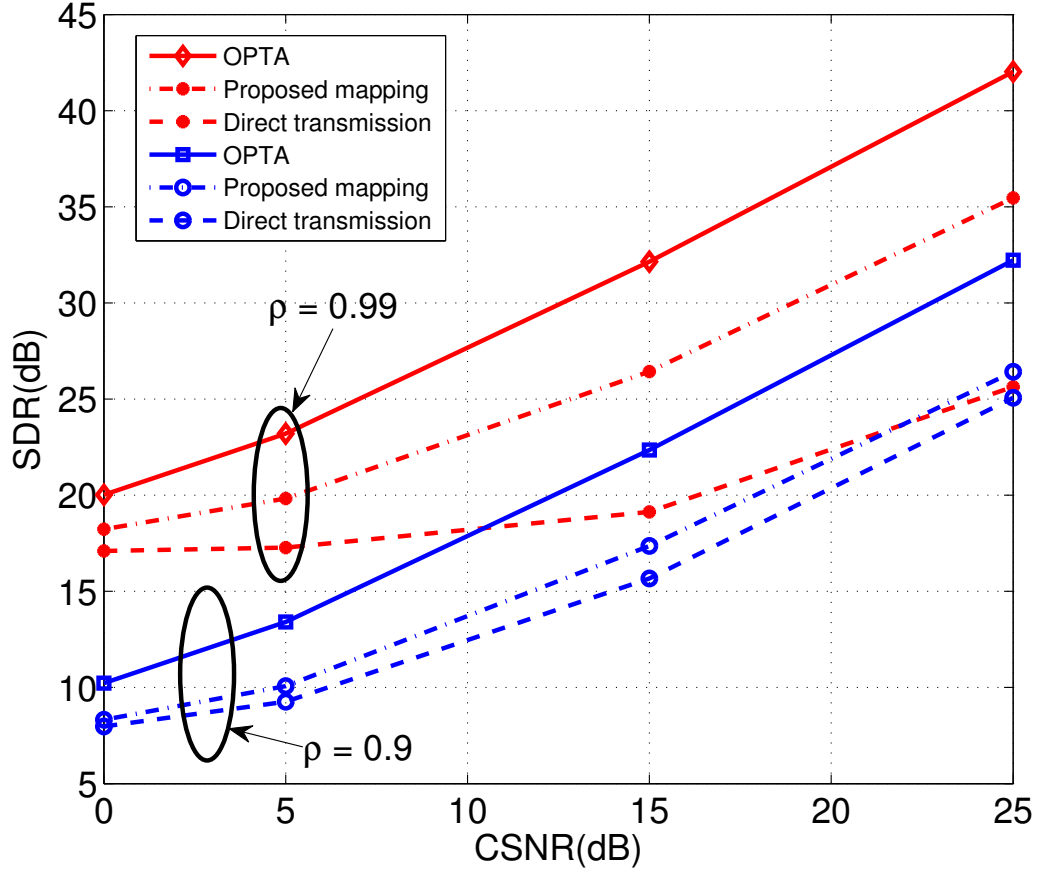


Figure 3.12: Performance evaluation for 1 : 1 systems when $\rho = 0.9$ and $\rho = 0.99$.

gains with respect to the system designed without considering the side information. It is interesting to note that the optimal value of r_{11} increases as the CSNR goes up. This is intuitive: since the SDR of the 1 : 1 system is always smaller than that of the 1 : 2 for the same CSNR, more power should be allocated to the 1 : 1 system in order to balance the distortion level in the 1 : 1 and 1 : 2 systems, thus minimizing the overall distortion.

We also evaluate the proposed simplified decoding method and compare it with the MMSE method. The results are shown in Fig. 3.18 and Fig. 3.19. During the simulation, we noticed the optimum T for the simplified decoding is different from the

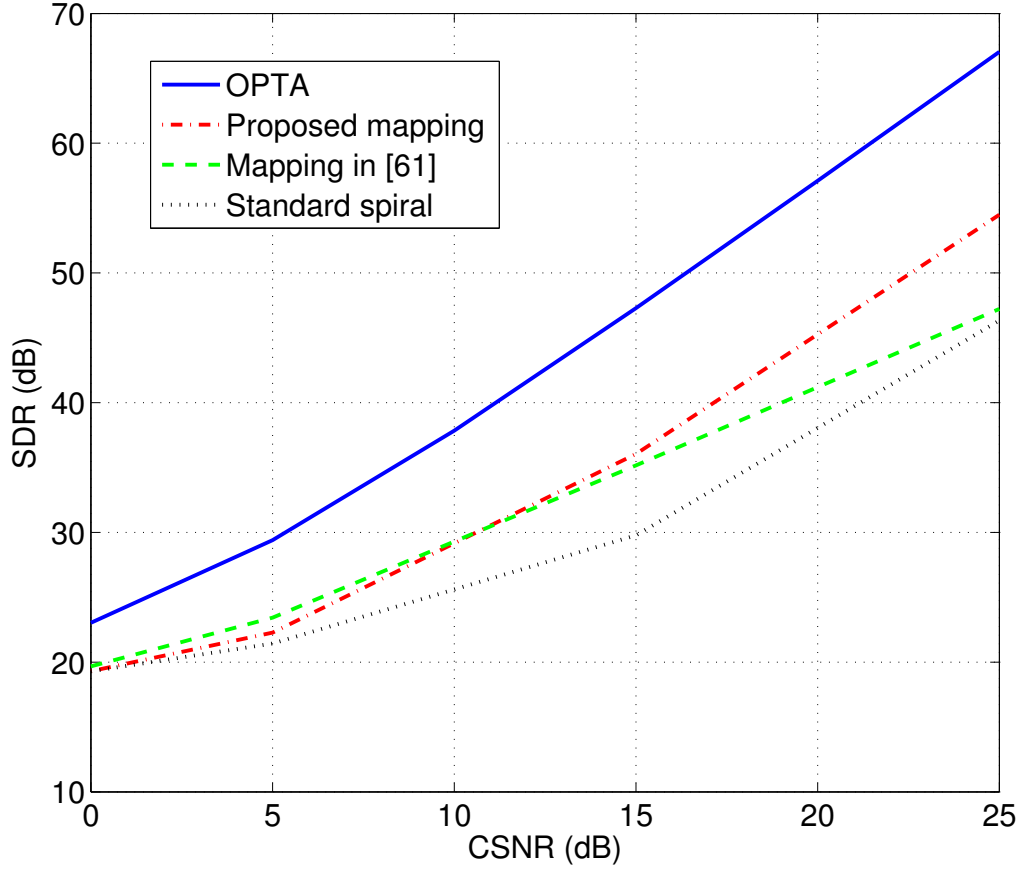


Figure 3.13: Performance evaluation for 1 : 2 systems when $\rho = 0.99$.

MMSE decoding for low CSNRs. Therefore, we also simulate the simplified decoding with the T optimized for MMSE decoding.

From the figures, we can see that irrespectively of the values of R_c and ρ , the results are similar: the simplified decoding is close to MMSE when CSNR is 0 dB, it is a little bit worse when CSNR is around 5 dB, and matches MMSE when CSNR is above 15 dB. Interestingly, when CSNR is above 15 dB, the optimum parameters for the simplified decoding are the same as for MMSE decoding. This means that we can replace the somewhat complex numerical MMSE decoding with a much simpler decoder without suffering any performance loss. Moreover, when CSNR and ρ increase,

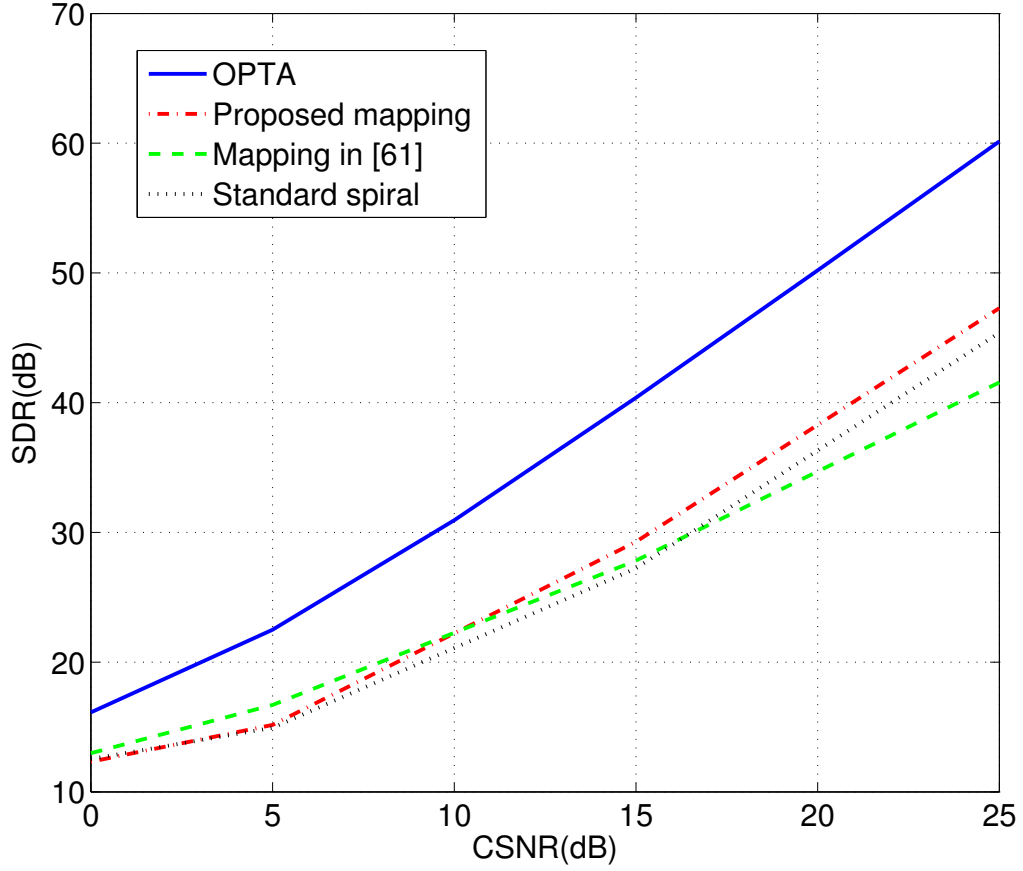


Figure 3.14: Performance evaluation for 1 : 2 systems when $\rho = 0.95$.

the advantage of using the simplified decoding method is greater, because the step needs to be smaller for the results to converge, with the consequent increase on complexity in time and space. When CSNR is below 0 dB, the simplified decoding performs poorly when using the optimum parameters for MMSE decoding but performs quite well if the proper parameters are used. In fact, T needs to be very small for the simplified decoding to work in low CSNR regime. When ρ is fixed and CSNR is small ($\frac{P}{\sigma_n^2}$ close to 0), (3.8) becomes

$$\text{SDR} \approx -10 \log_{10}(1 - \rho^2). \quad (3.19)$$

When T is very small, $\hat{X} \approx \tilde{X}$, which achieves (3.19). When CSNR is around 5 dB,

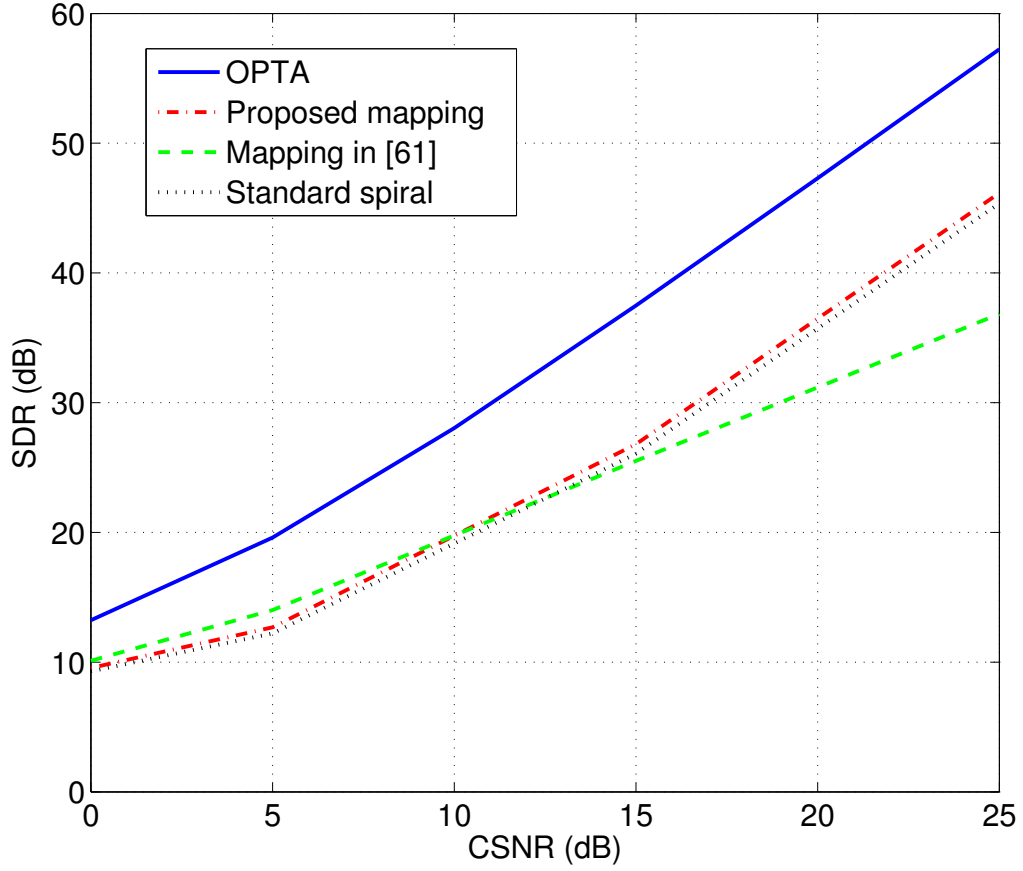


Figure 3.15: Performance evaluation for 1 : 2 systems when $\rho = 0.9$.

where Y and Z are equally important, simplified decoding suffers some degradation loss compared with MMSE decoding.

3.6 Conclusion

We have proposed a novel zero delay bandwidth expansion JSCC scheme for the transmission of memoryless Gaussian samples over AWGN channels when side information is available at the decoder. To take advantage of the side information at the receiver, we use a piece-wise linear function followed by the standard spiral bandwidth expansion mapping to construct a periodic space-filling curve. The proposed

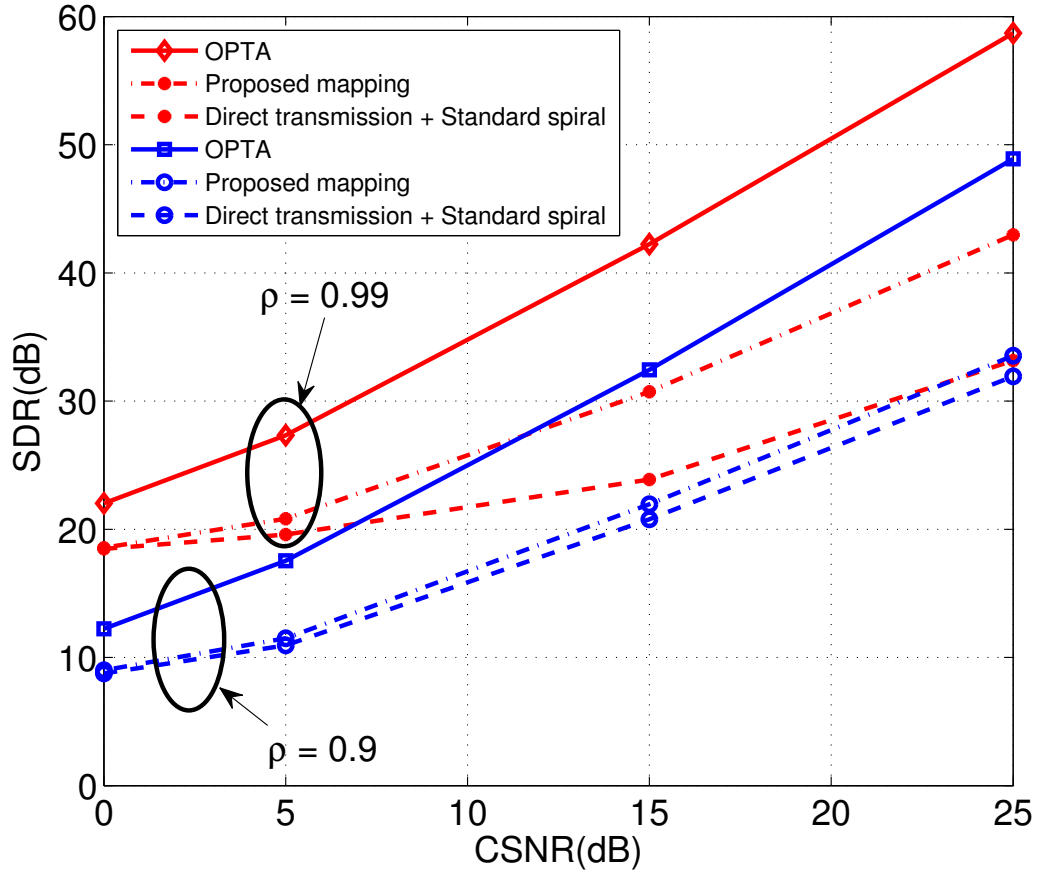


Figure 3.16: Performance evaluation for 6 : 10 systems when $\rho = 0.9$ and $\rho = 0.99$.

scheme manages to reduce the weak noise distortion without increasing the threshold distortion, which is a trade-off for bandwidth expansion schemes in both standard and Wyner-Ziv scenarios. In addition to MMSE decoding, we have proposed a simplified decoding algorithm which performs quite close to MMSE decoding with much less complexity. We also propose a coding scheme which combines the proposed 1 : M bandwidth expansion mapping with the 1 : 1 piece-wise linear mapping to construct a general rate bandwidth expansion mapping for the Wyner-Ziv scenario. Simulation results show that our scheme performs quite well for a wide range of CSNRs, while other existing schemes tend to perform worse for high values of CSNR and for highly

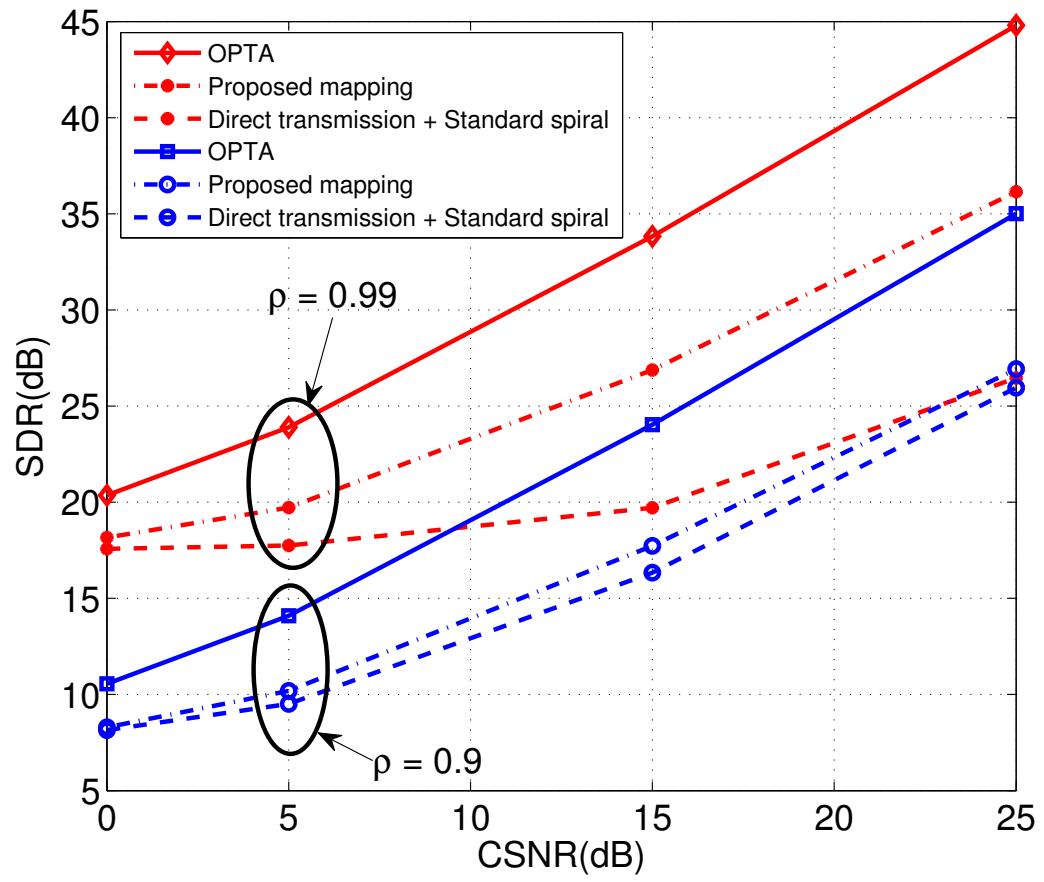


Figure 3.17: Performance evaluation for 9 : 10 systems when $\rho = 0.9$ and $\rho = 0.99$.

correlated side information.

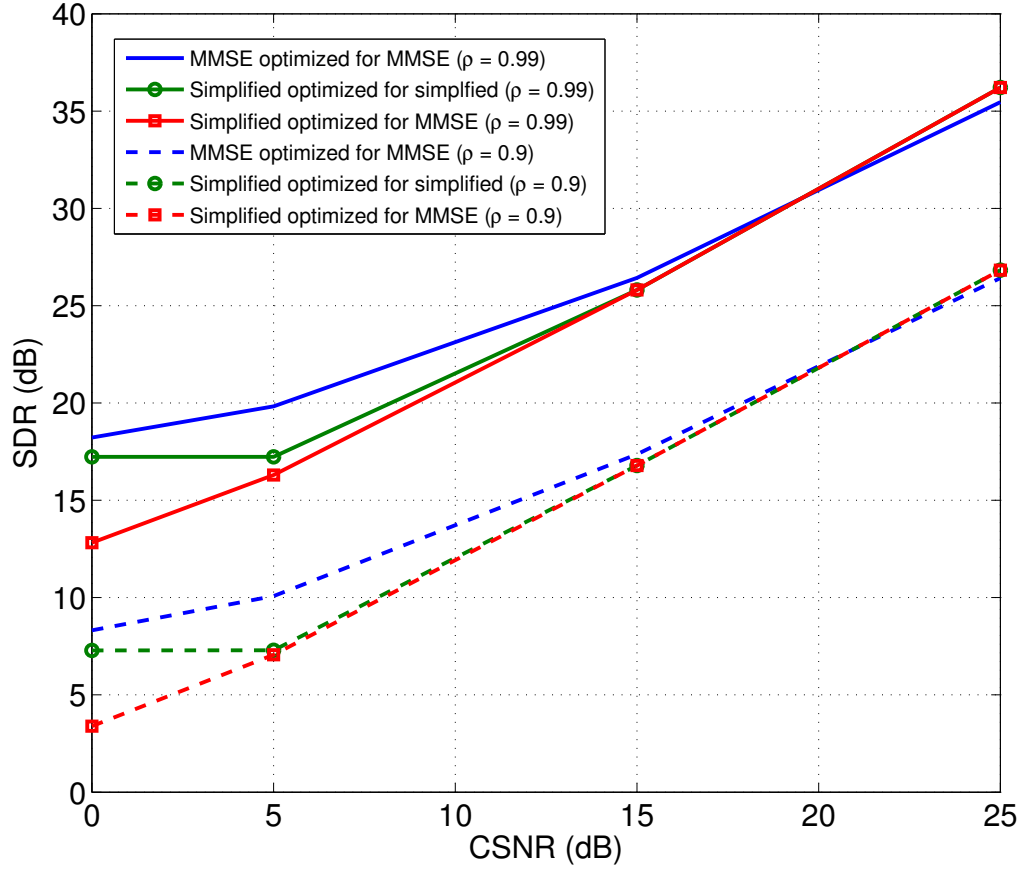


Figure 3.18: Performance comparison between simplified decoding and MMSE decoding for the 1 : 1 scheme when $\rho = 0.9$ and $\rho = 0.99$.

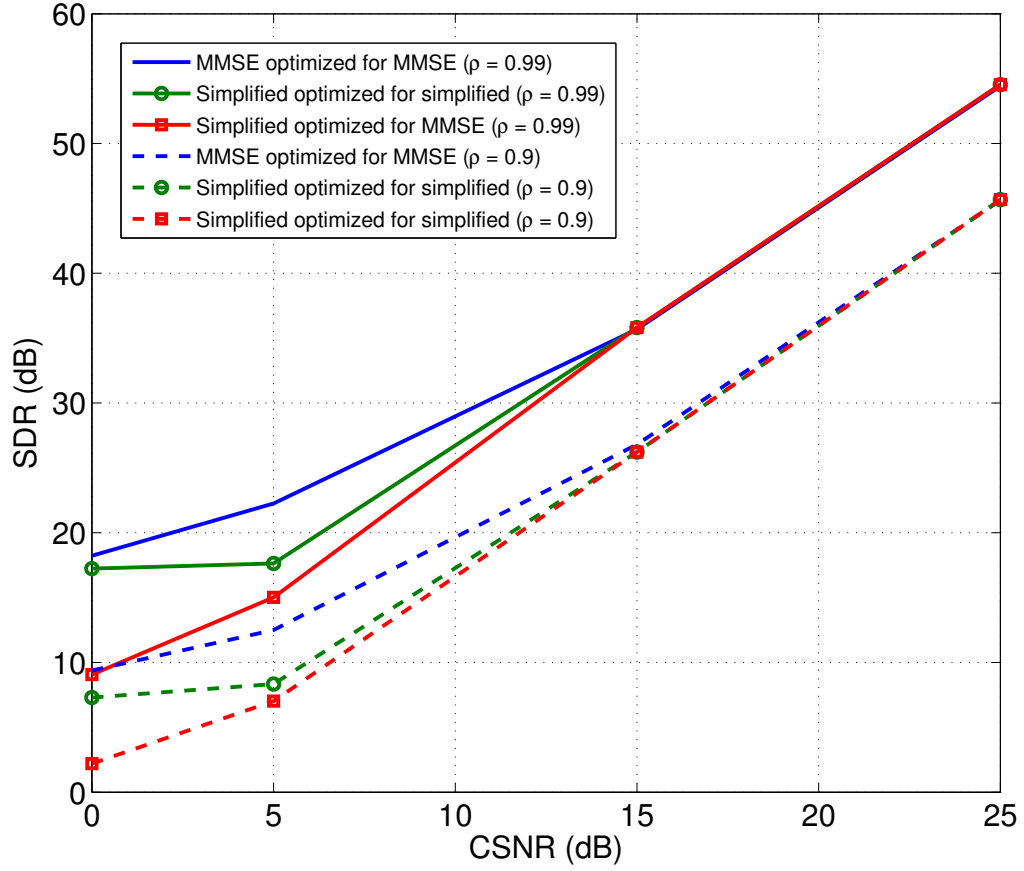


Figure 3.19: Performance comparison between simplified decoding and MMSE decoding for the 1 : 2 scheme when $\rho = 0.9$ and $\rho = 0.99$.

Chapter 4

NON-LINEAR MAPPINGS FOR TRANSMISSION OF CORRELATED SENDERS OVER SEPARATED NOISY CHANNELS

In this chapter, we consider the problem of transmitting correlated Gaussian senders over separated Gaussian channels using analog joint source-channel coding schemes. We propose a novel distributed joint source-channel coding scheme based on zero-delay analog mappings, which exploits the correlation between sources in an efficient way. Different from digital systems, the proposed system offers low delay, low complexity and high robustness. Simulation results show that the proposed scheme performs very close to the theoretical limits, it is very robust against signal to noise ratio mismatch and correlation mismatch, and outperforms existing zero-delay analog mappings, especially for high channel signal to noise ratios and highly correlated sources.

4.1 Introduction

In wireless sensor networks (WSNs), the problem of transmitting correlated measurements from sensors to the fusion center can be solved by distributed source-channel coding schemes in which the correlated measurements are encoded separately at each sensor and estimated jointly at the fusion center of the networks. Digital communication systems based on separation between source and channel coding are optimal for the transmission of correlated sources over separated noisy channels [83]. However, as explained in Chapter 1, analog communication systems have advantages over their digital counterparts. Different from the previous chapters in which we propose and apply analog joint source-channel coding techniques for point-to-point communication, in this chapter we extend the scope of our work to a multi-user scenario.

For multi-terminal communications, not much work on JSCC schemes has appeared in the literature, and most has been for multiple access channels. Schemes using Linear projection were proposed in [20, 21, 25, 60]. An iterative algorithm was proposed in [4] to design mappings for the transmission of correlated sources over separated channels. As in any other iterative optimization algorithms, local optima instead of global optima may be found. Moreover, the algorithm in [4] requires a huge lookup table, which means large delays and high complexity.

In this chapter, we focus on the design of zero-delay analog mappings using space-filling curves for transmitting quadratic Gaussian sources over separated Gaussian channels. The proposed scheme extends our previous work for Wyner-Ziv scenarios where a Gaussian source is transmitted through a point-to-point AWGN channel with correlated side information available at the receiver, which is described in detail in Chapter 3.

The remainder of this chapter is organized as follows. In Section 4.2, we describe the problem formulation and we also provide the derivation of the corresponding theoretical limits. Section 4.3 discusses the proposed schemes for the problem. Simulation results are presented in Section 4.4, and Section 4.5 concludes the chapter.

4.2 Problem Formulation and OPTA

4.2.1 Problem Formulation

The communication system based on analog JSCC over separated channels (or equivalently, with orthogonal multiple access) is depicted in Fig. 4.1. In a general multi-terminal communication system, there are M correlated sources, one at each terminal at the transmitter side. At terminal i , encoder $\alpha_i : \mathcal{R}^{K_i} \rightarrow \mathcal{R}^{L_i}$ maps a vector of source X_i ($[x_{i_1}, x_{i_2}, \dots, x_{i_{K_i}}]$) in $\mathcal{R}^{K_i}$ to Y_i ($[y_{i_1}, y_{i_2}, \dots, y_{i_{L_i}}]$) in $\mathcal{R}^{L_i}$. The bandwidth ratio for terminal i is defined as $R_{c_i} = K_i : L_i$ and represents the number of source symbols per channel symbol. Channels are orthogonal with transition probability $p(\hat{Y}_i|Y_i)$. At the receiver side, the reconstructed $\hat{X}_i$ s in $\mathcal{R}^{K_i}$ are jointly decoded from the M

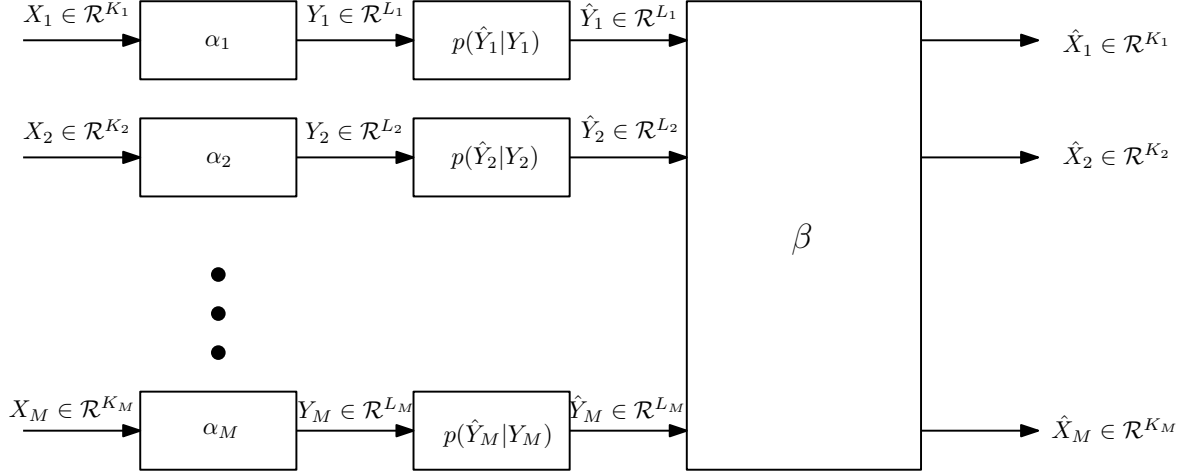


Figure 4.1: Communication system based on analog JSCC with orthogonal multiple access.

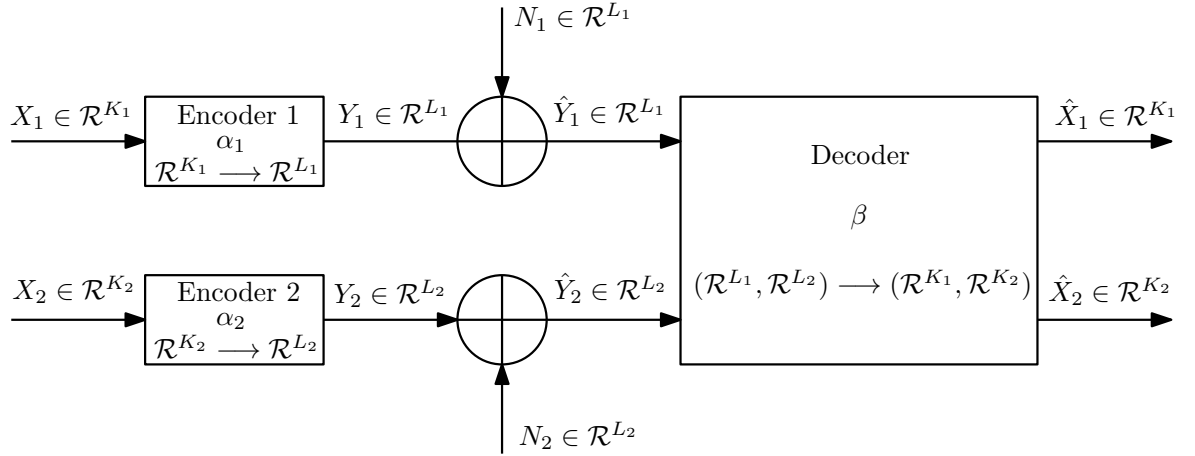


Figure 4.2: Quadratic Gaussian two-terminal analog JSCC communication system with orthogonal AWGN channels.

received vectors $([\hat{Y}_1, \hat{Y}_2, \dots, \hat{Y}_M])$ by the decoder function $\beta : (\mathcal{R}^{L_1}, \mathcal{R}^{L_2}, \dots, \mathcal{R}^{L_M}) \longrightarrow (\mathcal{R}^{K_1}, \mathcal{R}^{K_2}, \dots, \mathcal{R}^{K_M})$.

In this work, we focus on the quadratic Gaussian case ($M = 2$) where the sources at terminal 1 and terminal 2 are jointly Gaussian and the orthogonal multiple access channels are AWGN channels. The corresponding system is depicted in Fig. 4.2. The two sources x_1 and x_2 are assumed to be memoryless zero mean Gaussian random

variables modeled as $x_i = v + \omega_i \sim \mathcal{N}(0, \sigma_{x_i}^2)$, $i = 1, 2$, where $v \sim \mathcal{N}(0, \sigma_v^2)$ is the common information for both sources and $\omega_i \sim \mathcal{N}(0, \sigma_{\omega_i}^2)$ is unique to each source. v and ω_i are independent. Without loss of generality, we further assume that $\sigma_{\omega_1}^2 = \sigma_{\omega_2}^2$ which implies $\sigma_{x_1}^2 = \sigma_{x_2}^2 = \sigma_x^2$. The correlation coefficient $\rho = E[x_1 x_2] / \sigma_x^2 = \sigma_v^2 / \sigma_x^2$ denotes the correlation between x_1 and x_2 . The noise vector N_i in R^{L_i} is independent of x_i , with each noise sample $n_{k_i} \sim \mathcal{N}(0, \sigma_{n_i}^2)$, $i = 1, 2$. Without loss of generality, we assume $\sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_n^2$. As it is common in the literature, we use mean square error, MSE, as the distortion measure and the distortion per symbol for each source is defined as $D_i = \frac{E[\|X_i - \hat{X}_i\|^2]}{K_i}$, $i = 1, 2$. The total distortion is defined as $D = D_1 + D_2$ and the power constraint for each terminal is defined as $P_i = \frac{E[\|Y_i\|^2]}{L_i} \leq P_{max_i}$. The problem is to find α_1 , α_2 and β that minimize D subject to the average power constraint P_{max_1} and P_{max_2} .

4.2.2 Optimal Performance Theoretically Attainable (OPTA)

The OPTA is defined as the minimum achievable total distortion (D) for the given pair of channel signal to noise ratios $\text{CSNR}_i = 10 \log_{10}(\frac{P_i}{\sigma_n^2})$, $i = 1, 2$, and serves as the theoretical bound of the analog JSCC communication system. Since source channel separation holds for transmitting correlated Gaussian sources over separated AWGN channels [83], OPTA is derived by evaluating

$$R_1(D_1) < \frac{C_1}{R_{c_1}} \quad (4.1)$$

$$R_2(D_2) < \frac{C_2}{R_{c_2}} \quad (4.2)$$

$$R_1(D_1) + R_2(D_2) < \frac{C_1}{R_{c_1}} + \frac{C_2}{R_{c_2}} \quad (4.3)$$

where R_{c_i} is the bandwidth ratio defined in Section 4.2.1, $R_i(D_i)$ is the rate distortion function and C_i is the channel capacity.

When AWGN is considered, the channel capacity is given by [18]

$$C_i = \frac{1}{2} \log \left(1 + \frac{P_i}{\sigma_{n_i}^2} \right), \quad (4.4)$$

where $\sigma_{n_i}^2$ is the power of the channel noise and P_i is the average power per channel use.

The rate distortion function of quadratic Gaussian sources is [1]

$$R_1(D_1) \geq \frac{1}{2} \log^+ \left[\frac{1 - \rho^2 + \rho^2 2^{-2R_2(D_2)}}{D_1} \right] \quad (4.5)$$

$$R_2(D_2) \geq \frac{1}{2} \log^+ \left[\frac{1 - \rho^2 + \rho^2 2^{-2R_1(D_1)}}{D_2} \right] \quad (4.6)$$

$$R_2(D_2) + R_1(D_1) \geq \frac{1}{2} \log^+ \left[\frac{(1 - \rho^2) \beta(D_1, D_2)}{2D_1 D_2} \right] \quad (4.7)$$

where

$$\log^+ x = \max(\log x, 0). \quad (4.8)$$

$$\beta(D_1, D_2) = 1 + \sqrt{1 + \frac{4\rho^2 D_1 D_2}{(1 - \rho^2)^2}}. \quad (4.9)$$

4.3 Proposed Communications Scheme

In this section, we propose the distributed analog mappings for transmitting quadratic Gaussian sources over orthogonal AWGNs. The proposed system is depicted in Fig. 4.3.

4.3.1 Encoding

Standard analog mappings such as direct transmission and spiral-like 2 : 1 or 1 : 2 mappings described in Chapter 1 are optimal or near optimal for memoryless Gaussian sources over AWGN channel in point-to-point communications. However, as we will show in Section 4.4, these standard mappings do not perform well for distributed coding when sources at different terminals are correlated. Thus the question is: how

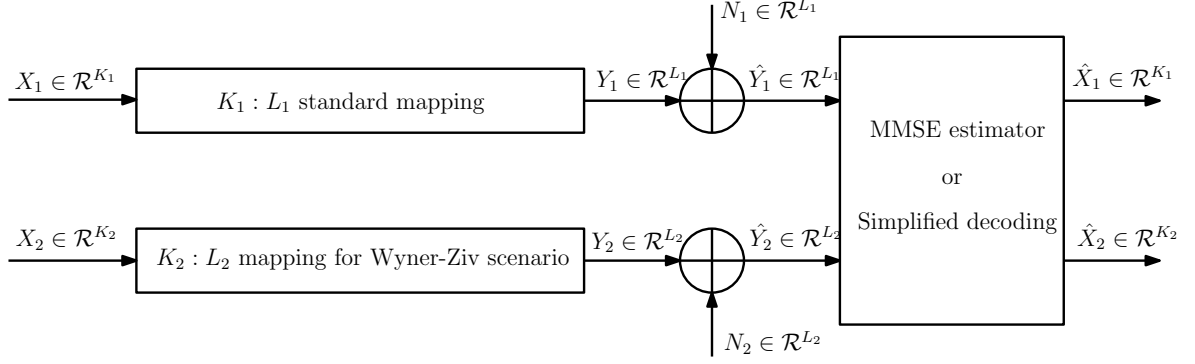


Figure 4.3: Proposed scheme: X_1 is encoded by standard mappings described in Chapter 1 and X_2 is transformed by the mapping designed for Wyner-Ziv scenario, X_1 and X_2 are jointly decoded by MMSE estimator.

can we exploit the correlation between x_{1_j} and x_{2_j} efficiently to design appropriate mappings? The way we approach this problem is by first considering the extreme case of $\sigma_{n_1}^2 = 0$, i.e., when x_1 is perfectly available at the receiver. In this case, we can think of x_1 as the “side information” of x_2 at the receiver and the problem is equivalent to the noisy Wyner-Ziv scenario considered in Chapter 3. In Chapter 3, we proposed periodic mappings to make use of the side information. For instance, the proposed 1 : 1 mapping for Wyner-Ziv scenario was defined as

$$y = \alpha(x) = ((x - \frac{T}{2}) \pmod{T} - \frac{T}{2})/c, \quad (4.10)$$

where T is the fundamental period of the piece-wise linear function, c is a constant used to satisfy the power constraint and $a \pmod{T}$ is the modulo operation with a as the dividend and T as the divisor. An example of the mapping when $T = 2$ is shown in Fig. 4.4. Notice that this mapping basically partitions the source space into segments of size T , and then transmits the difference between x and the centroid of the segment (in this case the mid point of the segment) where x belongs to. This mapping outperforms the linear transformation in the Wyner-Ziv scenario because it effectively avoids sending the redundant information (the centroid) already contained statistically in the “side information”. The optimum values of T decreases almost linearly with the correlation ρ .

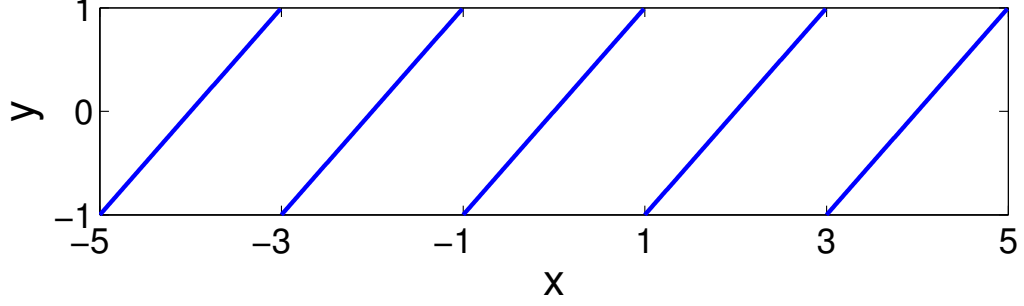


Figure 4.4: 1 : 1 periodic piece-wise linear mapping with $T = 2$

In the distributed problem considered here, $\sigma_{n_1}^2 > 0$, and $\hat{x}_1$ can be approximately represented as $\hat{x}_1 = x_1 + d = v + \omega_1 + d$, where $d \sim \mathcal{N}(0, D_1)$. The idea of applying the piece-wise linear mapping to x_2 still makes sense by thinking of the reconstructed $\hat{x}_1$ as imperfect “side information” of x_1 (different from the noisy Wyner-Ziv scenario where $D_1 = 0$, in the distributed case $D_1 > 0$). Notice that in the distributed case D_2 depends on D_1 since the “correlation” between x_2 and $\hat{x}_1$ is

$$\rho' = \frac{E[\hat{x}_1 x_2]}{\sigma_x \sqrt{\sigma_x^2 + D_1}} = \frac{1}{\sqrt{1 + D_1/\sigma_x^2}} \rho. \quad (4.11)$$

D_2 will be smaller if ρ' is greater (i.e., when D_1 is also smaller). As Y_2 should not contain any information of x_1 when α_2 with the optimum parameter is used, x_1 should be mapped as in point-to-point communications using the standard mapping described in Chapter 1. Symmetrically, we could also apply the piece-wise linear mapping to x_1 and think of $\hat{x}_2$ as “side information”, but we cannot apply the piece-wise linear mapping to both x_1 and x_2 , because the equivalent “side information” in that case would be weakly correlated with the corresponding source, which would result in a degraded performance. Without loss of generality, in the remainder of the paper we choose $\hat{x}_1$ as “side information” of x_2 .

When $K_2 = 1$ and $L_2 \geq 1$, we will use the mapping proposed in Chapter 3 as α_2 . When $K_2 > 1$ and $L_2 = 1$, the bandwidth reduction mapping proposed in [3] will be used as α_2 . The idea of the bandwidth reduction mapping is very similar to the

one that we proposed in Chapter 3. Instead of partitioning the source space in one dimensional space as in our work, the 2 : 1 bandwidth reduction mapping partitions the two dimensional source space into sets of regular hexagons with side length δ , and uses the standard 2 : 1 spiral-like curve to transmit the difference between the center of region and X_2 . The parameter δ behaves similar to the T in periodic mappings and the optimum δ also decreases almost linearly with the correlation ρ in the point-to-point Wyner-Ziv scenario.

4.3.2 Decoding

At the receiver side, the minimum mean square error (MMSE) estimator is used to obtain the estimates of both sources as

$$\begin{aligned} (\hat{X}_1, \hat{X}_2) &= E[X_1, X_2 | \hat{Y}_1, \hat{Y}_2] \\ &= \iint (X_1, X_2) P(X_1, X_2 | \hat{Y}_1, \hat{Y}_2) dX_1 dX_2 \\ &= \iint (X_1, X_2) \frac{P(\hat{Y}_1 | X_1) P(\hat{Y}_2 | X_2) P(X_1, X_2)}{P(\hat{Y}_1, \hat{Y}_2)} dX_1 dX_2, \end{aligned} \quad (4.12)$$

where $P(X_1, X_2)$ is the joint pdf of X_1 and X_2 . Unfortunately, we need to calculate the integral numerically due to the non-linearity of the mappings. The problem of the numerical method is that its complexity increases exponentially as K_i increases, and it also increases very fast as ρ and CSNR increase. When $R_{c_2} > 1$, it becomes infeasible to use the numerical method for high correlation and CSNRs. Next, we propose a simplified decoding method by modifying the simplified decoding algorithm proposed in Chapter 3.

As X_1 is mapped using a standard mapping and is nearly independent of Y_2 , we first estimate X_1 ($\hat{X}_1$) from $\hat{Y}_1$ by using MMSE (when $R_{c_1} = 1$) or ML. To estimate X_2 , we can use a procedure similar to that of in Section 3.3.3.2, but with the following modification:

1. Equation (3.14) should be changed to

$$\tilde{X}_2 = E[X_2 | \hat{X}_1] = \rho' \hat{X}_1, \quad (4.13)$$

where ρ' is the estimated correlation between x_2 and $\hat{x}_1$.

2. When the bandwidth reduction mapping in [3] is used, two indexes (k_i and k_j) for each hexagon are used, and (3.15) becomes

$$\begin{aligned} k_{i_0} &= \lfloor \tilde{x}_{2_1}/(3\delta) \rfloor \\ k_{j_0} &= \lfloor \tilde{x}_{2_2}/(\sqrt{3}\delta) \rfloor \end{aligned} \tag{4.14}$$

or

$$\begin{aligned} k_{i_0} &= 2 \times \lfloor \tilde{x}_{2_1}/(3\delta - 0.5) + 0.5 \rfloor \\ k_{j_0} &= 2 \times \lfloor \tilde{x}_{2_2}/(\sqrt{3}\delta - 0.5) + 0.5 \rfloor \end{aligned} \tag{4.15}$$

depending on which set of indexes represent the hexagon whose center is closer to $\tilde{X}_2$.

3. When the bandwidth reduction mapping in [3] is used, there are 6 neighboring hexagons around k_{i_0} and k_{j_0} , and we need to choose the value of k_i and k_j that maximizes $P(X_1, \tilde{X}_2)$.

4.3.3 Power allocation

If the total transmission power of all terminals, $P = P_1 + P_2$, is fixed, equal power allocation ($P_1 = P_2 = \frac{P}{2}$) does not necessarily yield the minimum total distortion (D_{min}) for given α_1 and α_2 even if $R_{c_1} = R_{c_2}$. The reason is that the proposed scheme is asymmetric since x_1 is transmitted using a standard mapping and x_2 is transmitted using periodic mapping).

Let us assume that $\hat{P}_1$ is the optimum power for terminal 1 and $\hat{P}_2$ is the optimum power for terminal 2 and $\hat{P}_1 \leq \hat{P}_2$. When there is no constraint on P_1 and P_2 , we will use $\hat{P}_1$ to transmit x_1 and $\hat{P}_2$ to transmit x_2 to achieve D_{min} . However, what can do we if P_1 needs to be greater than $\hat{P}_1$ and P_2 needs to be smaller than $\hat{P}_2$? We will show in the sequel that as long as $P_1 \geq \hat{P}_1$, $P_2 \leq \hat{P}_2$ and $P_1 + P_2 = P$, the proposed scheme can still achieve the minimum distortion D_{min} using P_1 and P_2 .

Assume terminal 1 transmits n samples of x_1 and terminal 2 transmits n samples of x_2 . Since x_1 and x_2 are identical distributed Gaussian random variables, D_{min} is achieved if we use α_2 to transmit x_1 using $\hat{P}_2$ and we also use α_1 to transmit x_2 using $\hat{P}_1$, which means that if we send k out of n samples of x_1 using α_1 and $\hat{P}_1$, k out of n

samples of x_2 using α_2 and $\hat{P}_2$, $n - k$ out of n samples of x_1 using α_2 and $\hat{P}_2$ and $n - k$ out of n samples of x_2 using α_1 and $\hat{P}_1$, D_{min} is still achieved. In this case, the average transmission power for terminal 1 is

$$P_1 = \frac{k\hat{P}_1 + (n - k)\hat{P}_2}{n}. \quad (4.16)$$

and the average transmission power of terminal 2 is

$$P_2 = \frac{k\hat{P}_2 + (n - k)\hat{P}_1}{n}. \quad (4.17)$$

If n is large enough, one can choose k to satisfy the power constraint while still achieving the same total distortion as with optimal $\hat{P}_1$ and $\hat{P}_2$. When $P_1 = P_2 = \frac{P}{2}$, we just need to make $k = \frac{n}{2}$. The idea is depicted in Fig. 4.5.

4.4 Simulation Results

In this section, we present simulation results for the proposed scheme and compare them with the corresponding theoretical limit (OPTA) and results from existing schemes. Specifically, we evaluate the performance of the proposed scheme for the cases in which $R_{c_1} = R_{c_2} = 1 : 1$ (Case A: bandwidth matching), $R_{c_1} = 1 : 2$, $R_{c_2} = 1 : 1$ (Case B: bandwidth expansion) and $R_{c_1} = 1 : 1$, $R_{c_2} = 2 : 1$ (Case C: bandwidth reduction). In case A, α_1 is the direct transmission mentioned in Chapter 1, and α_2 is the periodic piece-wise linear mapping proposed in Chapter 3. In case B, α_1 is the standard spiral-like curve mentioned in Chapter 1 and α_2 is the same as in case A. In case C, α_1 is the same as in case A and α_2 is the mapping proposed in [3]. For cases A and B, we use the numerical MMSE decoding, while for case C we use the simplified decoding described in Section 4.3.2

We first assume equal power constraint for the two terminals ($P_1 = P_2$) and we measure the performance in terms of signal to total distortion ratio defined as $\text{SDR} = 10\log_{10}(\frac{2\sigma_x^2}{D_1 + D_2})$ versus $\text{CSNR} = 10\log_{10}(\frac{P_1}{\sigma_n^2}) = 10\log_{10}(\frac{P_2}{\sigma_n^2})$. Similar to Chapter 3, we choose $\rho = 0.9$ and $\rho = 0.99$ for the simulations. We also apply the optimum power allocation described in Section 4.3.3 to further reduce the distortion. We use

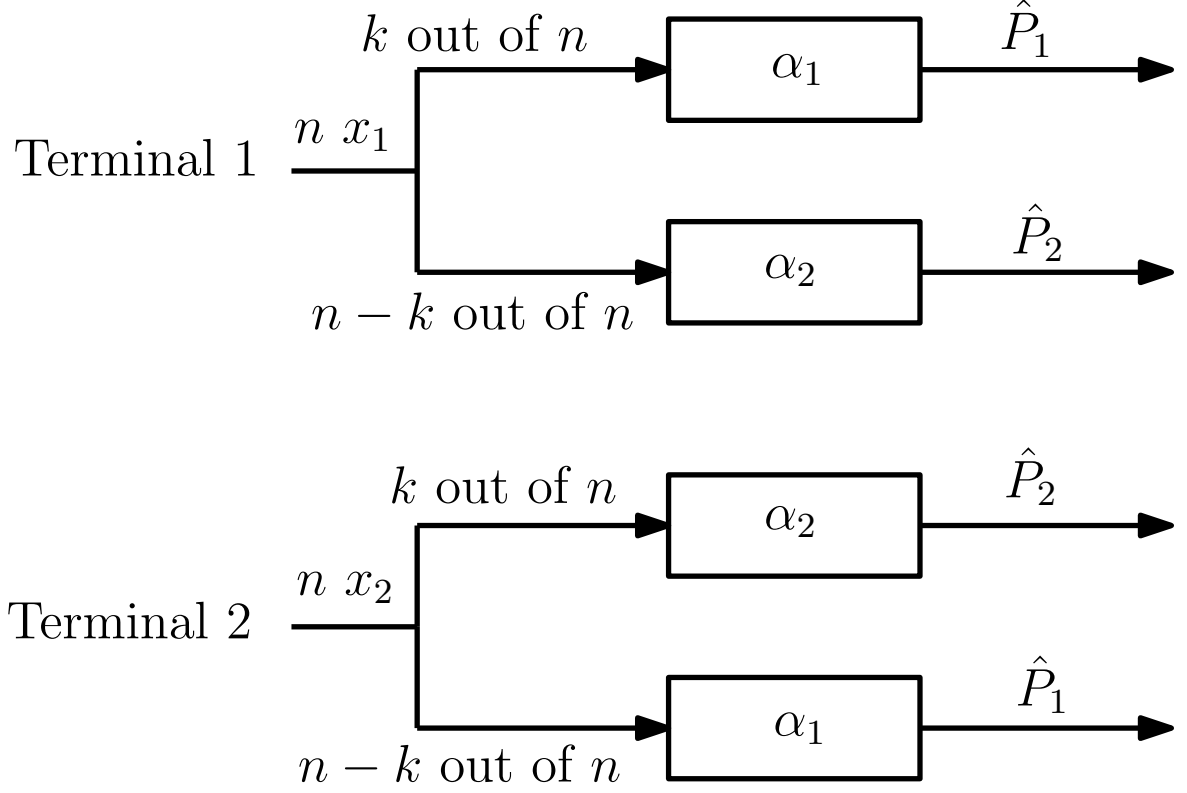


Figure 4.5: k out of n samples of x_1 are transmitted using α_1 and $\hat{P}_1$, k out of n samples of x_2 are transmitted using α_2 and $\hat{P}_2$, $n - k$ out of n samples of x_1 are transmitted using α_2 and $\hat{P}_2$ and $n - k$ out of n samples of x_2 are transmitted using α_1 and $\hat{P}_1$.

$k = \frac{n}{2}$ so that $P_1 = P_2$. The scheme achieves D_{min} while maintaining the same CSNR. In Fig. 4.6 and Fig. 4.7, we plot the results for case A, while the results for case B are plotted in Fig. 4.8 and Fig. 4.9, and the results for case C, they are plotted in Fig. 4.10 and Fig. 4.11.

For comparison, we include the performance obtained when optimized standard mappings ([47] and [68]) are applied to both x_1 and x_2 . Specifically, for case A, the optimum 1 : 1 linear mapping is used for both x_1 and x_2 , for case B, the optimal 1 : 2 spiral-like bandwidth expansion mapping is applied to x_1 and the optimal linear mapping is applied to x_2 . When comparing with the corresponding OPTA, we assume that both transmitter and receiver have perfect knowledge of ρ and CSNR, and thus all

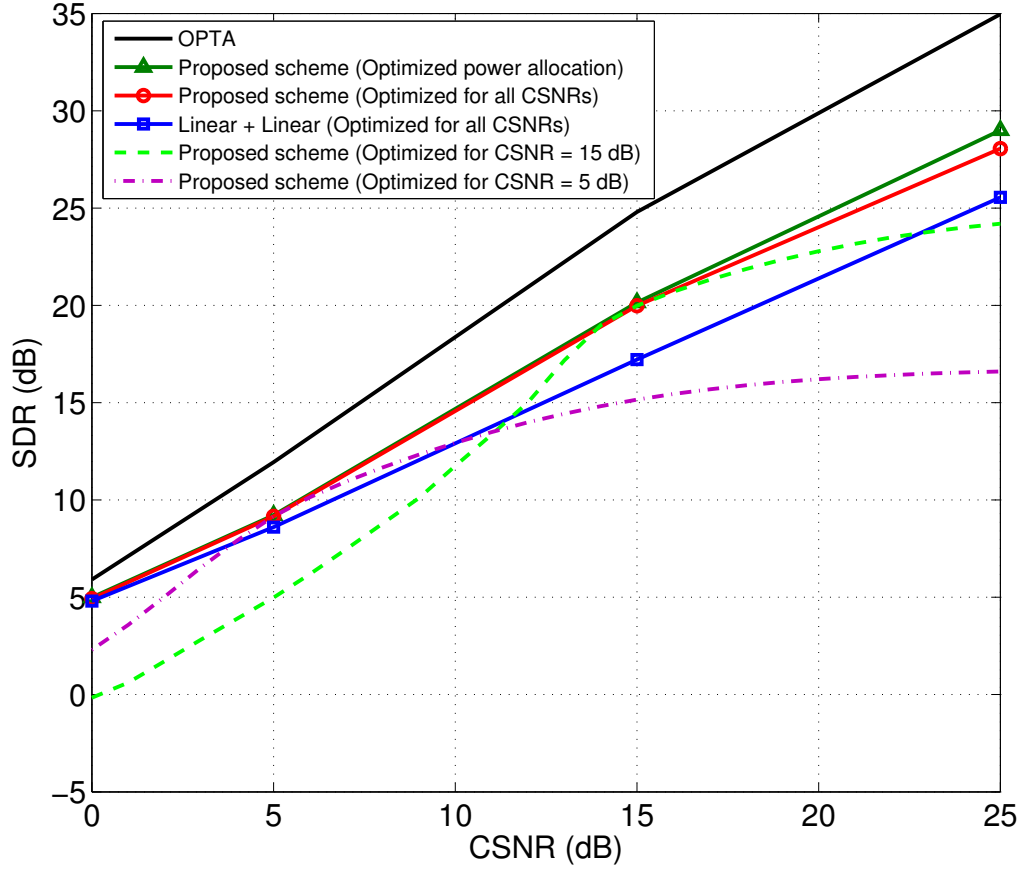


Figure 4.6: Performance evaluation for case A when $\rho = 0.99$.

the schemes are optimized for each analyzed ρ and each CSNR. The optimum values of T in the proposed scheme for case A are summarized in Table 4.1. Compared with Table 3.1, the optimum value of T depends not only on ρ but also on CSNR: the higher the ρ and the CSNR are, the smaller the optimum T is. This is expected since the optimum value of T is related to the actual correlation coefficients (between source and distorted side information) which depends on both ρ and CSNR. The optimum value of T in case B is similar to the one in case A and shown in Table 4.2. The optimum values of δ in the proposed scheme for case C are provided in Table 4.3. For low CSNRs, the optimum δ is very small and for high CSNRs δ follows the same rules as the optimum

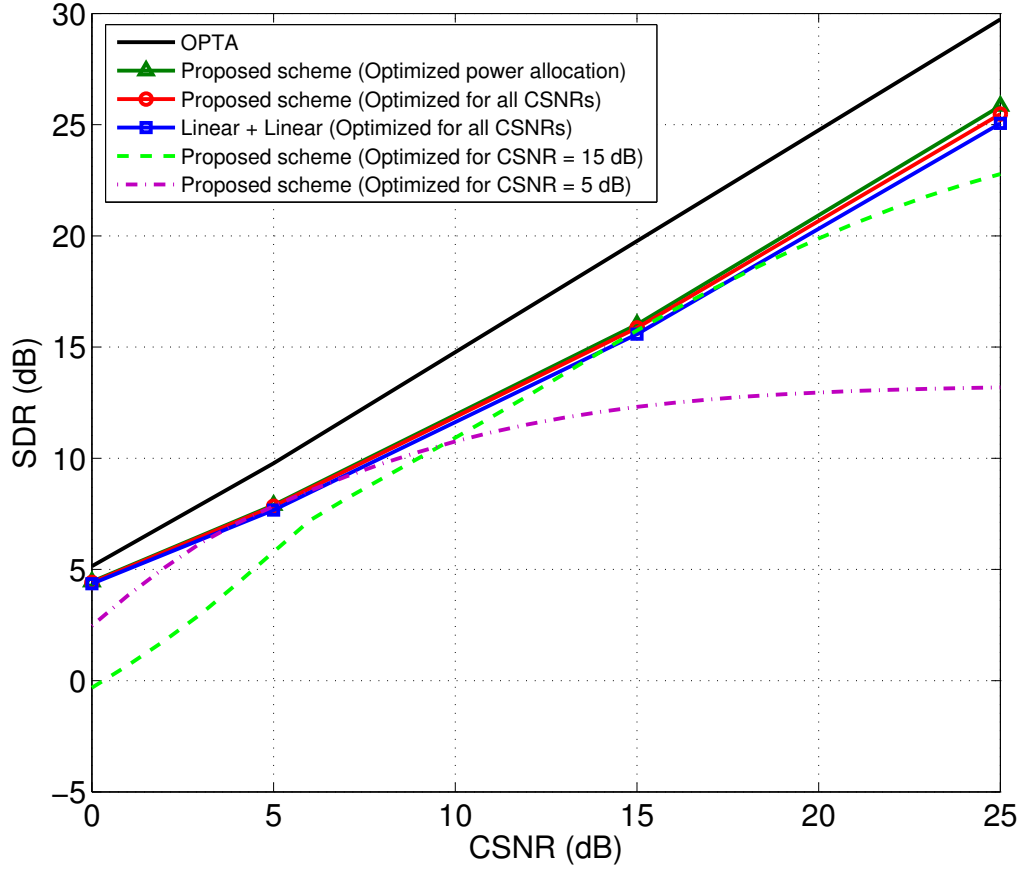


Figure 4.7: Performance evaluation for case A when $\rho = 0.9$.

Table 4.1: Optimized Parameter (T) of the proposed scheme for Case A

T	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	6.1	4.8	4.2	3.8
$\rho = 0.99$	6.2	3.6	1.7	1.2

T . This is because simplified decoding is used in case C, and in Chapter 3 we showed that the optimum parameter of the simplified decoding matches the ones for MMSE only for high CSNRs, and for low CSNRs the optimum parameter is very small.

From the figures, we clearly see that the proposed scheme outperforms the

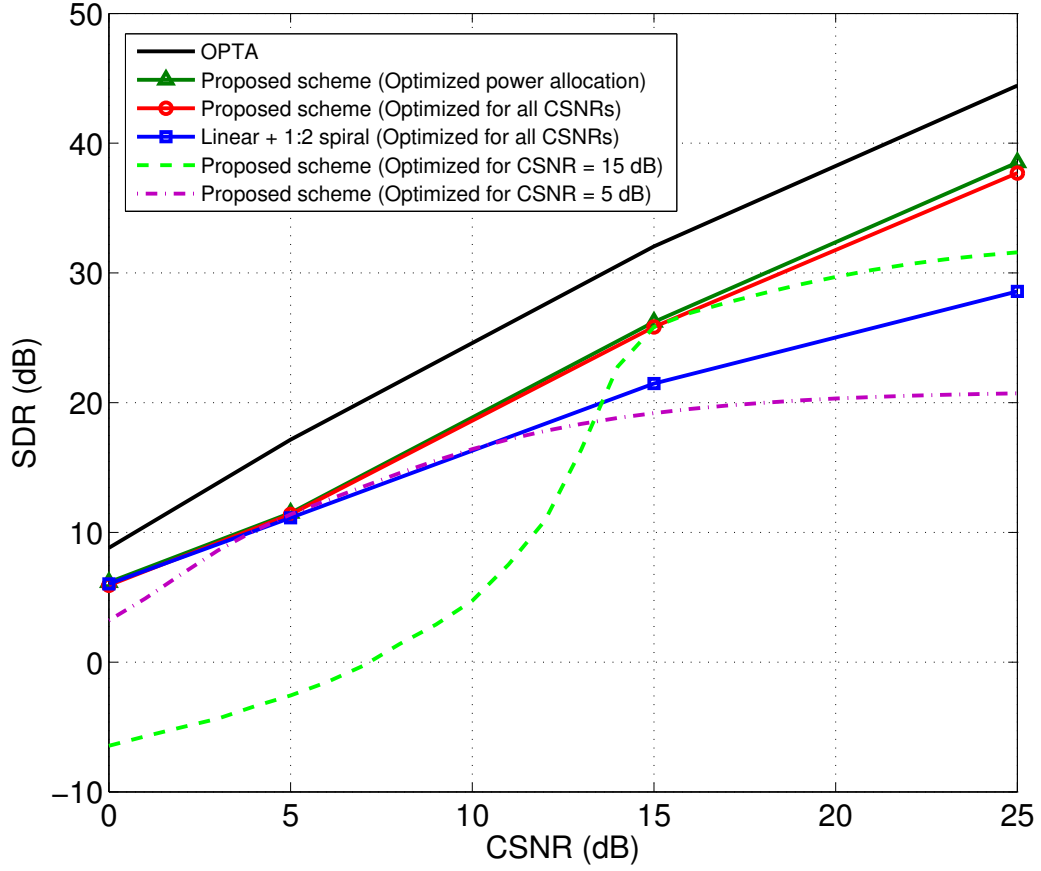


Figure 4.8: Performance evaluation for case B when $\rho = 0.99$.

Table 4.2: Optimized Parameter (T) of the proposed schemes for Case B

T	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	7.8	6.5	3.5	3.8
$\rho = 0.99$	8.2	7.8	1.3	1.1

other zero-delay systems, especially for large CSNRs and for highly correlated sources ($\rho = 0.99$). Moreover, the gap between the proposed scheme and OPTA is almost asymptotically constant, while other schemes present a much wider and fast increasing gap. Notice that the OPTA is actually the theoretical limit for the case when K_i and

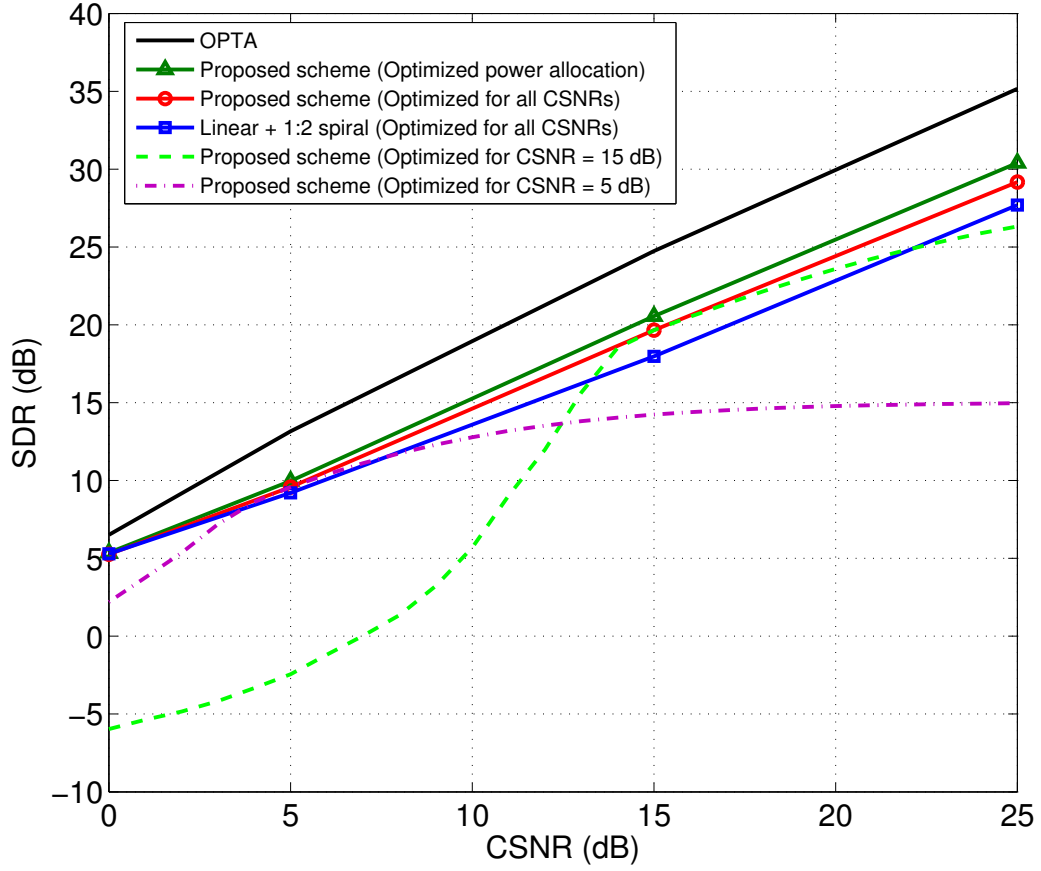


Figure 4.9: Performance evaluation for case B when $\rho = 0.9$.

Table 4.3: Optimized Parameter (δ) of the proposed schemes for Case C

δ	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	0.2	0.2	1.8	1.7
$\rho = 0.99$	0.2	0.1	0.7	0.6

L_i go to infinity. For zero-delay systems (K_i and L_i are small) in this paper, we can expect an even smaller gap from the actual theoretical limits. From Fig. 4.7 and Fig. 4.9, we notice that the performance of the proposed schemes when $\rho = 0.9$ are close to the schemes used for comparison. The reason is that the optimized T of the piece-wise

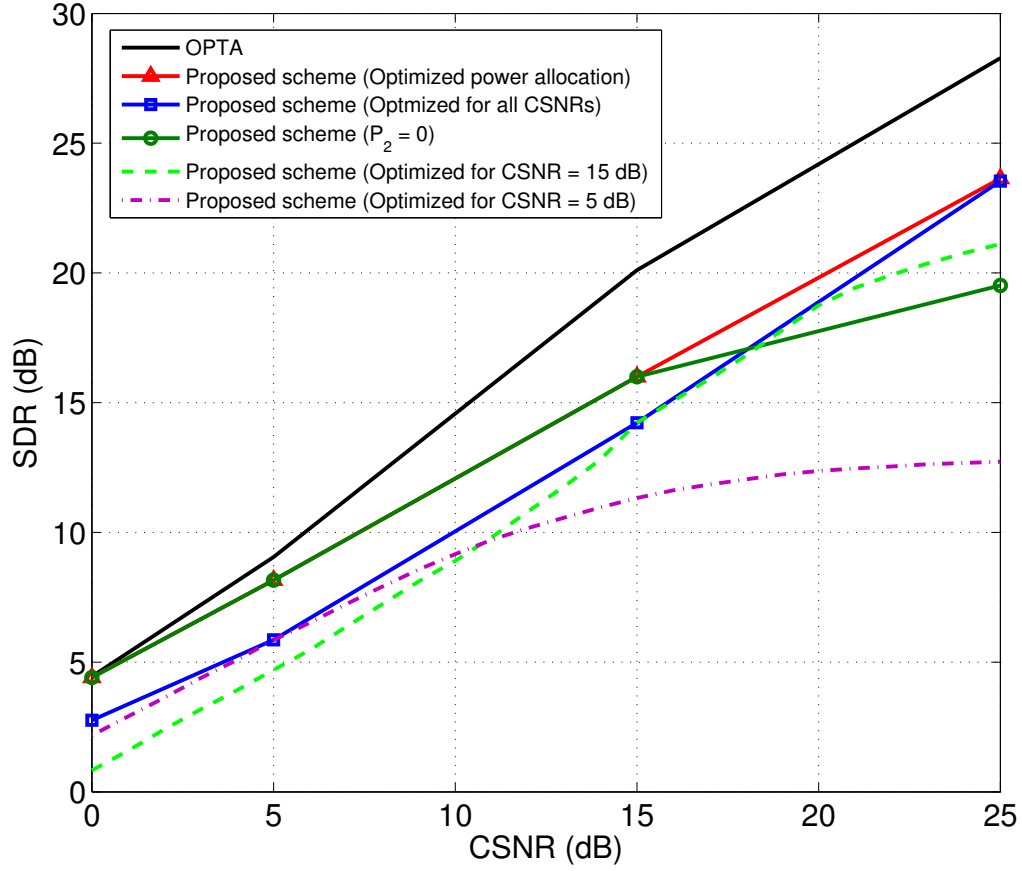


Figure 4.10: Performance evaluation for case C when $\rho = 0.99$.

linear mapping in this case is relatively large ($T > 3.8$), and as a result, source x_2 will most likely belong to the segment $([-T/2, T/2])$ and in this case, α_2 is practically a linear mapping.

With respect to the case of optimal power allocation. For case A, the improvement from the power allocation is almost limited to high CSNRs and high correlation. For case B, the improvement from the power allocation is more significant for lower correlation. Optimum power allocation tries to balance the distortion from α_1 and α_2 . For case C, as the simplified decoding is used, the optimum power allocation in this case is very interesting: $\hat{P}_2 = 0$ for low CSNRs, which means all the power is allocated

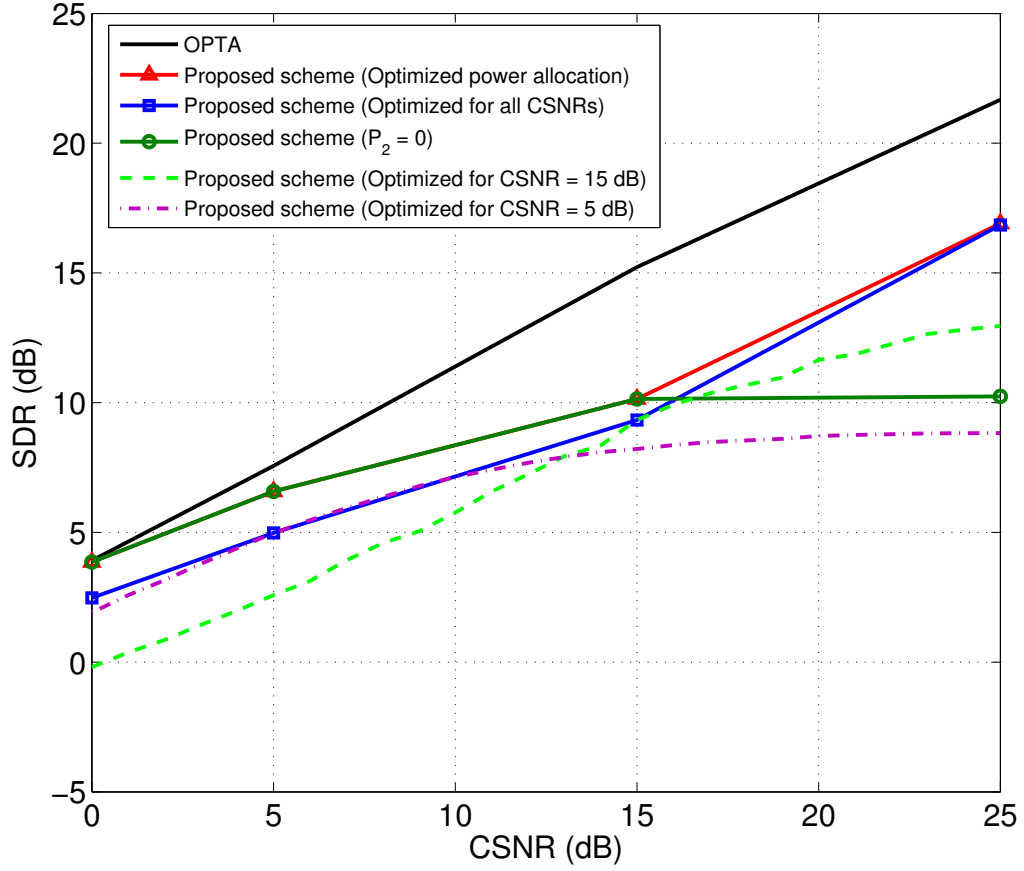


Figure 4.11: Performance evaluation for case C when $\rho = 0.9$.

to α_1 . In this case, $\hat{X}_2 = \tilde{X}_2$. We mentioned in Section 4.3 that D_2 depends on D_1 , and when using simplified decoding, the effect of D_1 on D_2 is more significant as ρ' and $\hat{X}_1$ all depend on D_1 . Therefore, when CSNR is low, a high ρ' and good $\hat{X}_1$ is the key.

We also evaluate the robustness of the proposed scheme to CSNR mismatch by using α_1 , α_2 , β and ρ optimized for CSNR = 5 dB and 15 dB. From all figures, we can see that the overall robustness characteristic of analog mappings under CSNR mismatching still holds quite well in the proposed scheme. Both the SDR degradation when the actual CSNR is below the actual CSNR and the SDR improvement when the

actual CSNR is above the optimized CSNR are graceful. Compared with case A, the robustness in case B under CSNR mismatch is worse. This is due to the fact that in case B α_1 also has a parameter Δ .

Finally, we evaluate the robustness of the proposed scheme to correlation mismatch. Specifically, α_1 , α_2 and β are optimized for $\rho = 0.9$ and 0.99 and the corresponding CSNR. When CSNR is low (5 dB), the proposed scheme shows good robustness to correlation mismatch as shown in Fig. 4.13, 4.15 and 4.17. However, when CSNR is high (15 dB), the robustness is relatively poor as shown in Fig. 4.12, 4.14 and 4.16. The reason is that when CSNR is low, the optimum values for T and δ are quite the same for all ρ , while when CSNR is high the optimum values for T and δ change dramatically, as shown in Table 4.1, Table 4.2 and Table 4.3, which increases the sensitivity to correlation mismatch.

4.5 Conclusion

We have proposed a distributed zero-delay JSCC scheme based on analog mappings for the transmission of quadratic Gaussian sources over separated noisy Gaussian channels. The proposed scheme exploits the correlation efficiently by applying specifically designed mappings to one of the two sources. We also propose a simplified decoding which greatly reduce the complexity of the proposed scheme as opposed to the numerical MMSE decoding. A power allocation strategy is proposed to minimize the distortion while still satisfying the power constraint. Simulation results show that the proposed schemes perform very close to the theoretical limits, outperform existing analog zero-delay mapping schemes, and is robust against CSNR mismatch and correlation mismatch.

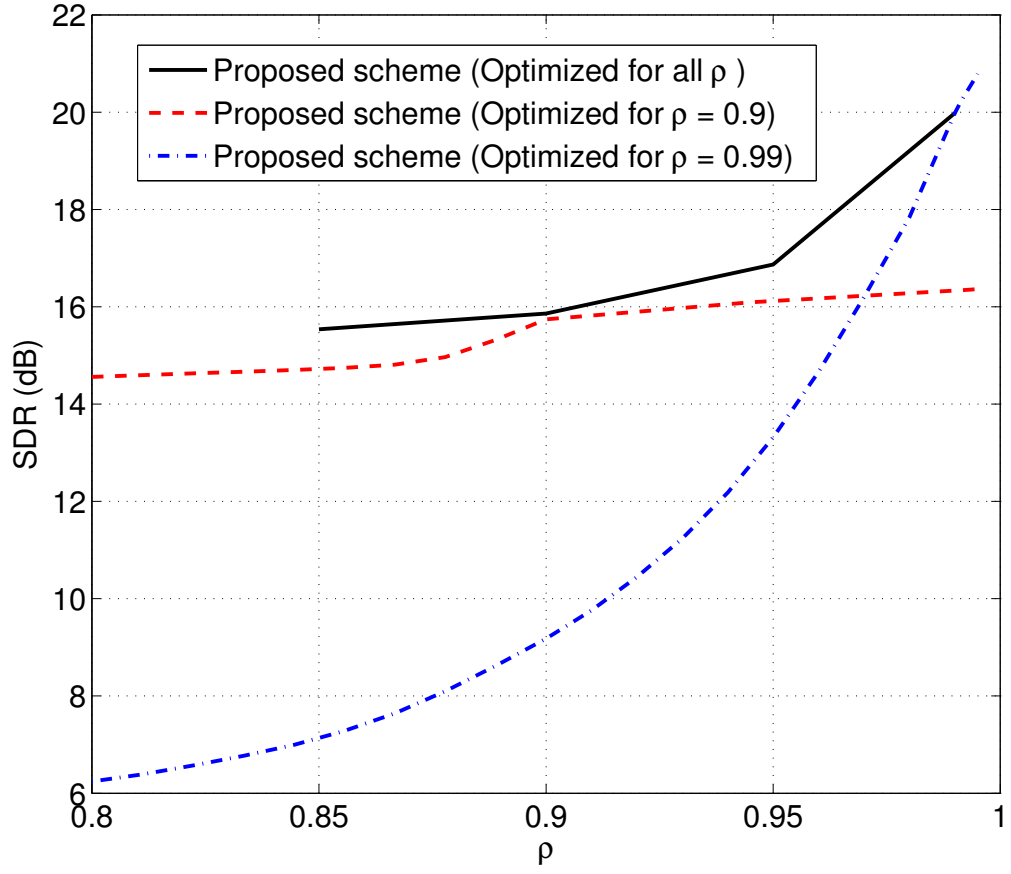


Figure 4.12: Robustness evaluation of the proposed scheme for case A under correlation mismatch when CSNR = 15 dB.

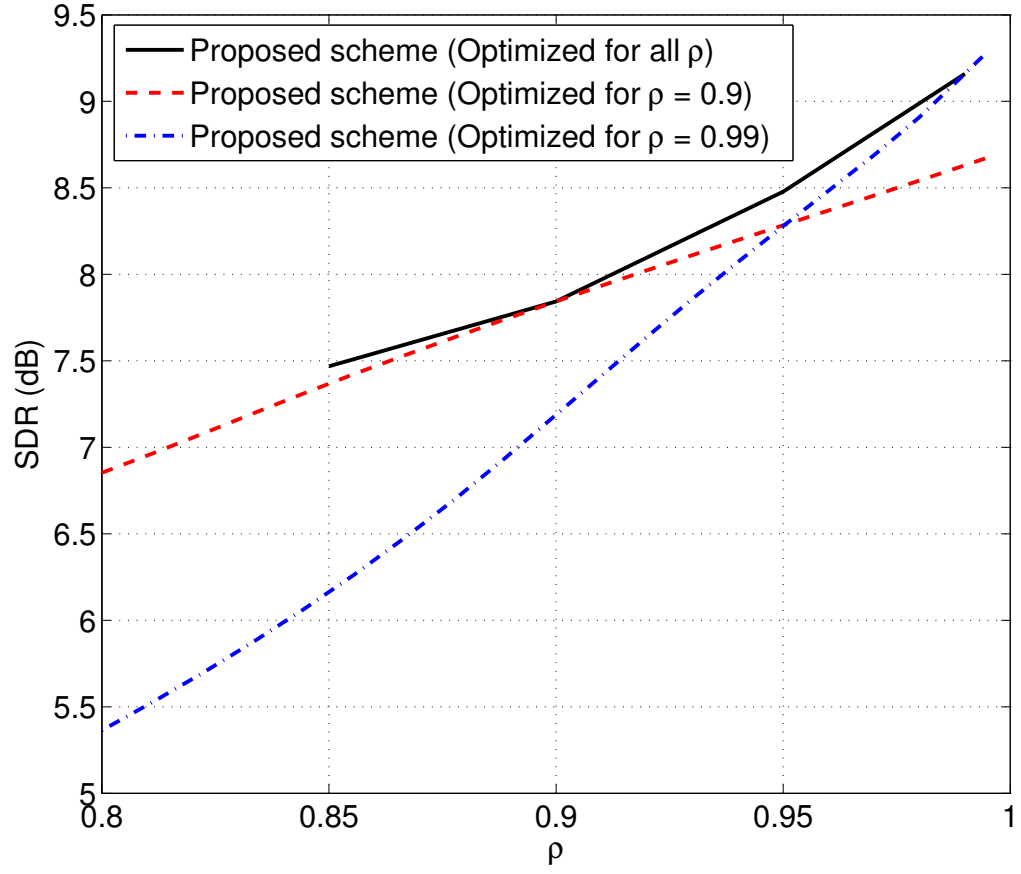


Figure 4.13: Robustness evaluation of the proposed scheme for case A under correlation mismatch when CSNR = 5 dB.

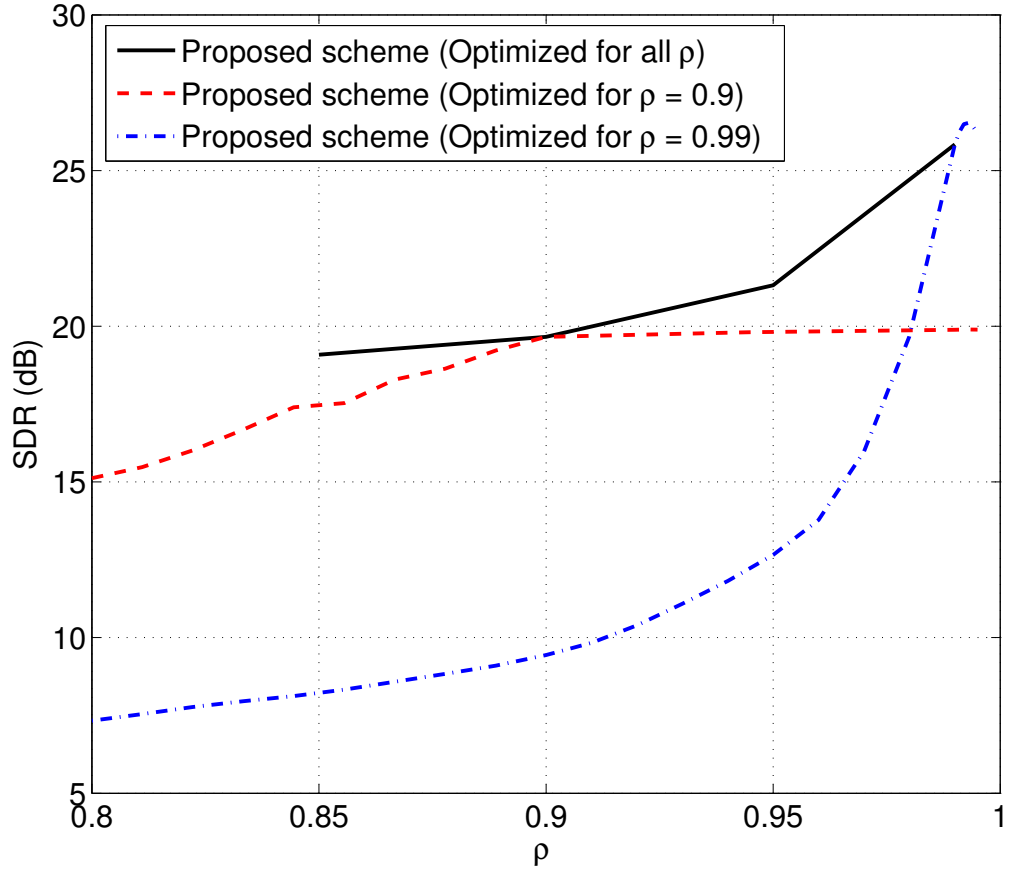


Figure 4.14: Robustness evaluation of the proposed scheme for case B under correlation mismatch when CSNR = 15 dB.

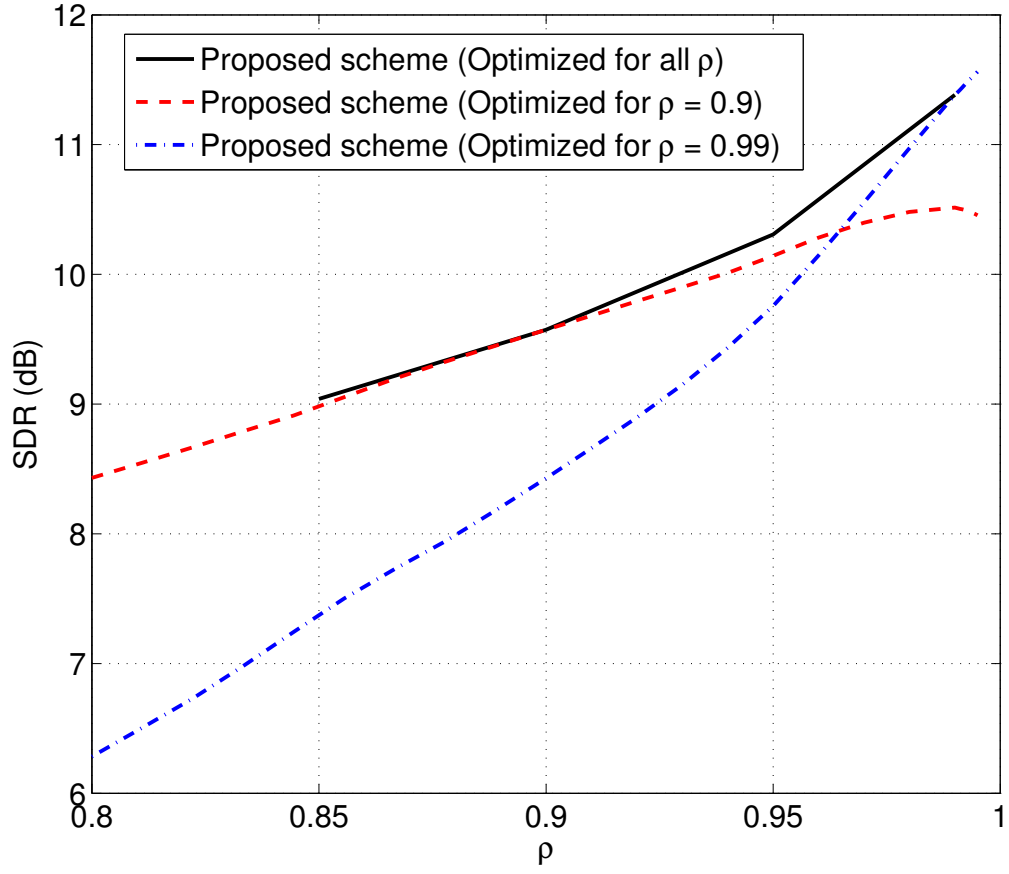


Figure 4.15: Robustness evaluation of the proposed scheme for case B under correlation mismatch when CSNR = 5 dB.

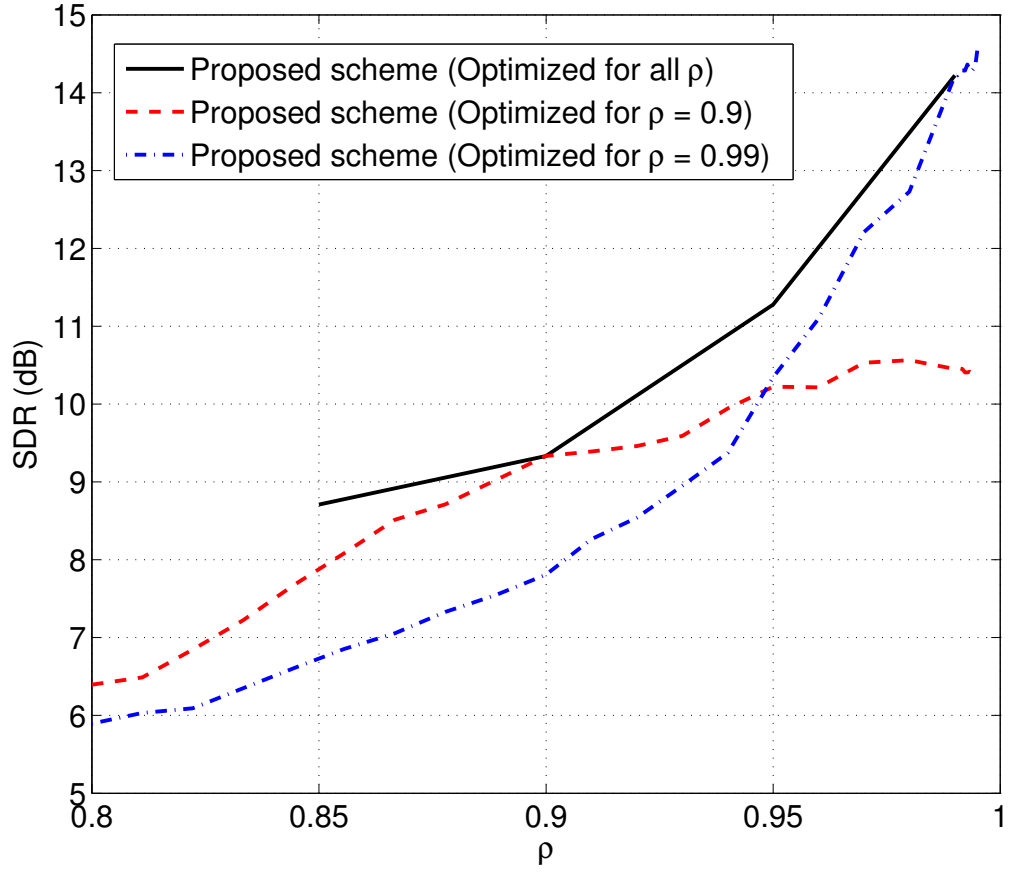


Figure 4.16: Robustness evaluation of the proposed scheme for case C under correlation mismatch when CSNR = 15 dB.

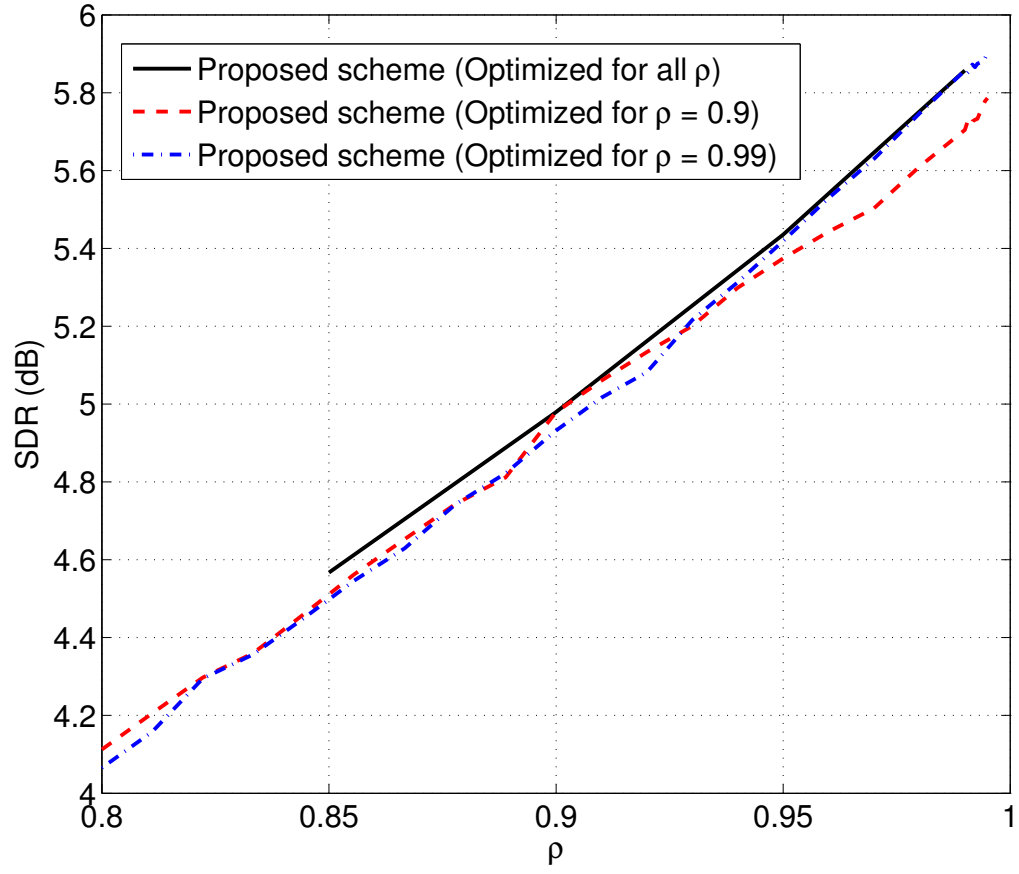


Figure 4.17: Robustness evaluation of the proposed scheme for case C under correlation mismatch when CSNR = 5 dB.

Chapter 5

NON-LINEAR MAPPINGS FOR THE MULTIPLE ACCESS CHANNEL

In this chapter, we extend our study of analog joint source-channel coding (JSCC) to multiple access channel (MAC) scenarios. In the first scenario, independent Gaussian sources are transmitted over a Gaussian MAC and side information is available at the receiver. In the second scenario, inter-correlated Gaussian sources are transmitted over a Gaussian MAC without side information. A distributed analog JSCC system based on the serial concatenation of Shannon-Kotel'nikov mappings and a CDMA-like access scheme is proposed. Compared with other orthogonal schemes where the rate of each user is determined only by the rate of the Shannon-Kotel'nikov mapping, in the proposed scheme the rate of each user is incorporated in the access scheme itself, which facilitates the design. Simulation results show that in both scenarios the proposed scheme outperforms systems based on the use of standard mappings. In the second scenario, the proposed scheme performs on par with the best existing analog schemes, which are specifically designed for a fixed transmission rate, but with the advantage of being more flexible in terms of transmission rate assignment for different users and easiness of adaption to different rates.

5.1 Introduction

In Chapter 3, we designed analog mappings for the transmission of a memoryless Gaussian source over an AWGN channel when side information is available at the receiver, while Chapter 4 dealt with the transmission of memoryless quadratic Gaussian sources over separated AWGN channels. However, the MAC setting is closer to reality in applications such as wireless sensor networks, since the channels to the central unit are shared by all the devices. Although most of the work on zero-delay analog mappings

deals with point-to-point communications, recent extensions to MAC environments have appeared in the literature. Previous work on analog JSCC for the MAC includes [23, 24, 27, 67] in which the quadratic Gaussian and Gaussian MAC case is treated. In [40, 43], an analog JSCC scheme based on CDMA for the transmission of independent Gaussian sources over a Gaussian MAC without side information is proposed. These works show promising results for the use of analog JSCC schemes in MAC environments. However, most of them focus on equal transmission rate, and the extension to other rates is not straightforward.

In this chapter, we propose a distributed zero-delay analog JSCC scheme for the transmission of Gaussian sources over a Gaussian MAC when side information is available at the receiver or when sources are inter-correlated. The proposed scheme first encodes each user's data by applying Shannon-Kotel'nikov mappings designed for point-to-point communications, and then utilizes a CDMA-like access scheme which converts the MAC into orthogonal channels and provides users with the flexibility to transmit at different rates. Different from standard CDMA, the input to the access scheme is discrete in time and continuous in amplitude, but the basic idea of channel orthogonalization still holds.

The remainder of this chapter is organized as follows. The formulation for the two MAC scenarios under consideration is presented in Section 5.2. Section 5.3 describes the proposed system and illustrates the CDMA-like access scheme using a Hadamard matrix. The theoretical limits are also discussed in Section 5.3. Simulation results are presented in Section 5.4, and Section 5.5 concludes the chapter.

5.2 Problem Formulation

In this section, we define the two MAC scenarios under consideration.

5.2.1 Scenario 1: Independent Gaussian sources with side information at the receiver

Fig. 5.1 depicts scenario 1. We consider a Gaussian MAC with M users transmitting memoryless independent Gaussian sources with $s_i \sim \mathcal{N}(0, \sigma_s^2)$, $i = 1 \dots M$. The corresponding side information symbol at the common receiver is modeled as $s'_i = s_i + u_i$, $i = 1, \dots, M$, where $u_i \sim \mathcal{N}(0, \sigma_u^2)$ is independent of s_i so that $s'_i \sim \mathcal{N}(0, \sigma_s^2 + \sigma_u^2)$. The correlation coefficient $\rho = E[s_i s'_i] / \sqrt{E[s_i^2] E[s'^2_i]} = \sigma_s / \sqrt{(\sigma_s^2 + \sigma_u^2)}$ denotes the correlation between s_i and $\hat{s}_i$. Each encoder consists of an encoding function α_i , which maps one or more source symbols to one or more channel symbols, y_i denotes channel symbol of user i and is transmitted over the Gaussian MAC with additive noise $n \sim \mathcal{N}(0, 1)$ (we assume the variance of n is 1 without loss of generality). The received signal is $r = \sum_{i=1}^M y_i + n$. At the common receiver, the decoder β_i reconstructs $\hat{s}_i$ from s'_i and r . As it is common in the literature, we use mean square error as the distortion measure and the distortion of user i is defined as $D_i = E[\|s_i - \hat{s}_i\|^2]$. The power constraint is defined as the total average power $P = \sum_{i=1}^M P_i \leq P_{max}$ where P_i is the average power of y_i . The problem is to find the encoders (α_i) and decoders (β_i) that minimize the distortion D_i subject to the power constraint.

5.2.2 Scenario 2: Quadratic (Correlated) Gaussian Sources without side information

Fig. 5.2 depicts scenario 2. Different from the previous case, we only consider a Gaussian MAC with two-users ($M = 2$). The quadratic Gaussian sources s_1 and s_2 are assumed to be memoryless zero mean Gaussian random variables modeled as $s_i = v + \omega_i \sim \mathcal{N}(0, \sigma_{s_i}^2)$, $i = 1, 2$, where $v \sim \mathcal{N}(0, \sigma_v^2)$ is the common information for both sources and $\omega_i \sim \mathcal{N}(0, \sigma_{\omega_i}^2)$ is unique to each source. v and ω_i are independent. Without loss of generality, we further assume that $\sigma_{\omega_1}^2 = \sigma_{\omega_2}^2$ which implies $\sigma_{s_1}^2 = \sigma_{s_2}^2 = \sigma_s^2$. The correlation coefficient $\rho = E[s_1 s_2] / \sigma_s^2 = \sigma_v^2 / \sigma_s^2$ denotes the correlation between s_1 and s_2 . Similar to scenario 1, encoder α_i separately maps s_i to y_i , which is transmitted over the Gaussian MAC. At the receiver, $\hat{s}_i$ is decoded only from the received signal r by

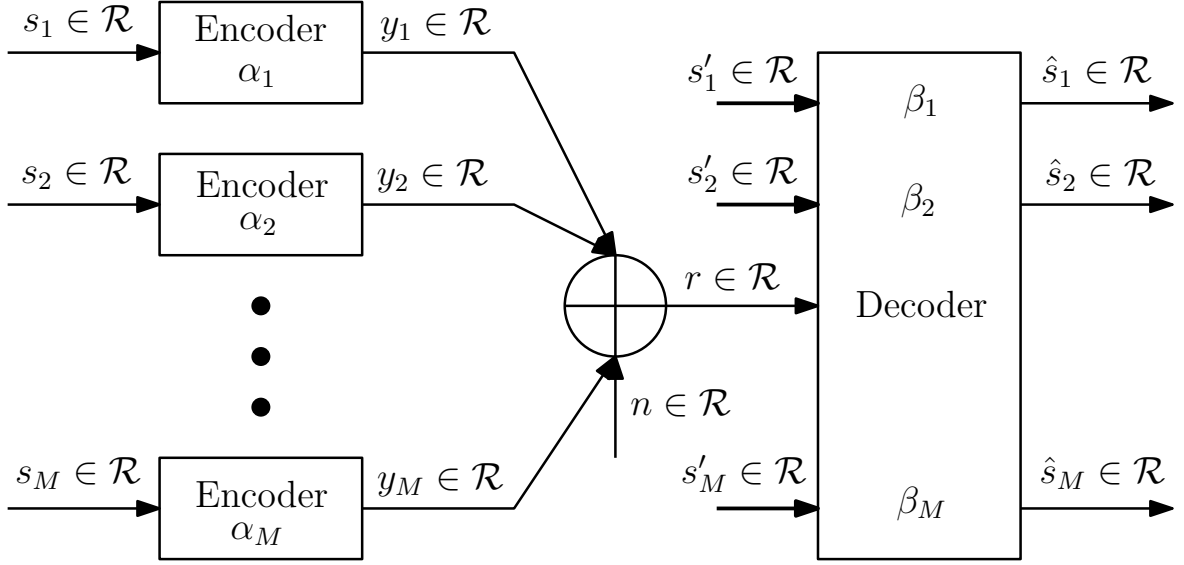


Figure 5.1: Independent Gaussian sources transmitted using an analog JSCC system over a Gaussian MAC when side information is available at the receiver.

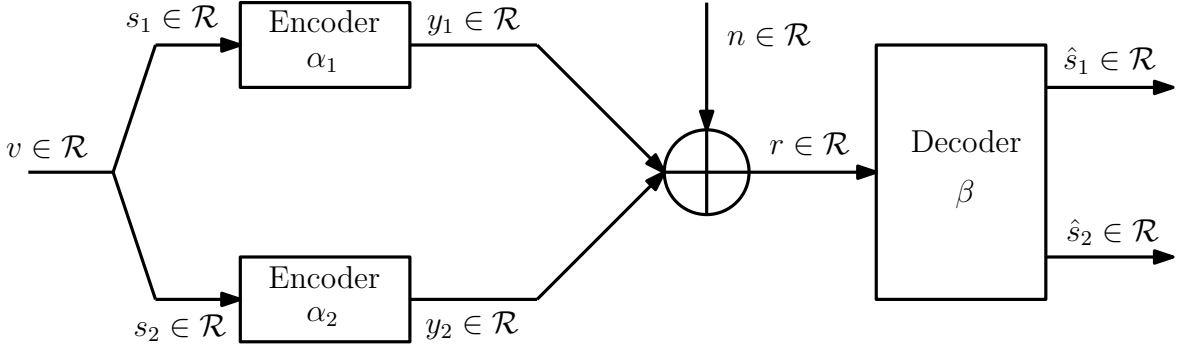


Figure 5.2: Analog JSCC communication system for the transmission of quadratic Gaussian sources over a Gaussian MAC. No side information is available at the receiver.

the decoding function β . The power constraints and distortion measures are defined as in scenario 1. The problem is to find α_1 , α_2 and β that minimize the total distortion $D = \sum_{i=1}^2 D_i$ subject to the power constraint.

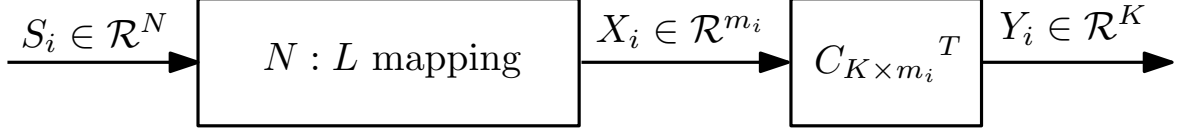


Figure 5.3: Encoder α_i . Sequence of source symbols S_i are first encoded by a Shannon-Kotel'nikov mapping and then transformed by sub matrix $C_{K \times m_i}^T$ of a CDMA-like access scheme codebook.

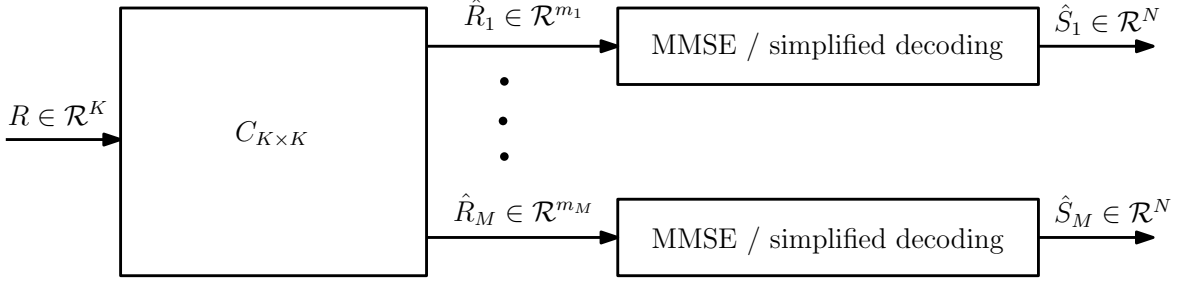


Figure 5.4: Decoder β . The received sequence is multiplied by codebook $C_{K \times K}$. For each user the decoding is performed as explained in Chapter 3 and Chapter 4 for point-to-point communication scenarios.

5.3 Proposed Communications Scheme

In Chapter 3, we have proposed analog JSCC schemes based on Shannon-Kotel'nikov mappings for transmitting a memoryless Gaussian source over an AWGN channel when side information is available at the receiver, while Chapter 4 considers the transmission of quadratic Gaussian sources over separated AWGN channels. Therefore, if we could manage to convert the MAC into orthogonal AWGN channels without reducing channel capacity, we would be able to encode the source using the mappings proposed in Chapter 3 or Chapter 4, depending on the scenario. Based on this idea, the proposed scheme combines the analog mappings proposed in Chapter 3 and Chapter 4 with a CDMA-like access scheme which converts the MAC into several orthogonal AWGN channels. The proposed encoder α_i is shown in Fig. 5.3. The decoder β is shown in Fig. 5.4. The CDMA-like access scheme is explained in detail next.

5.3.1 CDMA-Like Access Scheme

Let x_i^j denote the j th symbol user i wants to transmit prior to the access scheme. For M users to transmit signals over the MAC in K time intervals, we utilize a $K \times K$ orthogonal matrix (e.g., a Hadamard matrix), $C_{K \times K}$ ($K \geq M$). $C_{K \times K}$ is also known as the codebook. Assume user i wants to transmit m_i ($m_i < K$) symbols ($X_i = [x_i^1 x_i^2 \dots x_i^{m_i}]$) over K time intervals, then the codebook needs to be divided so that user i is assigned m_i columns and $\sum_{i=1}^M m_i = K$. Moreover, each user's columns are scaled by $\eta_i = \frac{1}{\sqrt{m_i}}$ for normalization purposes. The columns assigned to user i constitute the submatrix $C_{K \times m_i}$ so that $C_{K \times K}$ is given by

$$C_{K \times K} = [C_{K \times m_1} | C_{K \times m_2} \dots | C_{K \times m_M}], \quad (5.1)$$

where $C_{K \times m_i}$ is

$$C_{K \times m_i} = \begin{bmatrix} \underline{c}_i^1 \\ \underline{c}_i^2 \\ \vdots \\ \underline{c}_i^K \end{bmatrix} = \begin{bmatrix} c_i^1(1) & c_i^1(2) & \dots & c_i^1(m_i) \\ c_i^2(1) & c_i^2(2) & \dots & c_i^2(m_i) \\ \vdots & \vdots & \ddots & \vdots \\ c_i^K(1) & c_i^K(2) & \dots & c_i^K(m_i) \end{bmatrix} \quad (5.2)$$

At the k th interval, the transmitted symbol of user i is

$$y_i^k = X_i \underline{c}_i^{kT}, \quad (5.3)$$

where $\underline{c}_i^k$ is the k th row of $C_{K \times m_i}$ and $\underline{x}^T$ is the transpose of vector $\underline{x}$. The received signal at the k th interval is

$$r^k = \sum_{i=1}^M y_i^k + n^k = \sum_{i=1}^M X_i \underline{c}_i^{kT} + n^k, \quad (5.4)$$

where n^k is the k th noise symbol.

Note that $C_{K \times K}^T C_{K \times K} = D$ is a $K \times K$ diagonal matrix with M distinct values (m_i entries of value K/m_i). Therefore, the Gaussian MAC can be transformed into K orthogonal Single-Input Single-Output (SISO) AWGN channels by multiplying the

K received symbols $R = [r^1 r^2 \dots r^K]$ with $C_{K \times K}$ at the receiver. The equivalent j th received symbol of user i is:

$$\hat{r}_i^j = \frac{K}{m_i} x_i^j + N_{MAC} \underline{d}_i^j, \quad (5.5)$$

where $N_{MAC} = [n^1 n^2 \dots n^K]$ is the noise vector in K time intervals and $\underline{d}_i^j$ is the j th column of $C_{K \times m_i}$.

As the codebook $C_{K \times K}$ is normalized, the average power of x_i^j is P_i , and the capacity of the j th orthogonal AWGN channel of user i is

$$C_i^j = \frac{1}{2} \log_2 \left(1 + \frac{K P_i}{m_i} \right), \quad (5.6)$$

The scheme capacity is

$$C_{scheme} = \frac{1}{K} \left(\sum_{i=1}^M \sum_{j=1}^{m_i} C_i^j \right) = \frac{1}{K} \left[\sum_{i=1}^M \sum_{j=1}^{m_i} \left(\frac{1}{2} \log_2 \left(1 + \frac{K P_i}{m_i} \right) \right) \right], \quad (5.7)$$

where we have divided by K because the proposed system uses the MAC K times. The capacity of the Gaussian MAC is [18]

$$\begin{aligned} C_{MAC} &= \frac{1}{2} \log_2(1 + P) = \frac{1}{2} \log_2 \left(1 + \sum_{i=1}^M P_i \right) \\ &= \frac{1}{2} \log_2 \left[1 + \sum_{i=1}^M \sum_{j=1}^{m_i} \left(\frac{P_i}{m_i} \right) \right] \end{aligned} \quad (5.8)$$

It can be easily shown that C_{scheme} and C_{MAC} are equal if and only if

$$\frac{P_i}{m_i} = \frac{P_j}{m_j} \quad \forall j \neq i \text{ with } 1 \leq i, j \leq M. \quad (5.9)$$

(5.9) indicates the optimal allocation for the CDMA-like access scheme. For the two user case with a fixed m_1 and m_2 where $P = P_1 + P_2$, the optimal power for user i is

$$P_i^* = \frac{m_i}{m_1 + m_2} P. \quad (5.10)$$

Likewise, we can also derive the optimal m_1^* and m_2^* for a given P_1 and P_2 such that $m_1^* + m_2^* = K$.

$$m_i^* = \frac{P_i}{P_1 + P_2} K. \quad (5.11)$$

Since m_i is the number of columns assigned to user i , it should be an integer. Thus we approximate m_i^* as

$$m_i^* = \lceil \frac{P_i}{P_1 + P_2} \rceil K. \quad (5.12)$$

For the case of two users ($M = 2$), we can break up (5.7) to obtain the information rate achieved by each user

$$\begin{cases} R_1 = \frac{1}{K} \left(\sum_{i=1}^{m_1} \frac{1}{2} \log_2 \left(1 + \frac{K P_1}{m_1} \right) \right) \\ R_2 = \frac{1}{K} \left(\sum_{i=m_1+1}^K \frac{1}{2} \log_2 \left(1 + \frac{K P_2}{m_2} \right) \right). \end{cases} \quad (5.13)$$

Fig. 5.5 shows the maximal rates achieved by each user for the two user case when $P_1 = 8$ and $P_2 = 1$. The MAC capacity region is obtained from $R_1 + R_2 \leq C_{MAC}$. As indicated in (5.9), there exists a point in the curve where the CDMA-like access scheme achieves the MAC capacity.

In sum, if user i encodes s_i using Shannon-Kotel'nikov mappings with code rate $R_{c_i} = N/L$ and use the CDMA-like access scheme with parameters m_i and K , the transmission rate of user i , R'_i , which is the number of the number of source symbols per channel symbol is given by

$$R'_i = R_{c_i} \times \frac{m_i}{K}. \quad (5.14)$$

(5.14) makes explicit an important advantage of the proposed system over existing analog schemes for the MAC: the overall transmission rate does not only depend on R_{c_i} but also on m_i/K , which means that the rate can be controlled in the access scheme itself. This provides the proposed scheme with a lot of flexibility to adapt to different rates by adjusting either R_{c_i} or m_i/K or both. In the first MAC scenario, as the source symbols are independent for different users, R'_i can be set by adjusting m_i/K and R_{c_i} independently, so that the rates of different users are not necessarily equal. In the second MAC scenario, the total number of source symbols is the same for both users, and thus $R'_1 = R'_2$ is required. However, the proposed scheme still benefits from the

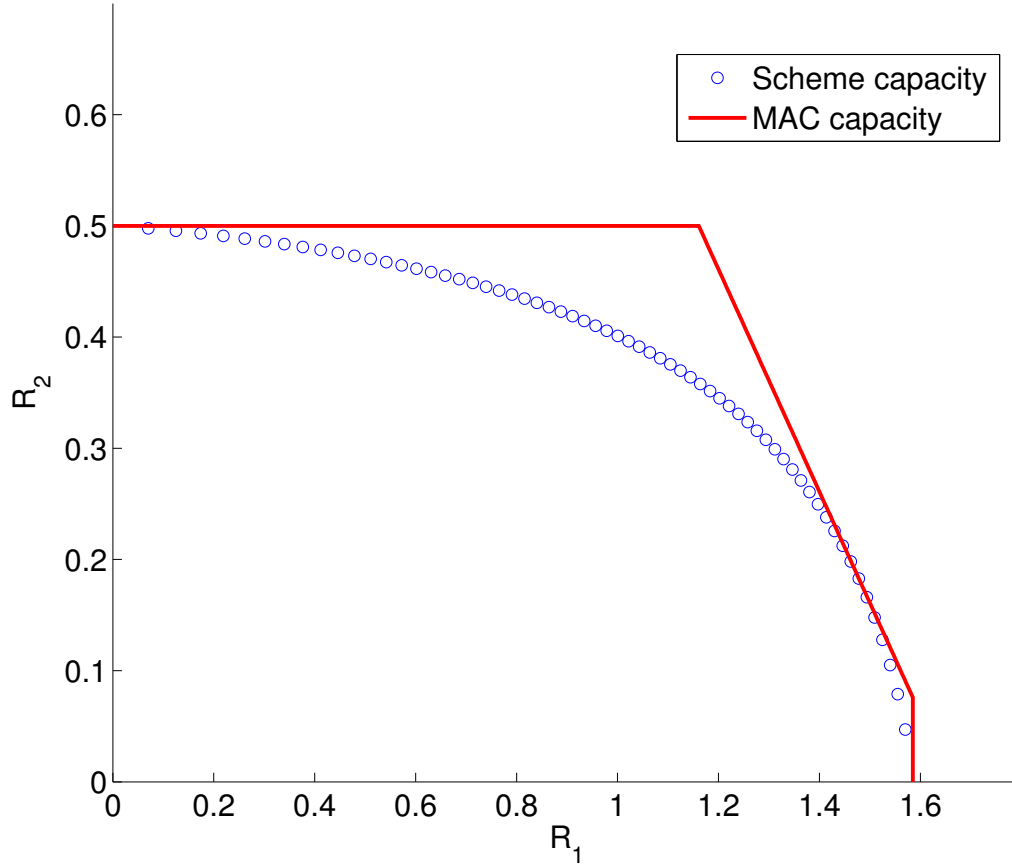


Figure 5.5: MAC capacity and the proposed scheme capacity for $P_1 = 8$, $P_2 = 1$ and the codebook size $K = 64$. Each point of the dotted curve is obtained from (5.13) by sweeping m_1 from 0 to 64.

fact that it can be easily adapted to different R'_i s, as opposed to existing schemes that work for a fixed R'_i .

5.3.2 Optimal Performance Theoretically Attainable (OPTA)

The OPTA serves as the theoretical bound of the analog JSCC communication system. In the case of the M -user MAC, the OPTA for the first scenario is derived by

evaluating [18]

$$\begin{cases} R(D_i)R'_i < C(P_i) \\ \sum_{i=1}^M R(D_i)R'_i < C(P), \end{cases} \quad (5.15)$$

where $R(D_i)$ is the rate distortion function of user i and $C(P_i)$ and $C(P)$ define the MAC capacity.

In the case of memoryless Gaussian sources with side information at the receiver (scenario 1), the rate distortion function of user i is [82]

$$R(D_i) = \begin{cases} \frac{1}{2} \log \frac{\sigma_s^2 \sigma_u^2}{(\sigma_s^2 + \sigma_u^2) D_i}, & 0 < D_i < \frac{\sigma_s^2 \sigma_u^2}{\sigma_s^2 + \sigma_u^2} \\ 0, & D_i \geq \frac{\sigma_s^2 \sigma_u^2}{\sigma_s^2 + \sigma_u^2}. \end{cases} \quad (5.16)$$

When a Gaussian MAC is considered, the capacity is [18]

$$\begin{cases} C(P) = \frac{1}{2} \log(1 + P) \\ C(P_i) = \frac{1}{2} \log(1 + P_i), \end{cases} \quad (5.17)$$

Since source-channel separation does not hold for the second MAC scenario, we cannot apply (5.15) to derive OPTA, instead, we can derive a lower bound of OPTA by equaling $R(D_i)R'_i$ to the capacity of each orthogonal channel in (5.13)

The rate distortion function of quadratic Gaussian sources is [1]

$$R_1(D_1) \geq \frac{1}{2} \log^+ \left[\frac{1 - \rho^2 + \rho^2 2^{-2R_2(D_2)}}{D_1} \right] \quad (5.18)$$

$$R_2(D_2) \geq \frac{1}{2} \log^+ \left[\frac{1 - \rho^2 + \rho^2 2^{-2R_1(D_1)}}{D_2} \right] \quad (5.19)$$

$$R_2(D_2) + R_1(D_1) \geq \frac{1}{2} \log^+ \left[\frac{(1 - \rho^2) \beta(D_1, D_2)}{2D_1 D_2} \right] \quad (5.20)$$

where

$$\log^+ x = \max(\log x, 0) \quad (5.21)$$

$$\beta(D_1, D_2) = 1 + \sqrt{1 + \frac{4\rho^2 D_1 D_2}{(1 - \rho^2)^2}}. \quad (5.22)$$

In sum, the OPTA for scenario 1 is obtained by plugging (5.16) and (5.17) into (5.15), while a the lower bound for the OPTA for scenario 2 is obtained by plugging (5.18), (5.19), (5.20) and (5.17) into $R(D_i)R'_i < C(P_i)$.

5.4 Simulation Results

In this section, we present simulation results for the two scenarios considered in the chapter.

5.4.1 Scenario 1

Although the proposed scheme works for any M in scenario 1, we choose $M = 2$ in our simulations. In the first set of simulations, we use the 1 : 1 periodic piece-wise linear mapping proposed in Chapter 3 followed by a CDMA-like access scheme of size $K = 8$ and three different values of m_1 and m_2 , specifically, $(m_1, m_2) = (1, 7)$, $(m_1, m_2) = (3, 5)$, and $(m_1, m_2) = (4, 4)$. In the second set of simulations, we use the 1 : 2 periodic spiral-like mapping proposed in Chapter 3, followed by the CDMA-like access scheme with the same specification as in the previous simulations. In order to minimize the distortion, we apply MMSE decoding. The optimal power allocation given in (5.10) is used for each CSNR $= 10\log_{10}(P_1 + P_2)$ and the parameters in the 1 : 1 periodic piece-wise linear mapping and the 1 : 2 periodic spiral-like mapping are optimized for each CSNR and ρ of interest. Tables 5.1 and 5.2 show the optimum values of the parameter T used for the 1 : 1 and 1 : 2 mappings. The performance of the proposed scheme is measured in terms of signal to distortion ratio: $\text{SDR} = 10 \times (\frac{m_1}{K}\log_{10}(\sigma_s^2/D_1) + \frac{m_2}{K}\log_{10}(\sigma_s^2/D_2))$ versus CSNR and compared with the corresponding OPTA given by

$$\text{SDR} < \frac{10}{R_c} \log_{10}(1 + P_1 + P_2) - 10\log_{10}(1 - \rho^2). \quad (5.23)$$

Notice that this limit does not depend on the specific values of m_1 and m_2 .

Table 5.1: Optimal parameter (T) for the 1 : 1 mapping

T	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	3.1	3.0	3.2	3.1
$\rho = 0.99$	0.9	1.0	1.0	1.0

Table 5.2: Optimal parameter (T) for the 1 : 2 mapping

T	CSNR = 0 dB	CSNR = 5 dB	CSNR = 15 dB	CSNR = 25 dB
$\rho = 0.9$	4.1	4.2	3.6	3.9
$\rho = 0.99$	1.4	1.1	1.1	1.2

Simulation results using the 1 : 1 periodic mapping for $\rho = 0.9$ and $\rho = 0.99$ are shown in Fig. 5.6 and Fig. 5.7, respectively. Fig. 5.8 and Fig. 5.9 plot the results using the 1 : 2 periodic mapping. As for the OPTA, we can see from the figures that the performance of the proposed scheme is the same irrespectively of the values of m_1 and m_2 . This is not surprising because when optimum power allocation is used, the CSNR of the orthogonal channels for user 1 and 2 are

$$\text{CSNR} = 10 \log_{10} \left(\frac{K P_i}{m_i} \right) = 10 \times \log_{10}(P) \quad (5.24)$$

which is the same as the total CSNR. For high correlations and high CSNRs, the proposed system also outperforms a scheme based on the concatenation of standard mappings and the CDMA-like access scheme (denoted in the figure as linear+linear for rate 1 : 1 and spiral+spiral for rate 1 : 2). This shows that with optimal power allocation, CDMA-like access schemes achieve the capacity of the Gaussian MAC. It is important to remark that the gap between the proposed scheme and OPTA is (almost) asymptotically constant. The robustness of the proposed scheme under CSNR mismatch is also shown in Figs. 5.6-5.9. We can see that the overall robustness characteristic of analog mappings under CSNR mismatch still holds quite well for the proposed scheme: when the real CSNR is lower/higher than the one used for the mapping design, the SDR degradation/improvement is graceful.

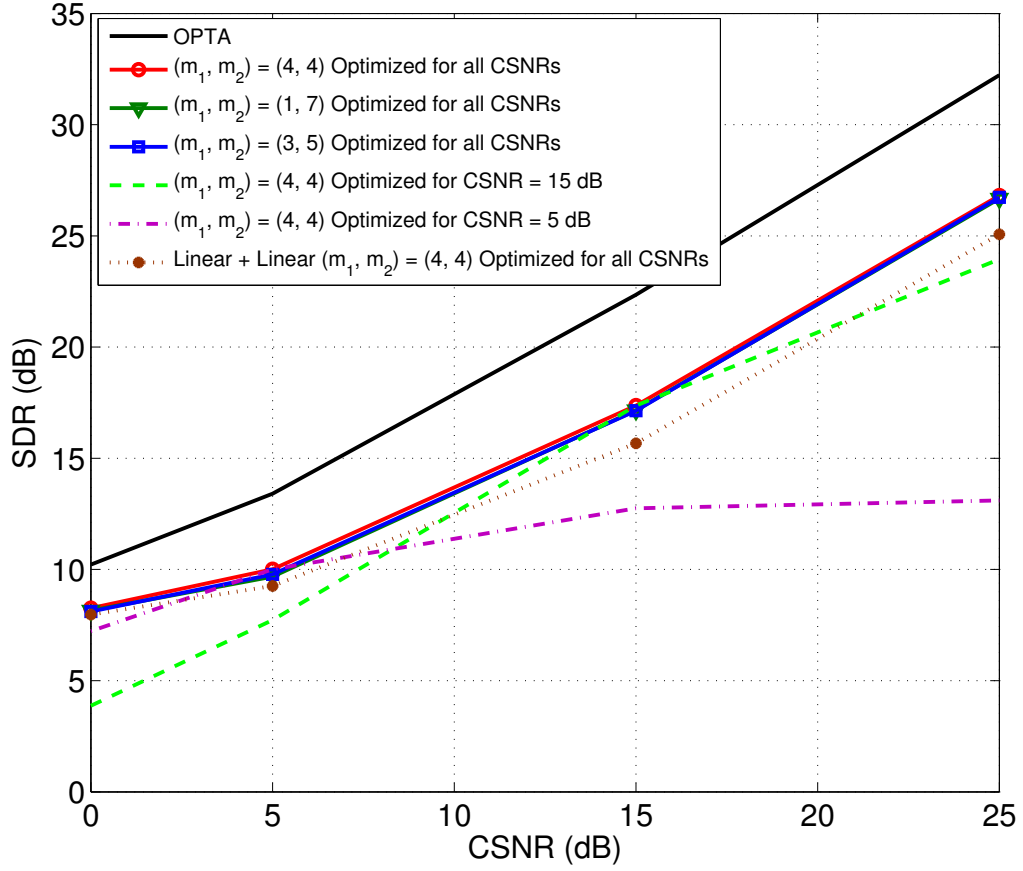


Figure 5.6: Performance evaluation for scenario 1 when $\rho = 0.9$: $M = 2$, $K = 8$ and different values of m_1 and m_2 . $R_{c_1} = R_{c_2} = 1 : 1$.

5.4.2 Scenario 2

For the transmission of quadratic (correlated) Gaussian sources without side information, we simulate the proposed system with three different combinations of Shannon-Kotel'nikov mappings and CDMA-like access scheme. As in the simulation for scenario 1, we use the optimal power allocation according to (5.10) and optimize the parameters for each analyzed CSNR and ρ .

In the first combination, $R_{c_1} = R_{c_2} = 2 : 1$ and $m_1 = m_2 = \frac{K}{2} = 1$, thus $R'_1 = R'_2 = \frac{m_i}{K} \times 2 = 1$. We use the standard $2 : 1$ bandwidth reduction mapping described in Chapter 1 to encode s_1 and the $2 : 1$ bandwidth reduction mapping

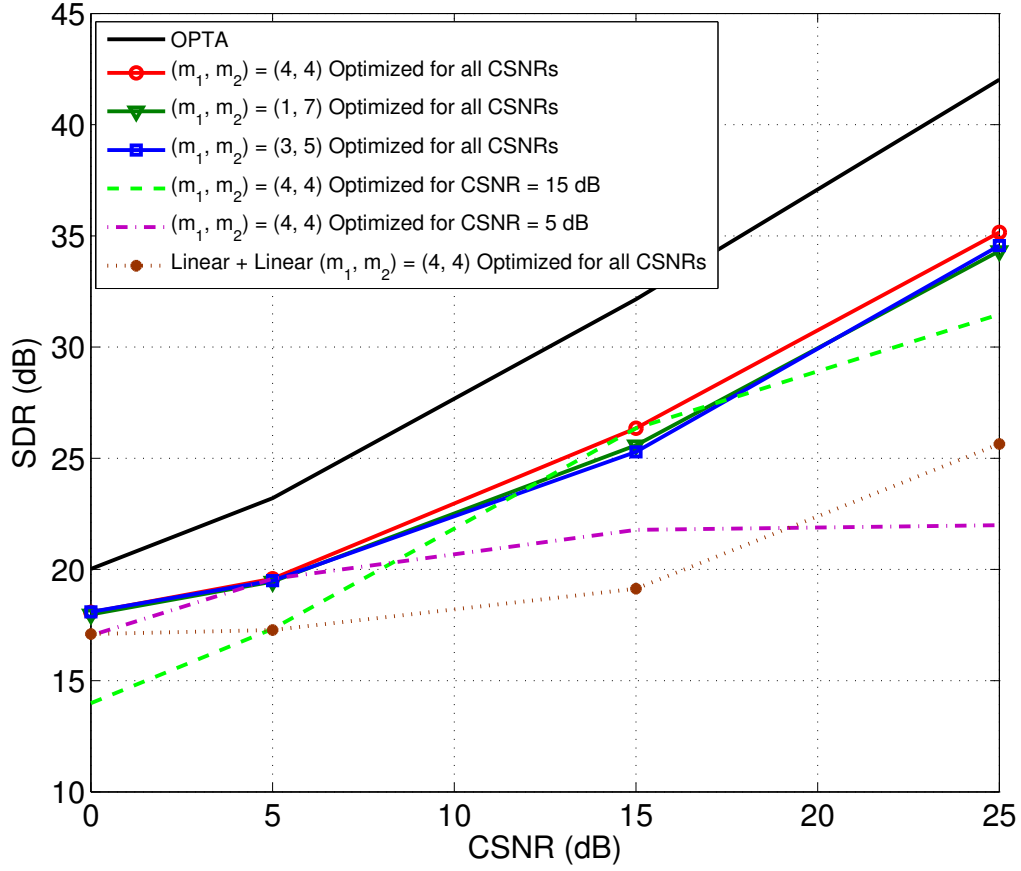


Figure 5.7: Performance evaluation for scenario 1 when $\rho = 0.99$: $M = 2$, $K = 8$ and different values of m_1 and m_2 . $R_{c1} = R_{c2} = 1 : 1$.

proposed in [3] to encode s_2 , and utilize the simplified decoding procedure described in Chapter 4. The optimum values of the parameter δ are shown in Table 5.3. Notice that the optimum parameter δ is nearly constant from CSNR = 30 dB to 50 dB, different to the result in Chapter 4 where the optimum parameter changes with CSNR. The reason is that in Chapter 4 the analyzed CSNR is relatively low < 30 dB, while in this chapter we simulate a higher CSNR to facilitate the simplified decoding. Thus, when CSNR is high enough, s_1 is almost error-free, the correlation between $\hat{s}_1$ and s_2 is very similar to ρ , and thus the variation of the optimum parameter is very small.

Fig. 5.10 shows the performance of the proposed scheme for $\rho = 0.95$. For

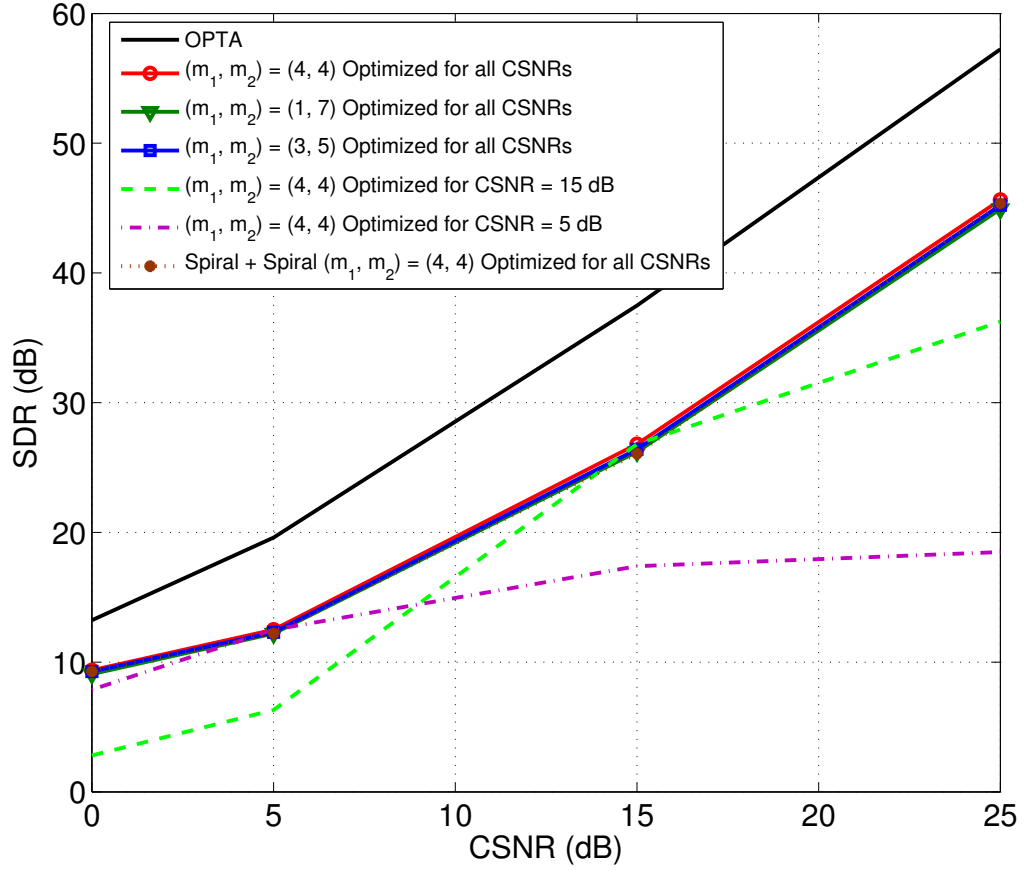


Figure 5.8: Performance evaluation for scenario 1 when $\rho = 0.9$: $M = 2$, $K = 8$ and different values of m_1 and m_2 . $R_{c_1} = R_{c_2} = 1 : 2$.

Table 5.3: Optimized parameter (δ) for the 2 : 1 mapping

δ	CSNR = 30 dB	CSNR = 35 dB	CSNR = 40 dB	CSNR = 45 dB	CSNR = 50 dB
$\rho = 0.95$	1.4	1.3	1.4	1.2	1.4

comparison, we also plot the result from [27], which, to our knowledge, provides the best results so far for transmitting quadratic Gaussian sources over a Gaussian MAC

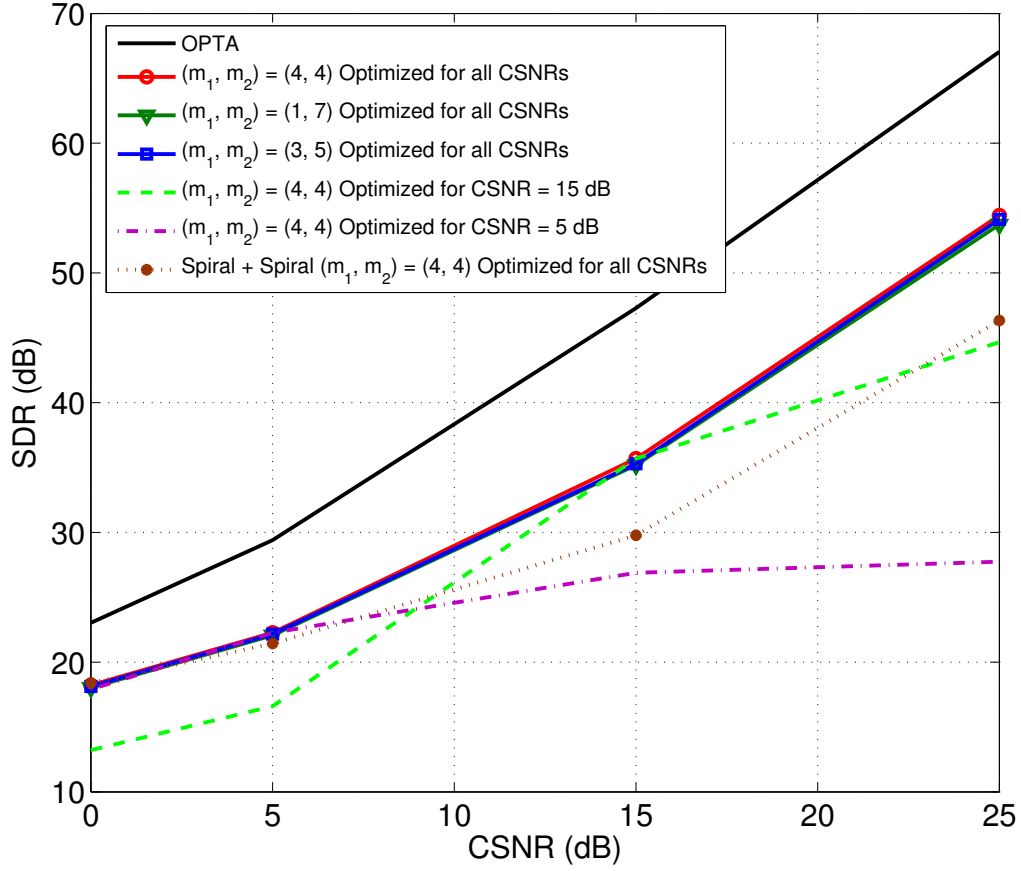


Figure 5.9: Performance evaluation for scenario 1 when $\rho = 0.99$: $M = 2$, $K = 8$ and different values of m_1 and m_2 . $R_{c1} = R_{c2} = 1 : 2$.

at rate 1, and the corresponding upper bound, which is given by [27]

$$\text{SDR} = \frac{2\sigma_s^2}{D} = \begin{cases} \left(\frac{\frac{P}{2}(1-\rho^2)+1}{P(1+\rho)+1} \right)^{-1}, & 0 < P < \frac{2\rho}{1-\rho^2} \\ \left(\frac{1-\rho^2}{P(1+\rho)+1} \right)^{-\frac{1}{2}}, & P \geq \frac{2\rho}{1-\rho^2}. \end{cases} \quad (5.25)$$

From Fig. 5.10, we can see that the proposed scheme achieves a performance similar to the one in [27], which was designed for specific rates, and exhibits a nearly constant gap to the theoretical limit. However, the proposed scheme is much easier to adapt to other transmission rates as we will see with more examples later. We also evaluate the robustness of our system to CSNR mismatch. The optimum parameters for CSNR

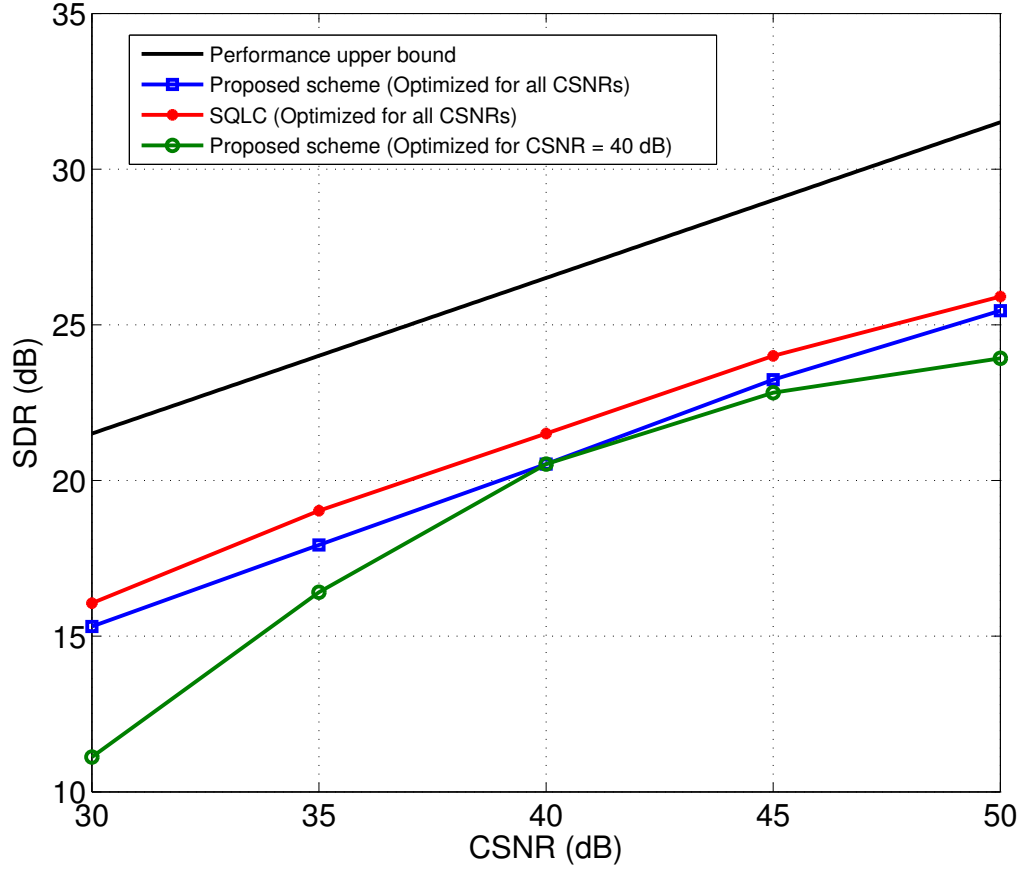


Figure 5.10: Performance evaluation for scenario 2 when $\rho = 0.95$. The CDMA-like access scheme is defined by $m_1 = m_2 = \frac{K}{2} = 1$. $R'_1 = R'_2 = 1$. The comparison scheme is the SQLC system in [27].

$= 40$ dB are used in the simulation, and the results are also included in Fig. 5.10. As in scenario 1, the proposed scheme shows very good robustness when the real CSNR is different from the one used in the design.

In the second combination, $R_{c1} = R_{c1} = 1 : 1$, $m_1 = m_2 = \frac{K}{2} = 1$, thus $R'_1 = R'_2 = \frac{m_i}{K} \times 1 = \frac{1}{2}$. We apply the standard linear mapping described in Chapter 1 to encode s_1 and use the $1 : 1$ periodic mapping proposed in Chapter 3 to encode s_2 . MMSE decoding is used. Fig. 5.11 shows the performance of the proposed scheme for $\rho = 0.95$. For comparison, we also include the lower bound for the OPTA described in

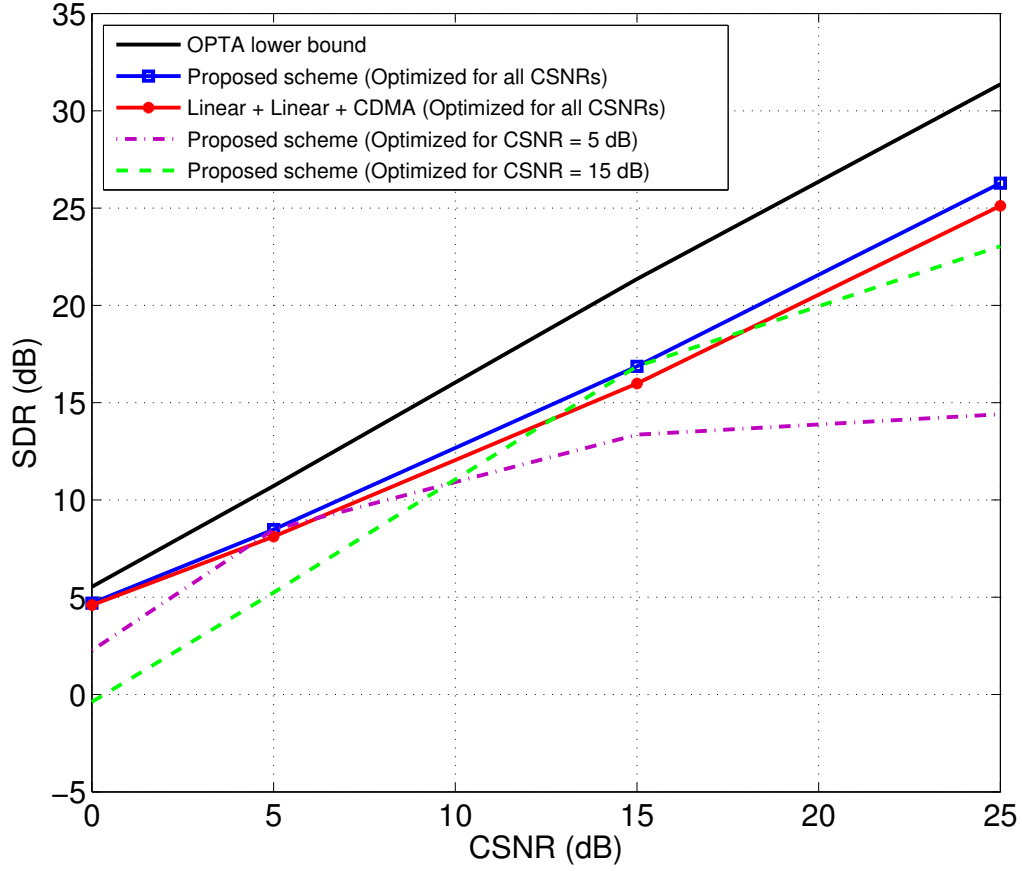


Figure 5.11: Performance evaluation for scenario 2 when $\rho = 0.95$. The CDMA-like access scheme is defined by $m_1 = m_2 = \frac{K}{2} = 1$. $R'_1 = R'_2 = \frac{1}{2}$.

Section 5.3.2 and a system that utilizes standard mappings for each of the users prior to the access scheme.

In the third combination, $R_{c1} = 1 : 1$, $R_{c2} = 1 : 2$, $m_1 = 1$, $m_2 = 2$, $K = 3$, thus $R'_1 = R'_2 = \frac{1}{3}$. We use the standard $1 : 2$ mapping described in Chapter 1 to encode s_1 and the $1 : 1$ periodic mapping proposed in Chapter 3 to encode s_2 . MMSE decoding is used. Fig. 5.12 shows the performance of the proposed scheme for $\rho = 0.95$.

We can see that for all the three combinations, when the CSNR is high the proposed system performs parallel to the theoretical limits, and outperforms the scheme that uses standard mappings to encode each of the users.

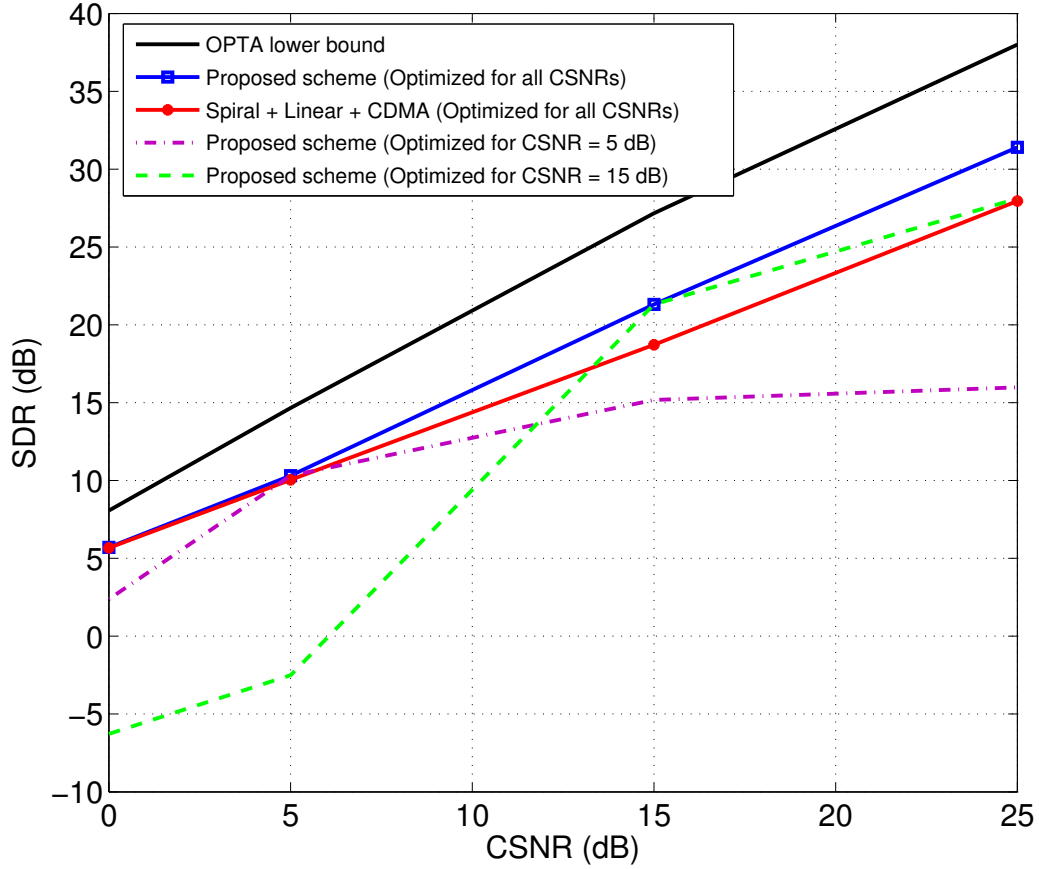


Figure 5.12: Performance evaluation for scenario 2 when $\rho = 0.95$. The CDMA-like access scheme is defined by $m_1 = 1$, $m_2 = 2$, $K = 3$. $R'_1 = R'_2 = \frac{1}{3}$.

5.5 Conclusion

We have proposed a zero-delay JSCC scheme based on analog mappings and a CDMA-like access scheme for two scenarios in MAC: the transmission of independent Gaussian sources over a Gaussian MAC with side information at the receiver and the transmission of quadratic (correlated) Gaussian sources over a Gaussian MAC without side information. Different from existing zero-delay schemes which are designed for a fixed transmission rate, the proposed scheme can be easily adapted to different rates. Simulation results show that the proposed system performs very close to the theoretical limits when the optimal power allocation is applied, and is robust against

CSNR mismatch.

Chapter 6

CONCLUSION AND FUTURE RESEARCH

In this dissertation, we have explored analog joint source-channel coding based on direct source-channel mappings in non-standard scenarios. The basic idea is to transmit discrete-time continuous-amplitude sources by directly mapping points in the source space to points in the channel space. Depending on the bandwidth ratio, which is defined as the number of source symbols per channel symbol, analog direct source-channel mappings are categorized into three types: bandwidth expansion mappings (bandwidth ratio < 1), bandwidth reduction mappings (bandwidth ratio > 1) and bandwidth match mappings (bandwidth ratio $= 1$). Compared with digital communications systems, analog direct source-channel mappings provide low delay, low complexity and robustness against varying channel conditions, and thus they are very appropriate for real-time communication systems subject to delay and complexity constraints. Analog direct source-channel mappings have so far been mostly applied to independent sources and point-to-point channels. However, as shown in this dissertation, they have great potential in more general scenarios.

In Chapter 2, bandwidth reduction mappings for the transmission of multivariate correlated Gaussian sources over point-to-point AWGN channels are investigated. PCCOVQ is utilized to obtain the optimal mapping for $2 : 1$, $3 : 1$ and $4 : 2$ bandwidth ratios. Simulation results show that linear mappings achieve OPTA for low CSNRs. For high CSNRs, the trained codebook is non-linear and it performs very close to OPTA. The $4 : 2$ mapping outperforms the $2 : 1$ mapping even though the bandwidth ratio is the same, as more correlation can be explored. Furthermore, the optimal signal space structure is not a spiral-like space filling curve (as for i.i.d. sources).

Chapter 3 considered the design of non-linear mappings for the transmission of independent Gaussian sources over a point-to-point AWGN channel when side information is available at the receiver (Wyner-Ziv scenarios). A $1 : 1$ scheme and a $1 : M$ scheme based on the use of Shannon-Kotel'nikov mappings in a periodic fashion is proposed. To reduce the complexity of the numerical MMSE decoder, a simplified decoding technique is proposed. A flexible rate scheme which combines the $1 : 1$ and $1 : M$ schemes is proposed to construct mappings with any bandwidth ratio between $1 : 1$ and $1 : M$.

In Chapter 4 and Chapter 5, analog direct source-channel mappings are extended to distributed scenarios where sources of multiple users are encoded separately and jointly decoded at the common receiver. Chapter 4 introduced the mappings for the transmission of correlated Gaussian senders over separated AWGN channels. An asymmetric encoding scheme is proposed where one of encoders is the standard mapping and the other is the mapping for the Wyner-Ziv scenario that was proposed in Chapter 3. The proposed mappings were shown to be robust against CSNR and correlation mismatch. Chapter 5 discussed analog mappings for the Gaussian MAC. A CDMA-like access scheme is used to convert the MAC into single-input single-output channels for each user. Compared with other orthogonal schemes where the rate of each user is determined only by the rate of the Shannon-Kotel'nikov mapping, in the proposed scheme the rate of each user is incorporated in the access scheme itself, which facilitates the design.

6.1 Future Work

There are multiple research directions that can be considered in future work, including:

- Generalizing the PCCOVQ method to any bandwidth ratio and to distributed scenarios.
- Exploring intra-correlation and inter-correlation at the same time.
- Designing analog mappings for the transmission of correlated sources over broadcast channels.

- Studying how sensitive the analog mappings are to non ideal carrier recovery and phase shift.
- Finding analog mappings for non-memoryless noisy channels.
- Exploring path diversity in multihop communication scenarios.

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