

STATE OF DELAWARE  
UNIVERSITY OF DELAWARE

DELAWARE GEOLOGICAL SURVEY

Robert R. Jordan, State Geologist

BULLETIN NO. 15

DIGITAL MODEL OF THE UNCONFINED AQUIFER  
IN CENTRAL AND SOUTHEASTERN DELAWARE



BY

RICHARD H. JOHNSTON  
HYDROLOGIST, U.S. GEOLOGICAL SURVEY

NEWARK, DELAWARE  
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PREPARED BY THE UNITED STATES GEOLOGICAL SURVEY  
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## CONVERSION OF MEASUREMENT UNITS

Factors for converting English units to metric units are shown to four significant figures. However, in the text the metric equivalents are shown only to the number of significant figures consistent with the values for the English units.

<u>English Unit</u>	<u>Multiply By</u>	<u>Metric Unit</u>
cubic feet per second (ft <sup>3</sup> /s)	0.02832	cubic meters per second (m <sup>3</sup> /s)
cubic feet per second per square mile [(ft <sup>3</sup> /s)/mi <sup>2</sup> ]	0.01093	cubic meters per second per square kilometer [(m <sup>3</sup> /s)/km <sup>2</sup> ]
feet (ft)	0.3048	meters (m)
feet per day (ft/d)	0.3048	meters per day (m/d)
feet squared per day (ft <sup>2</sup> /d)	0.0929	meters squared per day (m <sup>2</sup> /d)
gallons per minute (gal/min)	0.06309	liters per second (L/s)
gallons per minute per foot (gal/min)/ft	0.207	liters per second per meter (L/s)/m
inches (in)	25.4	millimeters (mm)
million gallons (Mgal)	3785	cubic meters (m <sup>3</sup> )
million gallons per day (Mgal/d)	3785	cubic meters per day (m <sup>3</sup> /d)
miles (mi)	1.609	kilometers (km)
square miles (mi <sup>2</sup> )	2.590	square kilometers (km <sup>2</sup> )

DIGITAL MODEL OF THE UNCONFINED AQUIFER  
IN CENTRAL AND SOUTHEASTERN DELAWARE

by

Richard H. Johnston

ABSTRACT

The unconfined aquifer in central and southeastern Delaware occurs as a southward-thickening blanket of fine to coarse sand. Transmissivity of the aquifer ranges from  $2,000 \text{ ft}^2/\text{d}$  ( $190 \text{ m}^2/\text{d}$ ) in the north to about  $22,000 \text{ ft}^2/\text{d}$  ( $2,000 \text{ m}^2/\text{d}$ ) in the south. At present (1975) ground-water withdrawal is light and widely distributed and no long-term decline in the water table has been observed. The unconfined aquifer is recharged almost totally by precipitation and discharge is principally by seepage to streams, bays, and the ocean.

A digital model was used to simulate flow in an approximate sense, in that only recharge by precipitation and discharge to surface-water bodies are represented. Winter conditions were simulated so that the ground-water evapotranspiration could be ignored. The model is a two-dimensional representation of the flow system which employs a finite-difference technique to solve the ground-water flow equation. The model was calibrated primarily by means of a steady-state analysis in which uniform areal recharge was assumed and discharge to streams and the sea was simulated. Calibration consisted of adjusting values of hydraulic conductivity throughout the model until observed water-level contours were duplicated and stream baseflows were approximated. Following calibration, approximately 70 percent of the computed heads differed from measured water table elevations by less than 1.5 ft (0.8 m) and so fell within the 5-foot (1.5 m) average annual fluctuation of the water table.

Agreement between base flows as computed in the steady-state calibration and base flows measured in the field is excellent except in one area (near Dover, Del.) where significant vertical leakage to the heavily pumped Cheswold artesian aquifer occurs.

The calibrated model suggests that the average transmissivity (T) of the unconfined aquifer is about 50 percent higher than values published previously by the author, which were calculated mostly from well-performance data. Except at a few sites where transmissivity (T) values were obtained from lengthy aquifer tests, input T values, taken from these earlier results, had to be increased during the calibration process. The revised transmissivity map, based on changes made during calibration, presented in this report, supersedes the earlier data.

The calibrated steady-state model was used to estimate base flow in ungaged streams and particularly ground-water discharge to tidal rivers.

The digital model was used to evaluate the effects of substantial increases in ground-water withdrawals in five selected areas. The decline in water levels and depletion in base flow were projected for a 30-year period using various withdrawal rates. In two areas, a seashore resort area and an irrigated farming area (where withdrawals are and will be mostly in the summer), water level declines are projected for very dry summer conditions where no recharge occurs.



## INTRODUCTION

### Scope and Purpose of the Investigation

The unconfined aquifer studied occurs as a blanket of sand across the southern three-quarters of Delaware and provides about one-half the ground water pumped in the State. The pumping rate (about 35 Mgal/d or 132,000 m<sup>3</sup>/d) is quite small if compared with the natural recharge or discharge from the aquifer (about 1,000 Mgal/d or 3,800,000 m<sup>3</sup>/d), and has caused little decline in the water-table elevation or in the base flow of streams. The greatest future demand for water supplies is expected to occur in the area between Dover and the seashore resorts. Within this area, the water-transmitting properties of the aquifer vary greatly but future pumpage will tend to follow development rather than be centered in hydrologically favorable areas. Evaluation of the effects of future pumping on water levels (and therefore on yield and cost of pumping wells) and on streamflow would be highly useful information to those concerned with water-supply management.

The purpose of the investigation was to evaluate the aquifer's potential for additional development. Digital simulation was selected as the most promising method to accomplish this. In particular, digital modeling was expected to provide:

- (1) A hydrologic description of the stream-aquifer system where data on aquifer properties are poor;
- (2) A capability to predict future water-level declines in areas where development of ground-water supplies are likely; and
- (3) A capability to predict the decline in base flow (fair-weather flow) of streams caused by pumping.

Digital simulation can solve the flow equations required to describe a stressed, complex, heterogeneous, regional stream-aquifer system. In contrast, the standard analytical methods of ground-water hydrology consider only a part of a complex aquifer problem. The advent of large, high-speed, digital computers made possible the solution of the flow equations used in aquifer simulation models. As a result, digital modeling is increasingly used to evaluate aquifers such as those described in this report.

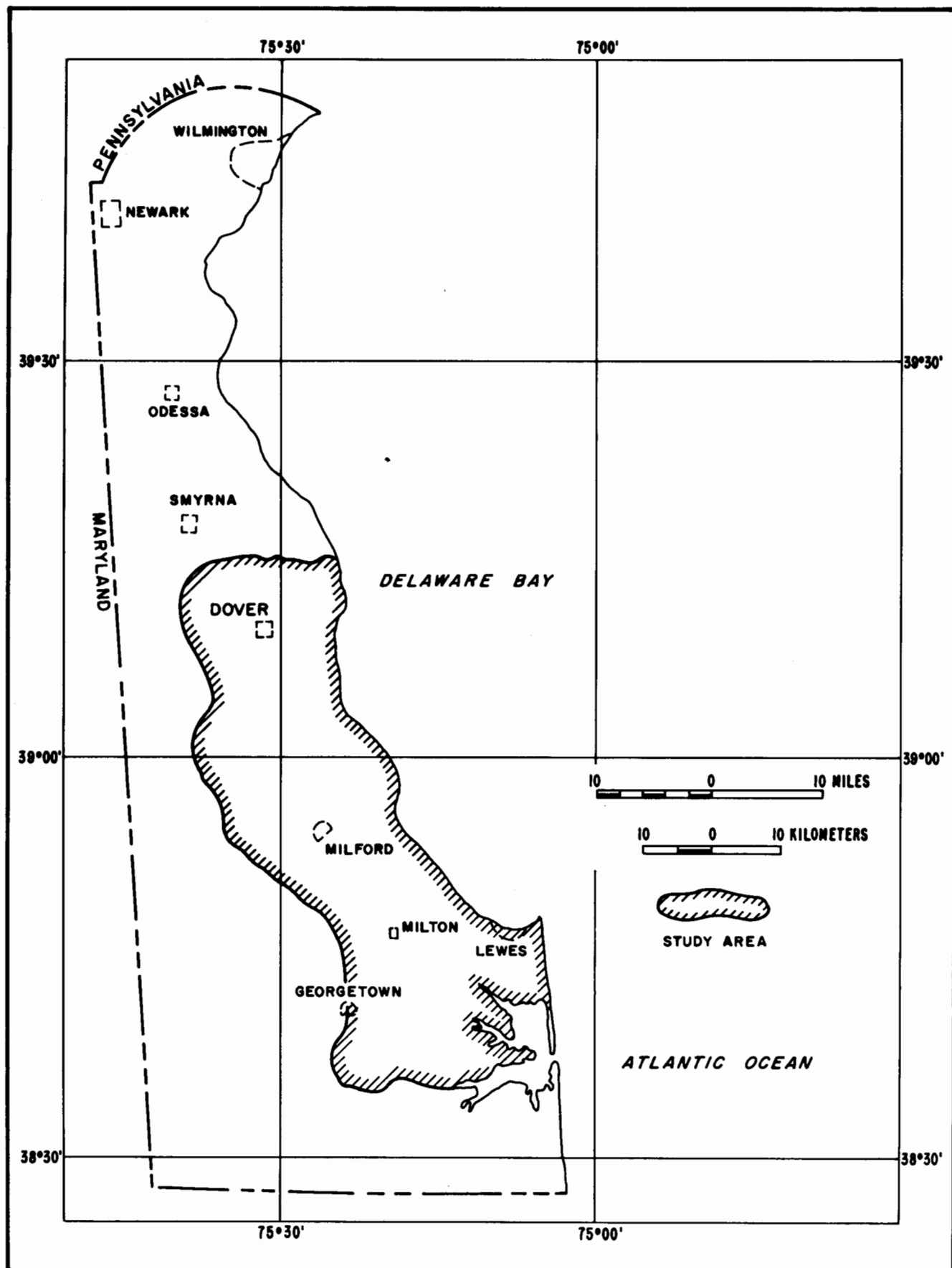


FIGURE 1. MAP OF DELAWARE SHOWING LOCATION OF STUDY AREA.

The area selected for modeling (Figure 1) comprises about 670 mi<sup>2</sup> (1,700 km<sup>2</sup>) in southeastern Delaware. Essentially, this is the area drained by Delaware Bay and the Atlantic Ocean between the Leipsic River on the north and Indian River and Rehoboth bays on the south.

### Acknowledgments

The study is part of an ongoing program of ground-water investigations made cooperatively by the U. S. Geological Survey and the Delaware Geological Survey. Special thanks are given to Robert R. Jordan, State Geologist of Delaware, and the staff of the Delaware Geological Survey who aided the study in many ways.

Digital simulation was made with the Burroughs 6700 computer located at the University of Delaware Computer Center. Special thanks are due to Robert Schaefer, Director of Academic Services at the Center, and his staff for providing assistance in computer programming. Costs of the computer runs were paid by the Delaware Department of Natural Resources, Division of Environmental Control.

Thanks are also given to G. D. Bennett, S. S. Papadopoulos, and P. C. Trescott of the U. S. Geological Survey, Reston, Va. for their many helpful suggestions during the design and calibration of the digital models.

## UNCONFINED AQUIFER-STREAM SYSTEM

### Hydrogeologic Setting

The unconfined aquifer is composed principally of fine to coarse sand which occurs as a southward thickening blanket across central and southern Delaware (Johnston, 1973). These sands represent several environments of deposition including fluviatile, estuarine, and near-shore marine, and probably several ages of deposition. In the northern two-thirds of the State the water-table aquifer is, in most cases, the Columbia Formation of Pleistocene age (Jordan, 1962, 1964, 1976). In some instances the Columbia may rest directly upon older sands of Miocene age and the entire sequence then functions as the water-table aquifer. In southern Delaware, Jordan and Talley (1976) have mapped the downdip extension of the Columbia Formation by means of cored wells and have termed these fluvial deposits the Beaverdam Formation. Jordan, (1962) proposed the name Omar Formation for the surficial

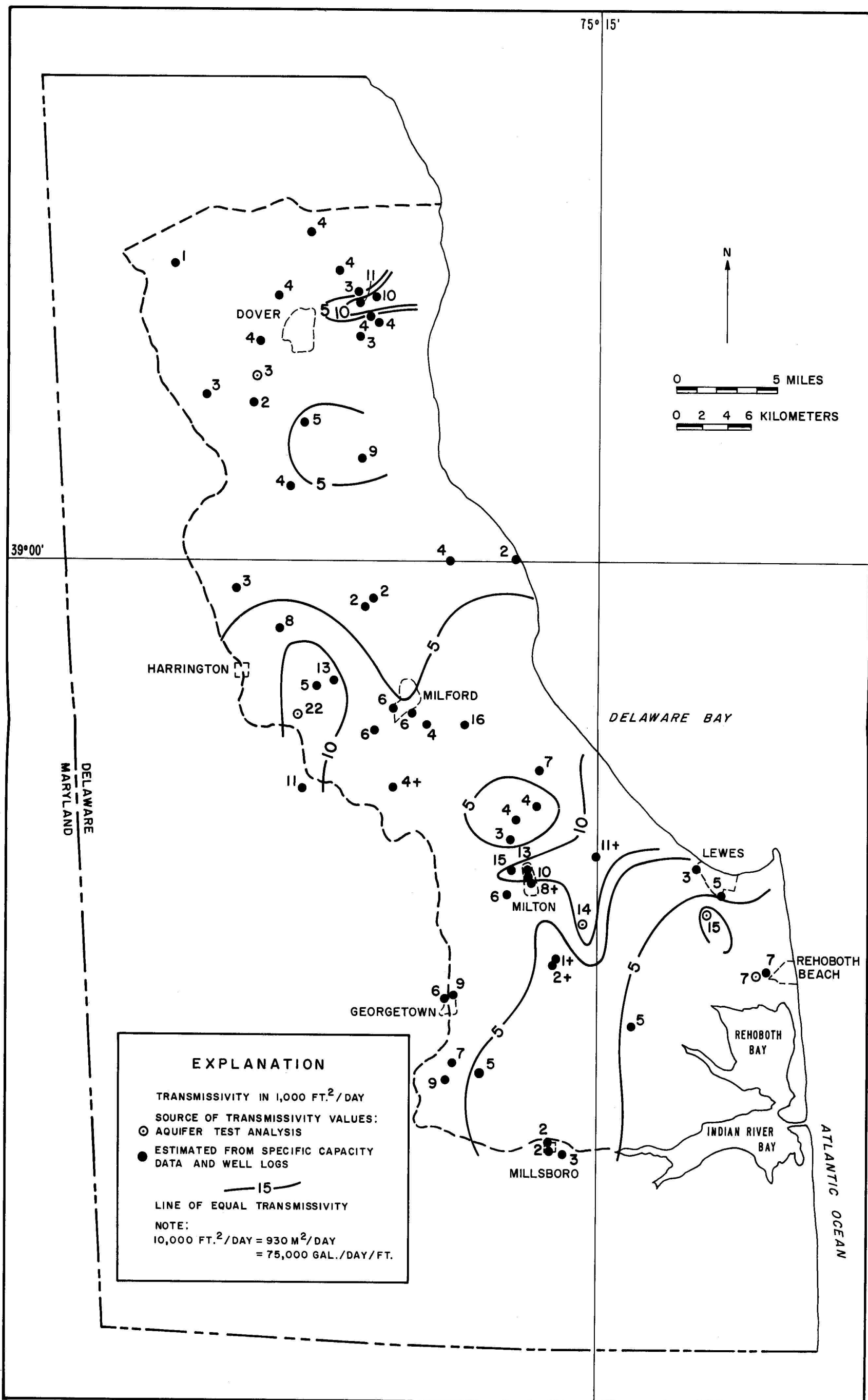


FIGURE 2. TRANSMISSIVITY OF THE UNCONFINED AQUIFER BASED ON FIELD AQUIFER TESTS AND WELL DATA (JOHNSTON, 1973).

deposits which overlie the Beaverdam in southern Delaware. Thus the Beaverdam and Omar Formations comprise the Columbia Group in the southern part of the State. Owens and Denny (1974) consider the Beaverdam to be Pliocene and the overlying sands and silts to represent fluviatile, estuarine, and back-barrier deposits of Pliocene and Pleistocene age. In general, they prefer the term "Pensauken Formation" for "Columbia Formation."

All the sandy deposits mentioned above behave hydrologically as a heterogeneous unconfined aquifer. Cushing and others (1973) applied the term "Quaternary aquifer" to these surficial sands. However, in this report they are simply referred to as the unconfined aquifer. The unconfined aquifer is hydraulically separated from underlying Miocene age artesian aquifers by extensive silty confining beds.

The saturated thickness of the unconfined aquifer ranges from about 15 feet (8 m) north of Dover to about 170 feet (52 m) near Milton. In general, there is a southward thickening of the aquifer across the model area (Johnston, 1973).

The transmissivity (T) of the unconfined aquifer is variable, reflecting changes in lithology (from fine to coarse sand with gravel lenses) and the southward increase in saturated thickness. Figure 2 is a previously published map (Johnston, 1973) showing the areal variation in transmissivity based on specific capacity data and a few aquifer tests. The T values estimated from specific capacity data are subject to considerable error because of variation in well construction and development that are difficult to evaluate. However, the T values obtained by aquifer-test analysis provide a few reliable control points. As discussed in a later section, T values had to be increased an average of 50 percent during model calibration. A revised T map, which supersedes Figure 2, will be discussed in the section entitled "SIMULATION RESULTS." Based on Figure 2, the average transmissivity is about 6,000 ft<sup>2</sup>/d (560 m<sup>2</sup>/d); however, the revised T map indicates an average T of about 9,000 ft<sup>2</sup>/d (840 m<sup>2</sup>/d).

Horizontal hydraulic conductivity ( $K_h$ ), based on values of T and saturated thickness at aquifer test sites, ranges from 50 to 250 ft/d (15 to 76 m/d). Vertical hydraulic conductivity ( $K_v$ ) is about one-tenth  $K_h$ . Analysis of two aquifer tests gave  $K_h:K_v$  ratios of 10:1 and 4:1 (Johnston, 1973). A later test (results unpublished) involving many observation wells and a more rigorous analysis of test data, provided a  $K_h:K_v$  ratio of 10:1. In the southern part of the model area where the upper section of the aquifer contains

fine sand and silt, this ratio probably exceeds 10:1. In areas where the aquifer is mostly medium to coarse sand, the ratio of 4:1 is probably more realistic.

The specific yield of the unconfined aquifer is about 0.15. This is an average value based on a calculation of a simplified hydrologic budget in which the computed values ranged from 0.11 to 0.17 (Johnston, 1973).

Specific capacities of large-diameter wells range from 5 to 100 (gal/min)/ft (1.0 to 21 (L/s)/m) with a mean value of 28 (gal/min)/ft (5.8 (L/s)/m). It is possible to construct wells yielding upwards of 500 gal/min (32 L/s) throughout most of the area.

### Regional Flow System

The streams of central and southern Delaware and the unconfined aquifer constitute a flow system which can be described as a thin blanket of sand containing widely spaced shallow drains. The streams in the model area penetrate only the upper few feet of the aquifer and receive about three-quarters of their flow from ground-water discharge. The percentage of streamflow derived from ground-water discharge ranges from about 50 percent for the St. Jones River at Dover (Station 01483600 on Figure 5) to about 90 percent for Beaverdam Creek near Milton (Station 01484270) (Johnston, 1973; 1976).

During base-flow conditions there is a close relationship between ground-water stage and streamflow. The hydraulic characteristics of the unconfined aquifer, transmissivity, specific yield, and aquifer size (distance from stream to ground-water divide), in conjunction with evapotranspiration rates determined the recession of streamflow. Consequently, flow in many streams can be estimated fairly accurately from observation well records. Curves relating base flow to ground-water stage and a general discussion of the aquifer-stream-flow relationship are presented in a separate report (Johnston, 1976).

Base flow and ground-water levels vary seasonally reflecting changes in aquifer storage, as well as variable rates of evapotranspiration and recharge. During most years, the period from mid-October to mid-April (non-growing season) is characterized by low evapotranspiration, frequent recharge to the aquifer, rising ground-water levels, and increasing base flow. The growing season (mid-April to mid-October) is characterized by high evapotranspiration, infrequent recharge,

and lengthy recessions of ground-water levels and base flow. Graphs showing seasonal fluctuations of ground-water levels and base flow for a 10-year period are presented in a separate report (Johnston, 1976, Figures 6, 9, 11, and 13).

It is noteworthy that there is very little difference between the mean ground-water stage during summer and winter. The average summer (May-September) water levels are only about 0.4 ft (0.1 m) lower than the winter (November-March) water levels (Johnston, 1973, p. 47). This suggests that ground-water discharge during summer (base flow plus evapotranspiration) is about equal to ground-water discharge during winter (all base flow). Thus the average winter base flow provides a good estimate of the long-term ground-water discharge as well as the long-term recharge rate. The average winter base flow of streams in the model area is  $1.03 \text{ (ft}^3\text{/s)/mi}^2$  ( $0.011 \text{ (m}^3\text{/s)/km}^2$ ). This is equivalent to a long-term recharge rate, or discharge rate, of 14 inches (356 mm) per year (Johnston, 1973).

No long-term change in ground-water levels has been recorded in observation wells tapping the unconfined aquifer. Pumpage from wells amounts to only about 4 percent of the total aquifer discharge and thus no measurable decline would be expected (Johnston, 1973). Furthermore, some of the pumped water is returned to the ground via septic tanks.

Leakage to and from the Miocene aquifers is negligible except in a small area north of Dover (Figure 1) where pumping from the Cheswold aquifer (Miocene age) is considerable (about 6 Mgal/d or  $22,000 \text{ m}^3\text{/d}$ ). Here the very low base flow of streams to the north of Dover strongly suggest that water is moving downward into the Cheswold rather than discharging to streams. The lack of downward leakage elsewhere in the area is suggested by water-balance studies (Mather, 1969) in which the computed runoff (calculated without considering leakage) was found to be similar to the measured runoff at gaging stations.

## DIGITAL AQUIFER MODEL

### Theory

The purpose of a digital aquifer model usually is to simulate the effects produced in an aquifer by pumping from wells considering such factors as variations in recharge and evapotranspiration rates, leakage through or from confining

beds, and leakage to or from streams and lakes. The information sought from a model would typically include changes in hydraulic head (drawdown) or changes in streamflow caused by pumping wells. Essentially, the model is used to solve the basic equation of ground-water flow, which is an expression of the continuity equation (principle of conservation of mass) that states:

inflow - outflow = rate of accumulation of storage.

For an unconfined aquifer where vertical flow is negligible, the flow equation may be stated as follows:

$$\frac{\partial}{\partial x} \left[ Kb \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ Kb \frac{\partial h}{\partial y} \right] = S_y \frac{\partial h}{\partial t} - w(x,y,t) \quad (1)$$

where

$h$  = hydraulic head

$x, y$  = rectangular coordinates

$K$  = hydraulic conductivity

$S_y$  = specific yield

$b$  = saturated aquifer thickness, which equals  $h - e$   
(hydraulic head in aquifer minus base of aquifer)

$w$  = net recharge per unit area (recharge minus discharge per unit area)

$t$  = time

For conditions of steady flow in an unconfined aquifer with negligible pumpage, constant rate of recharge, and all water discharging to streams, equation 1 may be written as follows:

$$\frac{\partial}{\partial x} \left[ Kb \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial y} \left[ Kb \frac{\partial h}{\partial y} \right] + w(x,y,t) = 0 \quad (2)$$

where all variables are the same as described above and

$$w(x,y,t) = r(x,y) - q(x,y)$$

where  $r(x,y)$  is recharged per unit area and

$q(x,y)$  is discharge per unit area to streams.



Equation 2 is simulated by the model under steady-state conditions. The equation forms the basis of steady-state model calibration and is further discussed in the sections on model concepts and steady-state simulation. Equation 1 is simulated by the model under transient conditions and is the basis for nonequilibrium calibration as well as for projecting water-level declines and streamflow depletion, as discussed in the sections on calibration and simulation results.

Analytical solutions for equations 1 or 2 are limited to a few cases representing very simple boundary conditions. However, there are a variety of numerical techniques which will provide approximate solutions. The most commonly used techniques involve the substitution of finite-difference approximations for the derivatives in the flow equation. For a mathematical discussion of these techniques, the interested reader should consult a standard text such as Von Rosenberg (1969) or Remson and others (1971). An excellent discussion of the derivation of finite-difference approximations on a physical basis from Darcy's law and the principle of continuity is given by Prickett and Lonquist (1971).

The finite-difference methods as applied to aquifer analysis involve the overlay of a grid on a map showing the regional extent of an aquifer. A discretized network of grid squares (or rectangles) with dimensions  $\Delta x$  by  $\Delta y$  is obtained. An individual volume or prism of aquifer has dimensions  $b\Delta x\Delta y$ .

Changes in head, inflow, and outflow at each discretized aquifer volume are calculated by a finite-difference equation for applied stresses such as pumpage and variable rates of recharge and evapotranspiration. Depending upon the type of aquifer problem, a numerical method is selected and finite-difference equations formulated. The model described here uses the iterative alternating direction implicit technique (IADI) as described by Pinder (1970) and later modified by Trescott (1973). This program is a highly versatile tool for aquifer analysis and the model or its variations are routinely used by hydrologists of the U.S. Geological Survey and other organizations. The model can be used to simulate confined or unconfined aquifer, exhibiting inhomogeneity and anisotropy, irregular aquifer boundaries, recharge, evapotranspiration, leakage from confining beds or streams, and pumping from wells. Trescott's (1973) program, which is written in Fortran IV, was modified slightly for simulations using the Burroughs 6700 system at the University of Delaware.

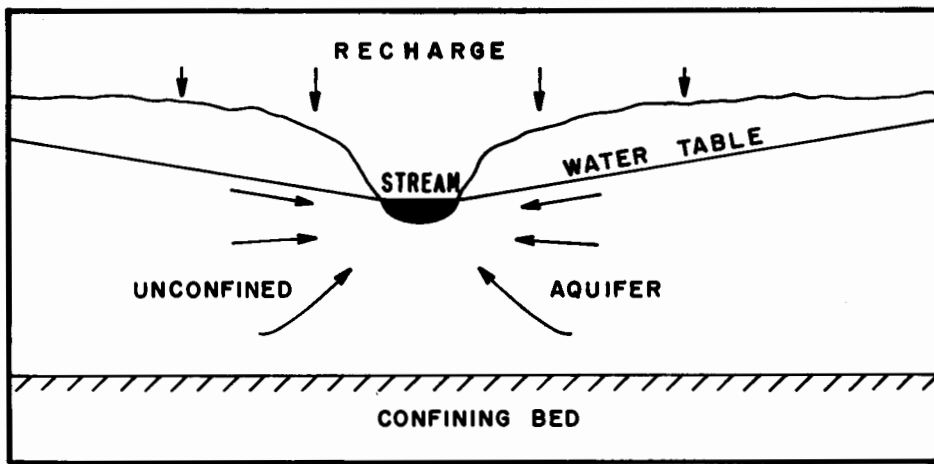
## Model Concepts, Boundaries, and Finite-Difference Grid

Over the long-term, steady-state conditions can be said to exist in the unconfined aquifer of central and southern Delaware. This is indicated by the relative constancy of the water table (during the past 20 years). Furthermore, rates of pumping from the aquifer are very small compared to natural rates of recharge and discharge to streams. Leakage to and from underlying artesian aquifers is negligible except in the area north of Dover and one small basin in southern Delaware. If only periods of a few months are considered, transient conditions exist with rising water levels and increasing discharge rates during winter and declining water levels and decreasing discharge rates during summer.

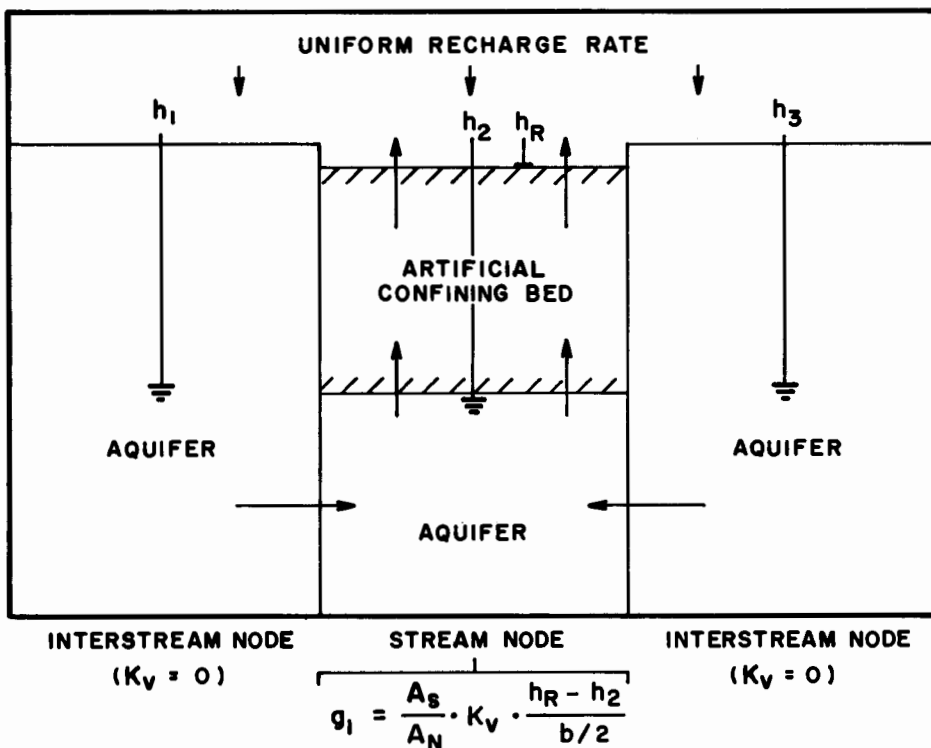
A model of the unconfined aquifer was designed which could accurately simulate the long-term steady-state condition as well as the short-term seasonal conditions. This calibrated model was then used for predicting water-level and streamflow declines caused by increased ground-water withdrawals.

Thus, modeling the unconfined aquifer involved three steps:

- (1) Design and calibration of a steady-state model in which the computed heads were compared with known steady-state water table elevations and computed outflow was compared with measured or estimated winter base flow of streams;
- (2) Design and calibration of a transient model involving no recharge. Heads computed with the steady-state solution are used as input and heads and ground-water discharge are computed every 30 days for a 5-month period of no recharge. The computed heads are compared with water-level recessions in observation wells and the computed discharge values are compared with known base-flow recession curves at gaging stations;
- (3) Predictive simulations in which the calibrated digital model was used to evaluate the effects on water levels and streamflow of large increases in ground-water withdrawals.



**A. NATURAL FLOW SYSTEM FOR UNCONFINED AQUIFER AND STREAMS.**



WHERE  $q_l$  = LEAKAGE,  $A_s$  = SURFACE AREA OF STREAM,  
 $A_N$  = SURFACE AREA OF NODE,  $K_v$  = VERTICAL HYDRAULIC  
 CONDUCTIVITY,  $h_R$  = RIVER HEAD (CONSTANT),  $h_1, h_2, h_3$  =  
 HEADS IN AQUIFER, AND  $b/2$  = THICKNESS OF ARTIFICIAL  
 CONFINING BED ( 1/2 AQUIFER THICKNESS ).

**B. SIMULATED LEAKAGE AT STREAM NODES IN DIGITAL MODEL.**

**FIGURE 3. MODEL CONCEPTION OF THE STREAM-AQUIFER FLOW SYSTEM.**

A two-dimensional areal model, with vertical leakage occurring only at stream nodes, is used to simulate the unconfined aquifer-stream system. This simulation assumes that the base of the unconfined aquifer is impermeable. Actually, the underlying confining bed is silt with an estimated hydraulic conductivity of about 1 ft/d (0.3 m/d). The conductivity of the unconfined aquifer is about two orders of magnitude greater (ranging from 50 to 250 ft/d or 15 to 76 m/d), and thus the assumption of two-dimensional flow is plausible under natural conditions. For pumping from the widely spaced wells (with limited cones of depression) in the unconfined aquifer, the model results should be reasonably correct. However, where pumpage from the underlying artesian aquifers is substantial (such as at Dover), the assumption of two-dimensional flow is invalid. A three-dimensional model of the Dover area involving the unconfined aquifer and underlying Cheswold (Miocene age) and Piney Point (Eocene age) aquifers is currently being developed.

Under natural conditions, ground water moves laterally to discharge points along the streams. Because the streams are partially penetrating into the aquifer, water must move with a vertical component in the immediate vicinity of the stream channels. Figure 3 shows the natural flow system and the simulated version of ground-water discharge to streams in the model.

The model uses an indirect method to compute ground-water seepage to streams. The computing routine calculates seepage through a confining bed, occurring over the entire area,  $\Delta x \Delta y$ , of each node into which seepage occurs. The seepage calculation is made using the expression

$$K_v \Delta x \Delta y \left( \frac{h_r - h}{m} \right)$$

where  $K_v$  is the vertical hydraulic conductivity of the confining bed,  $h_r$  the head above the confining bed,  $h$  the head within the aquifer at the node in question, and  $m$  the thickness of the confining bed.

In the aquifer, at locations immediately below a gaining stream, hydraulic head increases progressively from the bottom of the stream to the base of the aquifer. Assuming the aquifer to be homogeneous, the average or effective head should exist at one-half the distance between the stream and the bottom of the aquifer. Discharge into the stream was calculated by treating the upper half of the aquifer as a confining bed. The head above the "confining bed" was taken

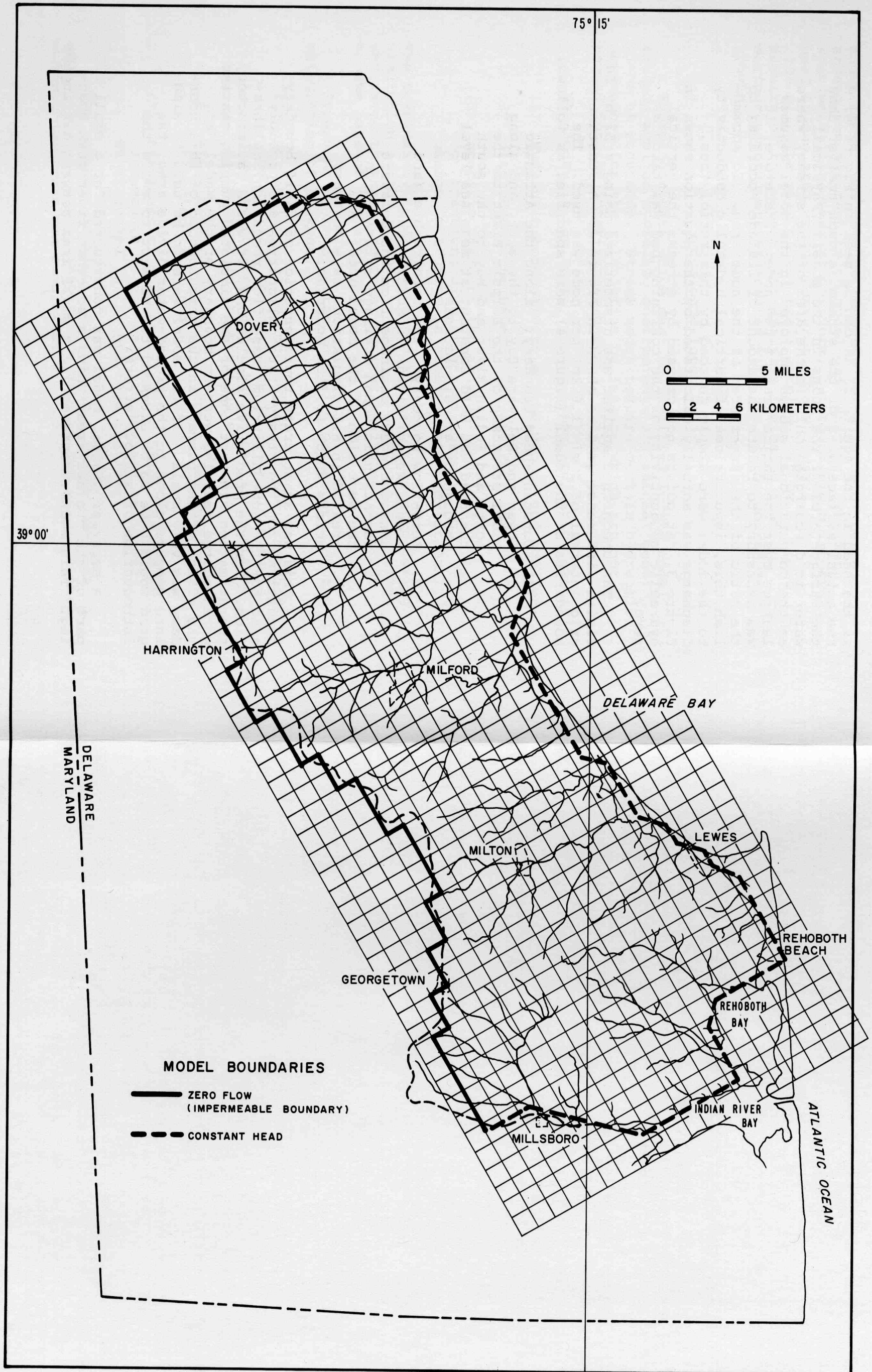


FIGURE 4. MODEL AREA WITH FINITE-DIFFERENCE GRID AND AQUIFER BOUNDARIES.

as the head in the aquifer. As can be seen in Figure 4, the actual surface area of the stream is much smaller than the surface area of the nodes in the model. Physically, water is discharging only in the area of the stream, whereas the computational scheme employed in the model assumes leakage over the entire area of the node. Therefore, it was necessary to reduce the amount of leakage according to the ratio of the stream area of the node area. To accomplish this, input values of vertical hydraulic conductivity to the model were simply reduced by this ratio; thus, discharge was actually calculated by the equation shown in Figure 3. At points not crossed by streams the vertical hydraulic conductivity of the confining bed was taken as zero.

The unconfined aquifer was discretized using a 52 by 20 finite-difference grid as shown in Figure 4. A constant grid interval of 1 mi (1.6 km) per node was used. The boundaries of the model (Figure 4) were specified as follows:

- (1) Constant-head boundary: along the Atlantic Ocean and Delaware Bay to the east, and along tidal stretches of the Leipsic River to the north and Indian River and Bay to the south the head was held constant at mean sea level throughout all simulations.
- (2) Zero-flow boundary: the topographic (and ground-water) divide separating the Chesapeake Bay drainage from the Delaware Bay-Atlantic Ocean drainage - located on the west side of the model area - was treated in all simulations as a boundary across which no flow occurred.

This was an accurate representation of field boundary conditions for the steady-state calibration and probably also for the nonequilibrium (base flow recession) calibration. For the predictive simulations, it was a satisfactory approximation, in that none of the simulated pumping centers were located near the western boundary of the model, and the effect of the pumpage in the neighborhood of this boundary was very small. If the model were to be used to simulate pumpage close to the ground-water divide area, the grid would have to be extended to the west to avoid the introduction of serious errors.

A separate digital model was constructed for a small part of the project area, using a somewhat finer mesh spacing. This model, representing a small area between Harrington



and Milford, was used to test the sensitivity of certain stream seepage results to model grid spacing. The mesh spacing utilized was not uniform, but the minimum interval, used in the area around Beaverdam Branch, was 1,000 ft (305 m). The size of the small model was 22 by 22 nodes. Results of the sensitivity analysis indicated that no serious errors were associated with use of the coarse mesh spacing.

### Hydrologic Input Data

Hydrologic data which must be specified in the model include aquifer hydraulic coefficients and initial conditions at each node. Stream nodes require certain hydraulic parameters for the calculation of leakage which are not needed for the inter-stream nodes.

All nodes in the model require these aquifer parameters:

- (1) Hydraulic conductivity of the unconfined aquifer;
- (2) altitude of the base of the aquifer; and
- (3) specific yield of the aquifer (required only for transient simulations as there is no change in storage in steady-state simulations).

The transmissivity is calculated for each time-step during simulation as the product of hydraulic conductivity times saturated thickness (current aquifer head minus altitude of the base of the aquifer).

Stream nodes require the following parameters for the calculation of leakage:

- (1) Vertical hydraulic conductivity of the streambed (the vertical conductivity of the aquifer reduced to compensate for the stream surface area, as discussed in the previous section);
- (2) river head or altitude of the stream surface at median flow; and
- (3) thickness of the confining bed below the stream (assumed to be one-half the aquifer thickness, as discussed in the previous section).

The recharge is also specified as a source function for each node. No evapotranspiration rate is specified because winter conditions are simulated.

Initial conditions are specified by assigning a "starting" potentiometric head at each node. For the steady-state simulations, any value of starting head may be assigned because the computed results are independent of initial values. However, for ease in analyzing steady-state model results, the starting heads are specified as the mean water-table altitudes. These head values, which do not differ from the average winter water-table by more than a few tenths of a foot, were obtained from 1:24,000 scale water-table contour maps published for Delaware (U.S. Geological Survey, 1964-65). If all hydrologic input data are correct, the steady-state model will reproduce the water-table surface as shown on the maps. Thus, the difference between starting heads and output heads is a measure of the accuracy of the model.

For the nonsteady calibration and predictive model runs, the starting heads were taken as the computed heads obtained in the steady-state calibration. These did not differ significantly from the measured mean water-table altitudes.

For the steady-state calibration runs, the hydraulic conductivity values specified at each node were obtained from the transmissivity map (Figure 2) and a saturated thickness map (Johnston, 1973, Figure 3). The saturated thickness map is considered to be quite accurate. However, as previously discussed, the transmissivity map varies greatly in accuracy. Thus, it was anticipated that conductivity values would be changed during model calibration. However, changes were made only within the range of hydraulic conductivity values obtained by aquifer test analyses (50 to 250 ft/d or 15 to 76 m/d).

Input values for the altitude of the base of the aquifer were obtained from a structure contour map which was based on several hundred geologic and driller's logs (Johnston, 1973, Figure 2). Accordingly, no changes in the input data on the base of the aquifer were anticipated, during model calibration.

A specific yield value of 0.15 was specified at all nodes for transient simulations.

For stream nodes, where upward leakage is calculated, values of vertical hydraulic conductivity ( $K_v$ ) were estimated to be one-tenth the horizontal hydraulic conductivity ( $K_h$ ). Inasmuch as the input values of  $K_h$  are suspect in many parts of the model area, input values of  $K_v$  obtained using a 10:1 ratio are also questionable. Accordingly, changes in  $K_v$  values were anticipated during model calibration.



River head values assigned at the stream nodes were the average stream altitude as shown on 1:24,000 scale topographic maps or, if available, the stream altitude at median flow, as obtained from gaging station data. The river heads are very reliable for steady-state model simulations but less so for transient simulations, where the stream stage is varying. Nevertheless, the difference between stream stage at high base flow and low base flow is less than 1 foot for the small streams.

As discussed in the section on model concepts, a hypothetical confining bed, equivalent to one-half the aquifer thickness is used at the stream nodes to compute leakage. If this concept is valid then the input values of the confining bed thickness should be considered accurate because of the good control on aquifer thickness.

A uniform recharge rate of 14 in (356 mm) per year was specified at all nodes. As discussed earlier, this value represents the average ground-water runoff during winter months for central and southern Delaware. Undoubtedly, recharge rates vary throughout the model area; however, insufficient data exist to define these local variations. No change in the recharge was anticipated, or made, during the steady-state simulations.

## CALIBRATION

Digital aquifer models must closely simulate the natural flow of ground water if they are to be usable. The process of determining and improving the ability of a model to do this is termed calibration. Digital aquifer models are most effectively calibrated by simulating the known history of pumping from wells and comparing head declines computed by the model with actual declines as measured in wells. In this study, however, this approach cannot be used because withdrawals from the unconfined aquifer have been very small and no measurable decline of the water table has occurred to date (1975).

However, a steady-state simulation of the aquifer-stream system can be made. Heads computed by the model can be compared with the known steady-state altitude of the water table and computed leakage at stream nodes can be compared with the measured base flow of streams. Also, a transient calibration can be made by simulating a period of no recharge using the output from the steady-state simulation as the initial condition. Head declines and changes in leakage at stream nodes

computed in this transient simulation may then be compared with actual water-level declines in wells and base-flow recessions in streams.

### Steady-State Simulation

An approximation of the differential equation describing two-dimensional steady flow in a homogeneous aquifer was given by Stallman (1962, p. 138). For a finite-difference mesh having a uniform grid spacing ( $\Delta x = \Delta y = \text{constant}$ ), the head distribution around a particular node in a discretized areal model is as follows:

$$h_1 + h_2 + h_3 + h_4 - 4h_0 + \left(\frac{W}{T}\right) \Delta x \Delta y = 0, \quad (3)$$

where:

- $h_0$  = head at a particular node,
- $h_1, h_2, h_3, h_4$  = heads at the 4 surrounding nodes,
- $\Delta x = \Delta y$  = grid spacing,
- $T$  = transmissivity, and
- $W$  = steady rate of recharge per unit area.

This relationship indicates that an increase in the recharge rate will increase the head differences between nodes. Conversely, an increase in transmissivity will decrease the head differences (or water-table gradients will be lessened). It is apparent that an infinite number of  $W$  and  $T$  values will satisfy with  $W/T$  ratio needed for a given head distribution. Therefore, both  $T$  and  $W$  cannot be varied during calibration, otherwise the calibration process is meaningless.

Hydraulic conductivity ( $K$ ) is specified in the model rather than  $T$  (which equals  $Kb$ ) because saturated thickness ( $b$ ) varies according to head. As previously discussed, values for the uniform recharge rate ( $W$ ) were considered to be more reliable than the values used for ( $K$ ). For this reason,  $W$  was held constant during the calibration procedure and  $K$  was varied until the computed heads closely reproduced the steady-state water table. Vertical conductivity ( $K_z$ ) of the confining beds below streams was also changed during calibration, but always in the ratio of 10:1 (that is,  $K/K_z = 10$ ). In making changes in the  $K$  values, regional

hydrology was considered and much of the specific-capacity data were eventually ignored. Changes in K were made only within the range of 50 to 250 ft/d (15 to 76 m/d) - the range of K values obtained by aquifer test analyses (Johnston, 1973, Table 2). K values were increased an average of 50 percent from the initial to the final calibration runs.

As mentioned in the previous section, input heads are the steady-state water-table altitudes. Therefore, draw-downs computed by the model (input head minus output head) represent head error. Thus, calibration becomes a process of minimizing the drawdown (or head) error. Early simulations resulted in negative head errors throughout a large part of the model area; in other words, computed heads were higher than the steady-state water-table elevations. This suggested that the input values of K were generally too low. Noteworthy was a close agreement between input heads and computed heads in the vicinity of aquifer test sites. Thus the input K values (as well as the input recharge rate) were considered correct at these sites.

The calibration criterion selected for the model was that head errors should be less than the average annual fluctuation of the water table (about 5 ft or 1.5 m). Specifically, the head errors should lie within the range of +2.5 to -2.5 ft ( $\pm 0.8$  m) and the mean head error should approach zero. For various reasons all nodes in the model cannot be realistically expected to meet this criterion. Head errors at nodes adjacent to the model boundaries may be caused by the computational method rather than errors of input data. At stream nodes, the starting values of aquifer head were set equal to the stream surface altitudes (which differ slightly from the actual aquifer heads). This was done so that there would be no leakage to or from streams at the beginning of the simulation. The criterion finally selected for model calibration was that the standard deviation of the head errors at inter-stream nodes should be less than 2.5 ft (0.8 m). This means that 70 percent of the head values (or 2 standard deviations) will occur within the 5-foot (1.5 m) annual range of the water table.

Figure 5 shows a graph of head error distribution for an early simulation compared with the final simulation when calibration was completed. The early simulation (K values based on the published transmissivity map) was characterized by negative head errors with a standard deviation of 3.8 and a negative mean error of -2.2 ft (-0.7 m). The final calibration run shows a mean error close to zero (+0.3 ft or +0.7 m) and a standard deviation of 2.3 which meets the

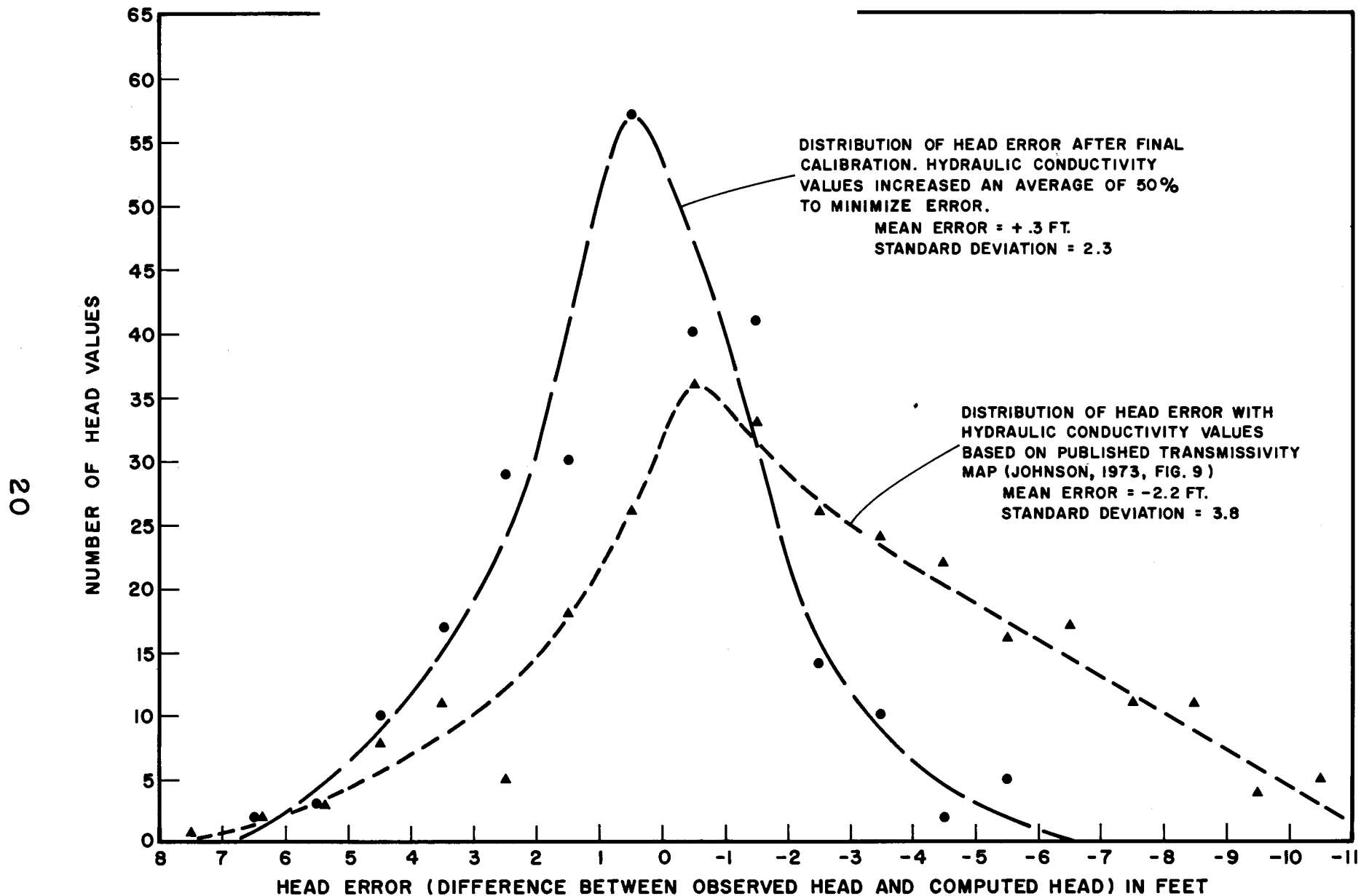


FIGURE 5. GRAPHS SHOWING DISTRIBUTION OF HEAD ERROR RESULTING FROM CHANGING THE HYDRAULIC CONDUCTIVITY MATRIX IN MODEL SIMULATIONS.

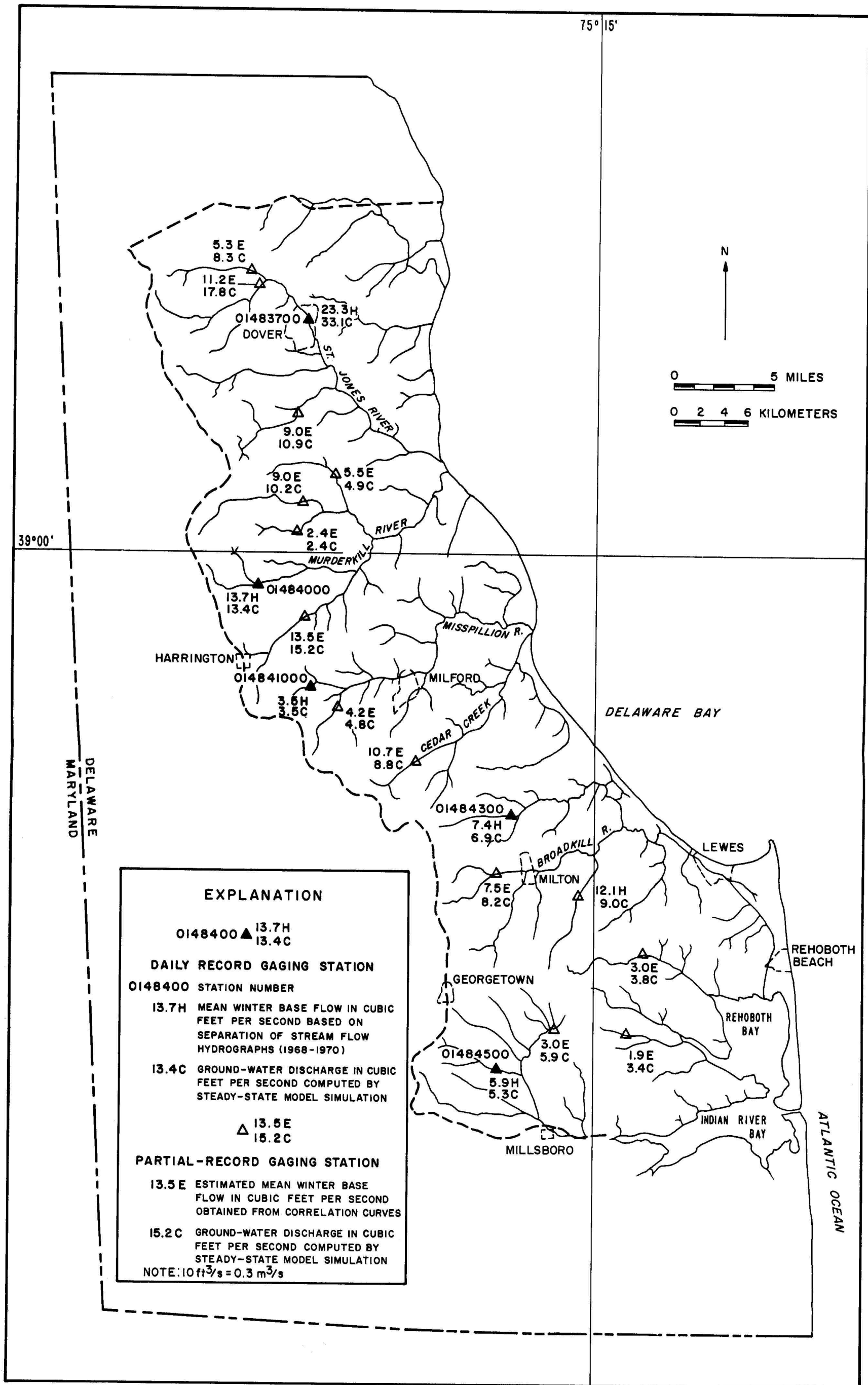


FIGURE 6. COMPARISON OF MEAN WINTER BASE FLOW AT STREAM-GAGING STATIONS WITH GROUND-WATER DISCHARGE COMPUTED BY STEADY-STATE MODEL SIMULATIONS.

stated criteria. The large negative head errors observed in the early simulation were essentially eliminated in the final calibration. The distribution of conductivity values used in the final calibration provides the basis for a revised transmissivity map that is presented in the following section on simulation results.

The calibrated model should reproduce the distribution of base flow throughout the model area. The model, with its computed head distribution, in effect, defines the drainage area for streams during base flow conditions. Figure 6 shows a comparison of ground-water discharge computed by the model and winter base flows, where available. Base flow data for the period 1968-70 were used because precipitation was near normal and streamflow about average during the 3 years.

Five continuous-record gaging stations are located within the model area. At these stations, winter base flow could be estimated fairly accurately by separation of the streamflow hydrographs (Johnston, 1973, p. 41-45). It is noteworthy that ground-water discharge computed by the model at 4 of the 5 streams is within 10 percent of the mean winter base flow.

The poor agreement occurred in the St. Jones River basin near Dover (Station 01483700 in Figure 6) where the model-computed discharge exceeds the field value by about 50 percent. The lack of agreement is due to large ground-water withdrawals from the deeper Cheswold (Miocene) artesian aquifer. These withdrawals have produced an extensive cone of depression with head differences between the Cheswold and unconfined aquifer of as much as 100 ft (30 m). As a result, a significant amount of water which would naturally discharge to the St. Jones River and its tributaries is leaking downward from the unconfined aquifer to the Cheswold aquifer. It is noteworthy that the difference between the model-computed and field values of ground-water discharge ( $10 \text{ ft}^3/\text{s}$  or  $0.28 \text{ m}^3/\text{s}$ ) is about the same as the current 6 Mgal/d ( $23,000 \text{ m}^3/\text{d}$ ) pumping rate from the Cheswold (equivalent to  $9 \text{ ft}^3/\text{s}$  or  $0.25 \text{ m}^3/\text{s}$ ).

The assumption of two-dimensional flow made in designing the model is thus invalid in the Dover area. Because the water-table contours were matched in the St. Jones River basin using flow to the river that is higher than the actual base flow, the model transmissivity values are too high. However, the effect of these slightly high T values on the total transmissivity distribution in the study area is minor.

The 2-D model cannot, of course, be used to make predictive simulations in the St. Jones River basin. However, the 2-D model is useful in pinpointing the area of leakage from the unconfined aquifer to the Cheswold. (See further discussion in the section on model results.)

The assumption of two-dimensional flow in the model is also invalid in Beaverdam Creek basin south of Milton where there is significant upward leakage from the Manokin artesian aquifer under natural conditions (Johnston, 1973, p. 60). The model-computed discharge is  $9 \text{ ft}^3/\text{s}$  ( $0.25 \text{ m}^3/\text{s}$ ) compared to the field base flow estimate of  $12.1 \text{ ft}^3/\text{s}$  ( $0.34 \text{ m}^3/\text{s}$ ) as shown in Figure 6. The difference between the model and field values ( $3 \text{ ft}^3/\text{s}$  or  $0.09 \text{ m}^3/\text{s}$ ) provides a rough estimate of the natural leakage rate. The model values of transmissivity are slightly low in the basin because water-table contours are matched using a flow to Beaverdam Creek that is less than the actual base flow.

At the partial-record stations shown in Figure 6, winter base flow was estimated by use of correlation curves. These curves, which relate a few base-flow discharge measurements to concurrent flows at a continuous record station were provided by K. R. Taylor (written commun., October, 1972). A discussion of the preparation and use of the correlation curves is given in Cushing, Kantrowitz, and Taylor (1972, p. 26-29 and Figure 14). The curves were used to transfer mean winter base flow from a continuous record station to a partial-record station. Winter base flow values obtained with the correlation curves are estimates. However, these values are accurate enough to indicate any parts of the model area where serious errors exist. Figure 6 shows that at 10 of the 12 streams (excepting the two tributaries of the St. Jones River), the model-computed values are within 30 percent of the estimated winter base flow values.

#### Transient Simulation with No Recharge

A quasi-transient calibration was made by simulating periods of no recharge. The purpose was to compare the recession of water levels and base flow, as determined in the field, with computer-generated values. This transient simulation is independent of recharge rate and provides a further check on the aquifer parameters. A specific yield value, which is not required for the steady-state simulation, is needed for the transient simulation and a uniform value equal to 0.15 is used in the model. The heads computed with the steady-state simulation are used as the initial conditions

and heads and ground-water discharge are calculated for a 150-day period of no recharge. This period was selected because during most years, there is a continuous recession of ground-water levels and base flow for 3 or 4 months during the summer; during drought years, the recession may last 5 or 6 months.

During the 3-year period 1968-70 used for calibration of the model, a period of low rainfall and no recharge occurred from July to September 1970. After heavy rains in June and July, the base flow of the streams was relatively high (about equivalent to mean winter base flow) and ground-water levels were approximately at mean stage. Attempts to duplicate the ensuing recession of water levels and base flow were partially successful.

The model-generated values of ground-water discharge agreed closely with base flow data except where pumpage or evapotranspiration were substantial. On the other hand, the computed heads did not agree closely with the measured water levels at some observation wells. The reason for the poor match at some wells is probably related to: (1) node spacing in the model and (2) the use of an average specific yield value for all nodes in the model. The model computes an average head for a 1 mile square nodal area rather than at a specific site. Depending upon the location of the well site with respect to streams and ground-water divides, the computed head may differ from the measured head by several feet. As previously noted, an attempt to overcome this scaling problem was made by enlarging a small area of the model to 1,000 ft (305 m) grid spacing and repeating the simulation. The agreement between observed and computed heads was improved; however, the observed water-level recession could not be closely matched. It is probably unrealistic to expect good duplication of water-level recessions at individual wells without an accurate knowledge of areal variations in specific yield. However, the fact that the model can accurately duplicate the base-flow recession curves (see following discussion) suggests that the use of an average specific yield is valid on a regional basis.

Values of ground-water discharge computed by the model at the five continuously-gaged streams are shown graphically in Figure 7. Streamflow hydrographs for June-September 1970 for these streams are shown in Figures 8, 9, 10, and 11. Superimposed on the hydrographs are the recession curves of ground-water discharge generated by the model.



As can be seen, a very close match exists between measured streamflow at Beaverdam Branch and Stockely Branch and model-generated recession curves. This suggests that:

- (1) The aquifer parameters ( $K$ ,  $K_z$ , and  $S_y$ ) are reliable for these basins, and
- (2) ground-water evapotranspiration is probably small (neither basin is swampy, and ground-water levels are 5 to 50 feet (1.5 to 3.0 m) below land surface nearly everywhere in these basins in summer.

Similar conditions exist in Sowbridge Branch basin except that there is a small pond with regulated flow at the outlet to the basin. However, the general trend of the hydrograph recessions closely follows the computer-generated curve (Figure 11).

The hydrograph for the Murderkill River departs below the computer-generated curve (Figure 9). Black Swamp in the head-waters of the basin is probably characterized by appreciable water loss due to ground-water evapotranspiration. The difference between the measured flow and computer values (3 to 4 ft<sup>3</sup>/s or 0.08 to 0.11 m<sup>3</sup>/s) is probably a good estimate of ground-water evapotranspiration.

The hydrograph for the St. Jones River departs considerably below the computer-generated curve (Figure 10). As discussed in the section on steady-state simulation, water which would normally discharge to the river is probably leaking downward to the Cheswold aquifer. The Cheswold pumping averaged about 6 Mgal/day (23,000 m<sup>3</sup>/d) or about 9 ft<sup>3</sup>/s (0.25 m<sup>3</sup>/s) during 1970 but may have been higher during the summer when water demands are highest. Figure 10 indicated that the difference between the computer-generated curve and the base-flow recession curve ranges from 14 ft<sup>3</sup>/s (0.4 m<sup>3</sup>/s) at the high base-flow to about 9 ft<sup>3</sup>/s (0.25 m<sup>3</sup>/s) at the low-flow end. Thus, most of the disparity between the two curves can be accounted for by the Cheswold pumping.

## SIMULATION RESULTS

The unconfined aquifer model was useful for several purposes. The model permitted the preparation of a revised transmissivity map for the unconfined aquifer based on changes made during model calibration. The model helped to

identify an area of substantial vertical leakage to the heavily pumped Cheswold (Miocene age) aquifer. Estimates of ground-water discharge and net fresh-water flow in the tidal reaches of rivers were made with the model. The model was also used to project the decline of water levels and streamflow in five selected areas where increased withdrawals of water are likely.

The technique used to identify the area of substantial leakage to the Cheswold aquifer is described in a separate report (Johnston and Leahy, 1977). Briefly, the Cheswold aquifer is characterized by a regional cone of depression encompassing 140 mi<sup>2</sup> (363 km<sup>2</sup>) due to pumping in the Dover area. The model results indicate that water losses from the unconfined aquifer occur within a small area, and these losses cannot be accounted for except by downward leakage. This small area is in the St. Jones River basin north of Dover. The winter base flow in the St. Jones basin is about 10 ft<sup>3</sup>/s (0.28 m<sup>3</sup>/s) less than the model-computed value. The 10 ft<sup>3</sup>/s (0.28 m<sup>3</sup>/s) or 6.5 Mgal/d (25,000 m<sup>3</sup>/d) difference is equivalent to the ground-water pumpage from the Cheswold and suggests that virtually all leakage from the unconfined aquifer to the Cheswold aquifer is occurring within the St. Jones River basin (32 mi<sup>2</sup> or 83 km<sup>2</sup>). Furthermore, most of the leakage is occurring within the two tributary basins of the St. Jones (Figure 5) which comprise about 25 mi<sup>2</sup> (65 km<sup>2</sup>).

#### Revised Transmissivity Map of the Unconfined Aquifer

The average transmissivity of the unconfined aquifer is apparently higher than estimated from existing well data. Values of hydraulic conductivity used for the initial model runs were obtained from the transmissivity map shown in Figure 2. The early model runs showed clearly that conductivity values would have to be increased substantially in some areas to calibrate the model.

The average transmissivity (T) of the unconfined aquifer is about 6,000 ft<sup>2</sup>/d (560 m<sup>2</sup>/d) if the T values shown in Figure 2 are assumed to be reliable. However, final model calibration suggests that the average T is about 9,500 ft<sup>2</sup>/d (880 m<sup>2</sup>/d). A transmissivity map of the aquifer based on final model calibration is shown in Figure 12.

Comparison of the pre-modeling transmissivity map (Figure 2) and the revised T map (Figure 12) shows major differences. In particular, the transmissivity in parts of southern Delaware is substantially higher than the T estimated

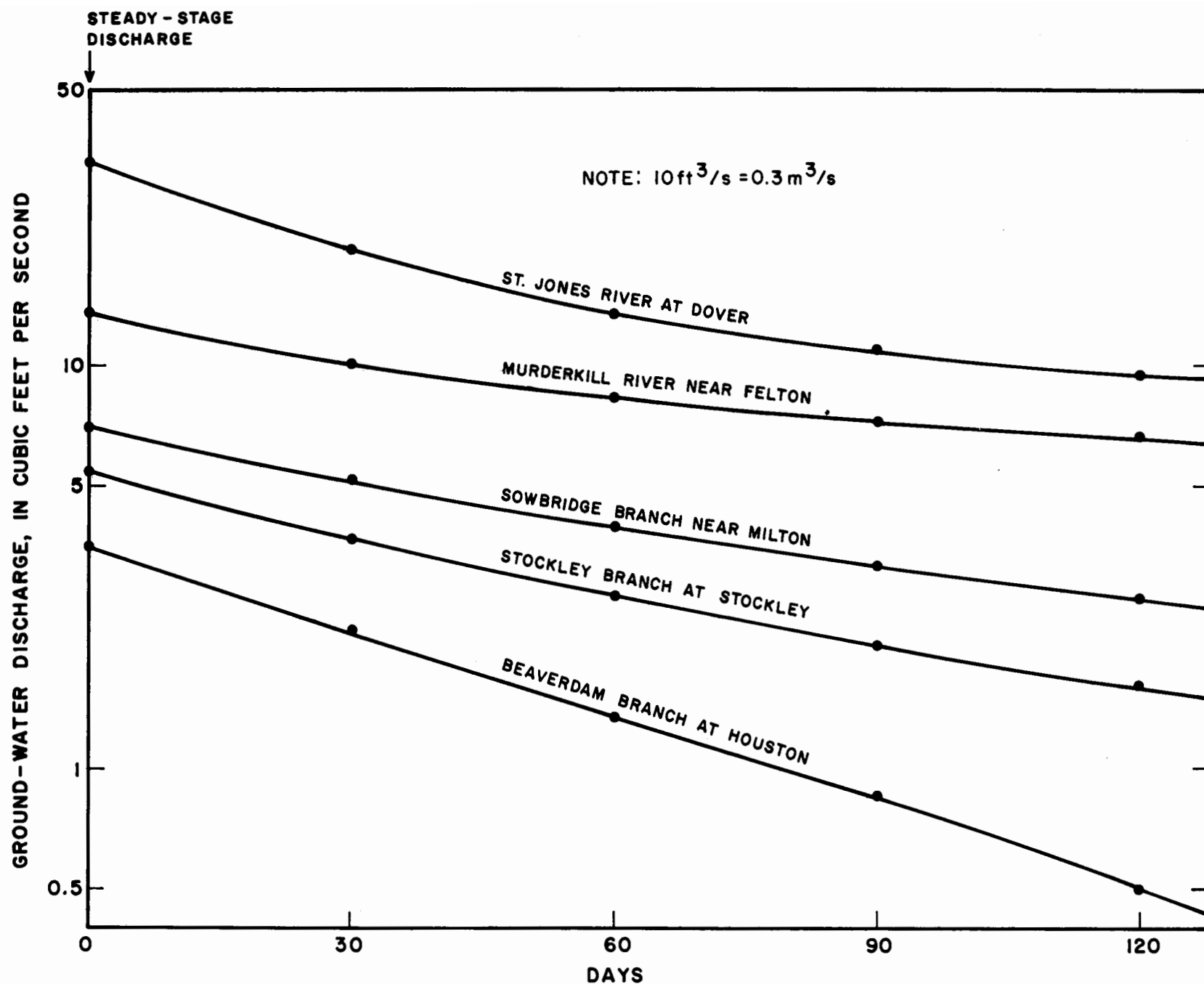


FIGURE 7. GROUND - WATER DISCHARGE AT 5 STREAMS BASED ON TRANSIENT MODEL SIMULATION OF 150 DAY PERIOD WITH NO RECHARGE.

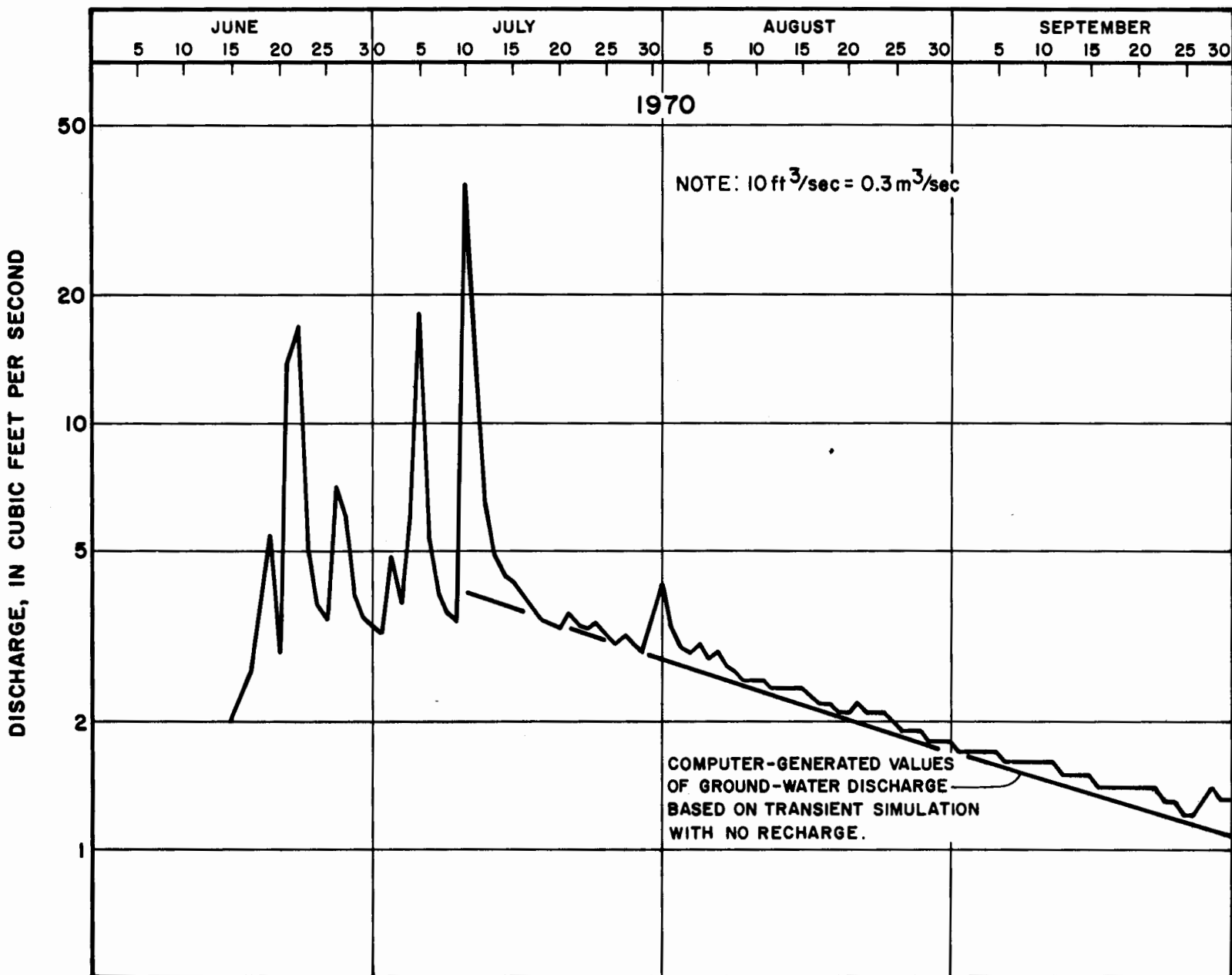


FIGURE 8. COMPARISON OF STREAMFLOW HYDROGRAPH FOR BEAVERDAM BRANCH AT HOUSTON, DEL. AND COMPUTED GROUND-WATER DISCHARGE USING DIGITAL MODEL.

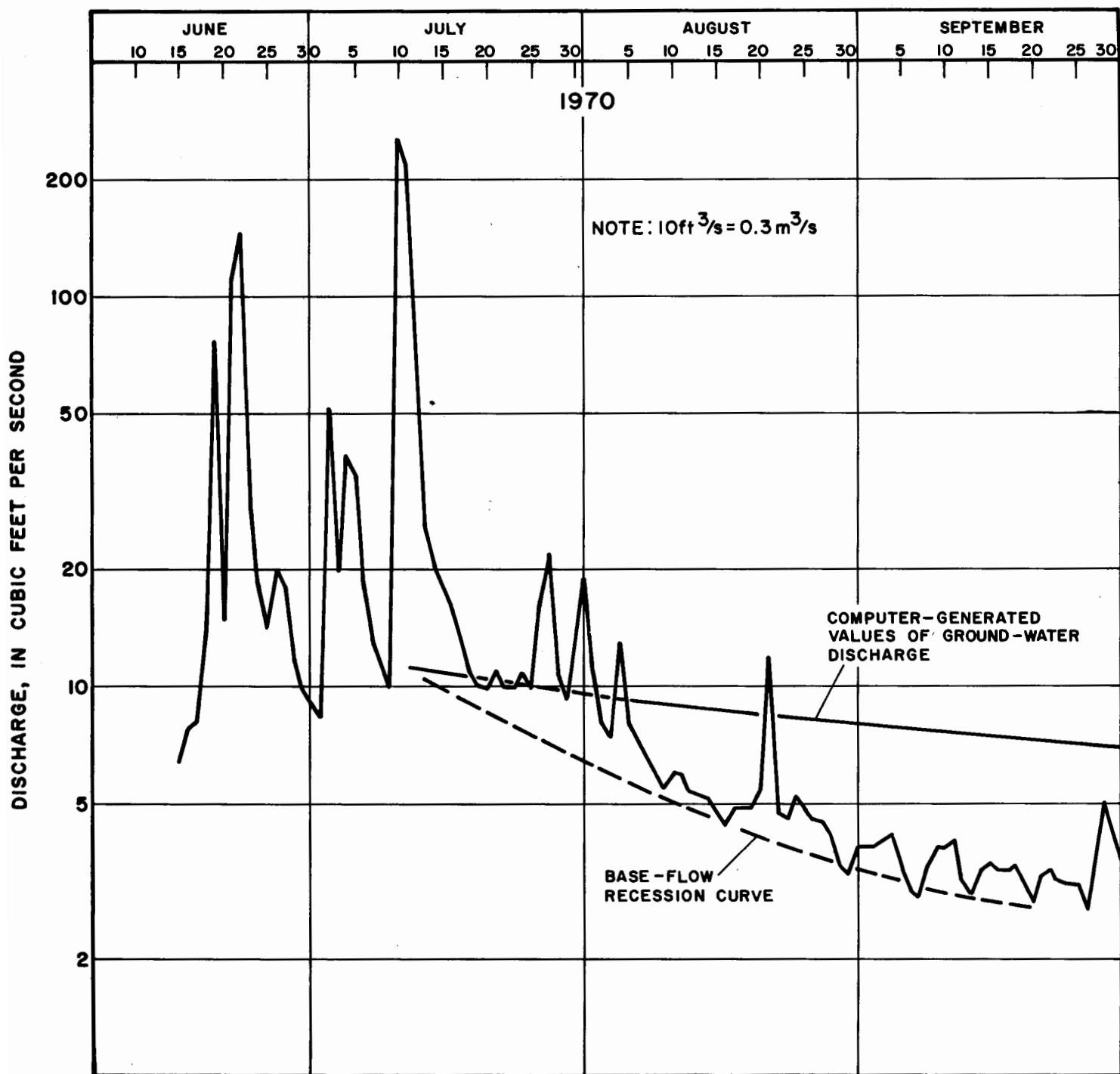


FIGURE 9. COMPARISON OF STREAMFLOW HYDROGRAPH FOR THE MURDERKILL RIVER NEAR FELTON, DEL. AND COMPUTED GROUND-WATER DISCHARGE USING DIGITAL MODEL.

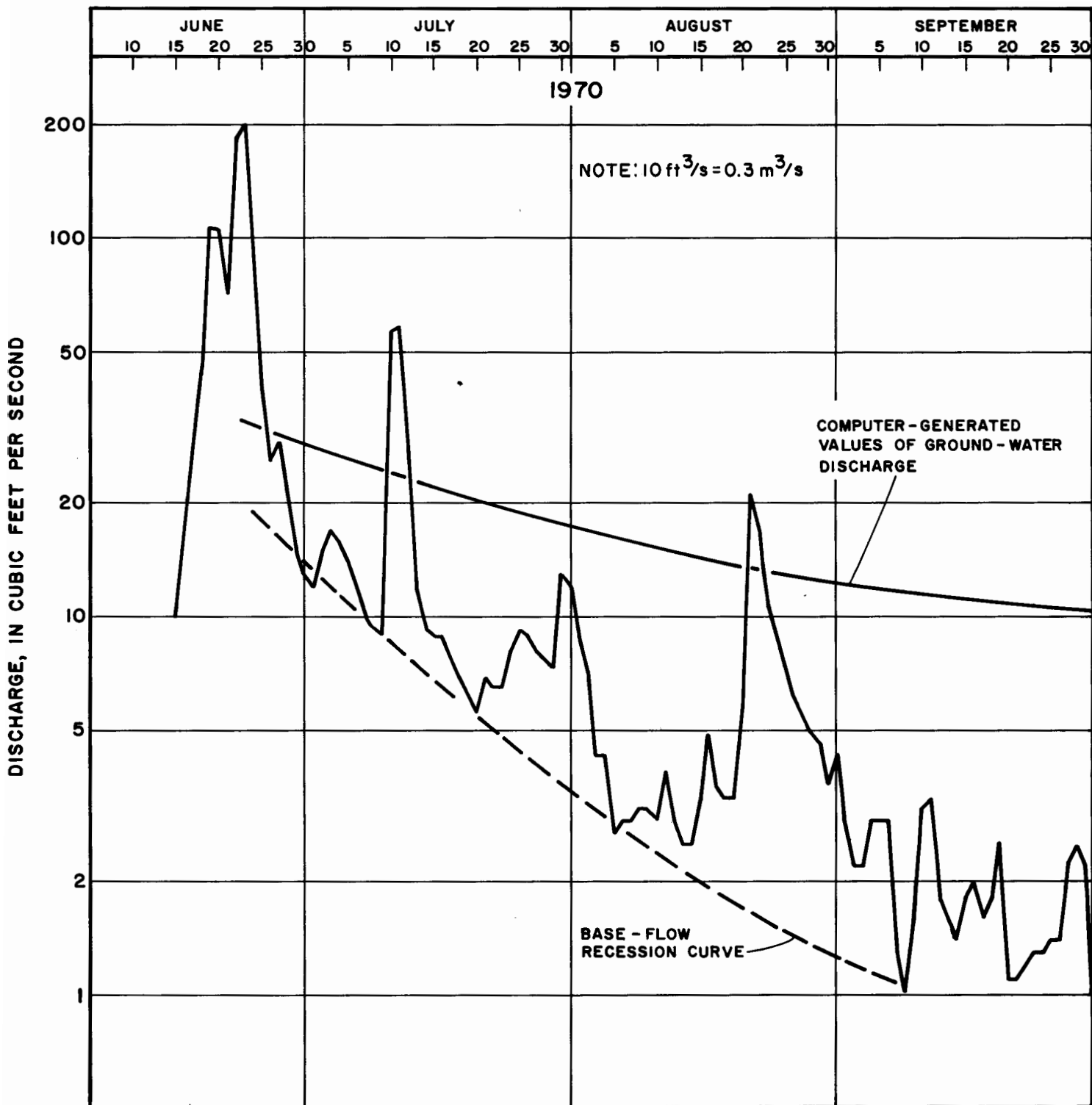


FIGURE 10. COMPARISON OF STREAMFLOW HYDROGRAPH FOR THE ST. JONES RIVER AT DOVER, DEL. AND COMPUTED VALUES OF GROUND-WATER DISCHARGE USING DIGITAL MODEL.

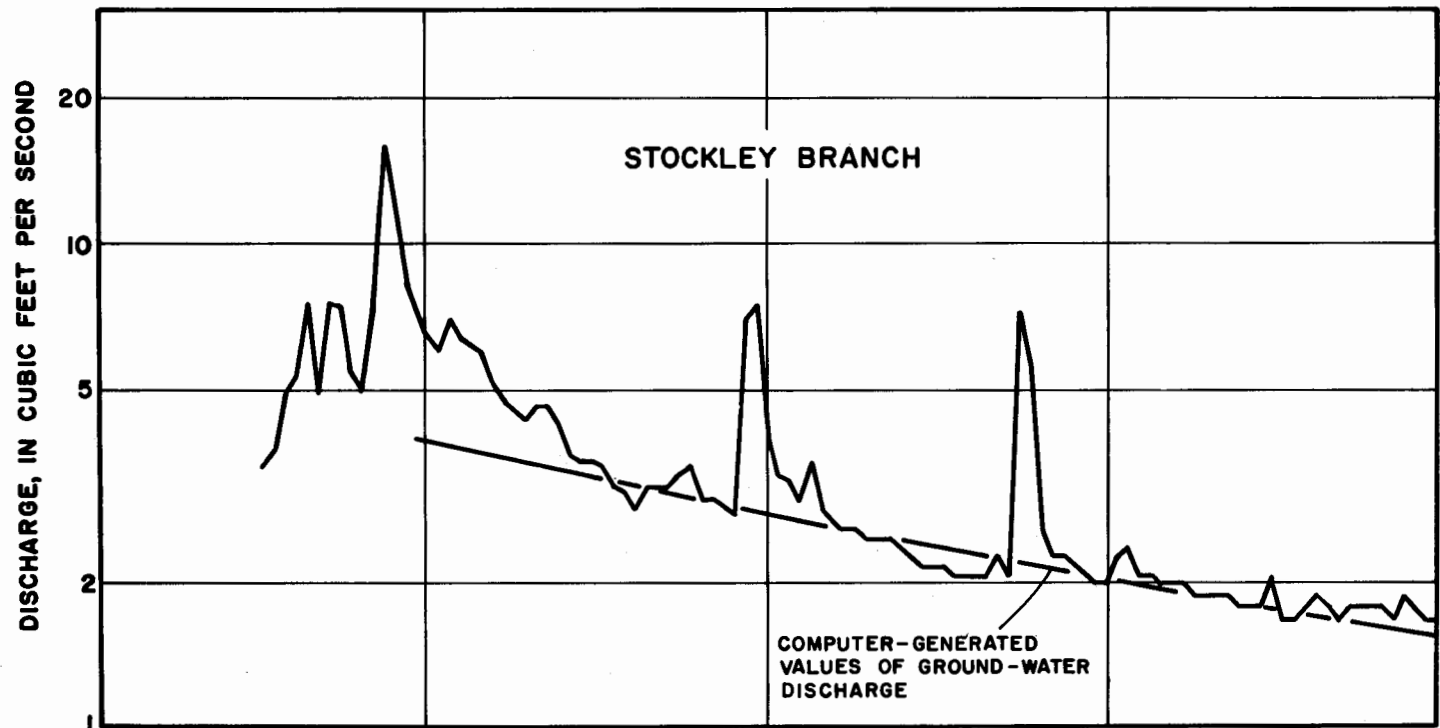
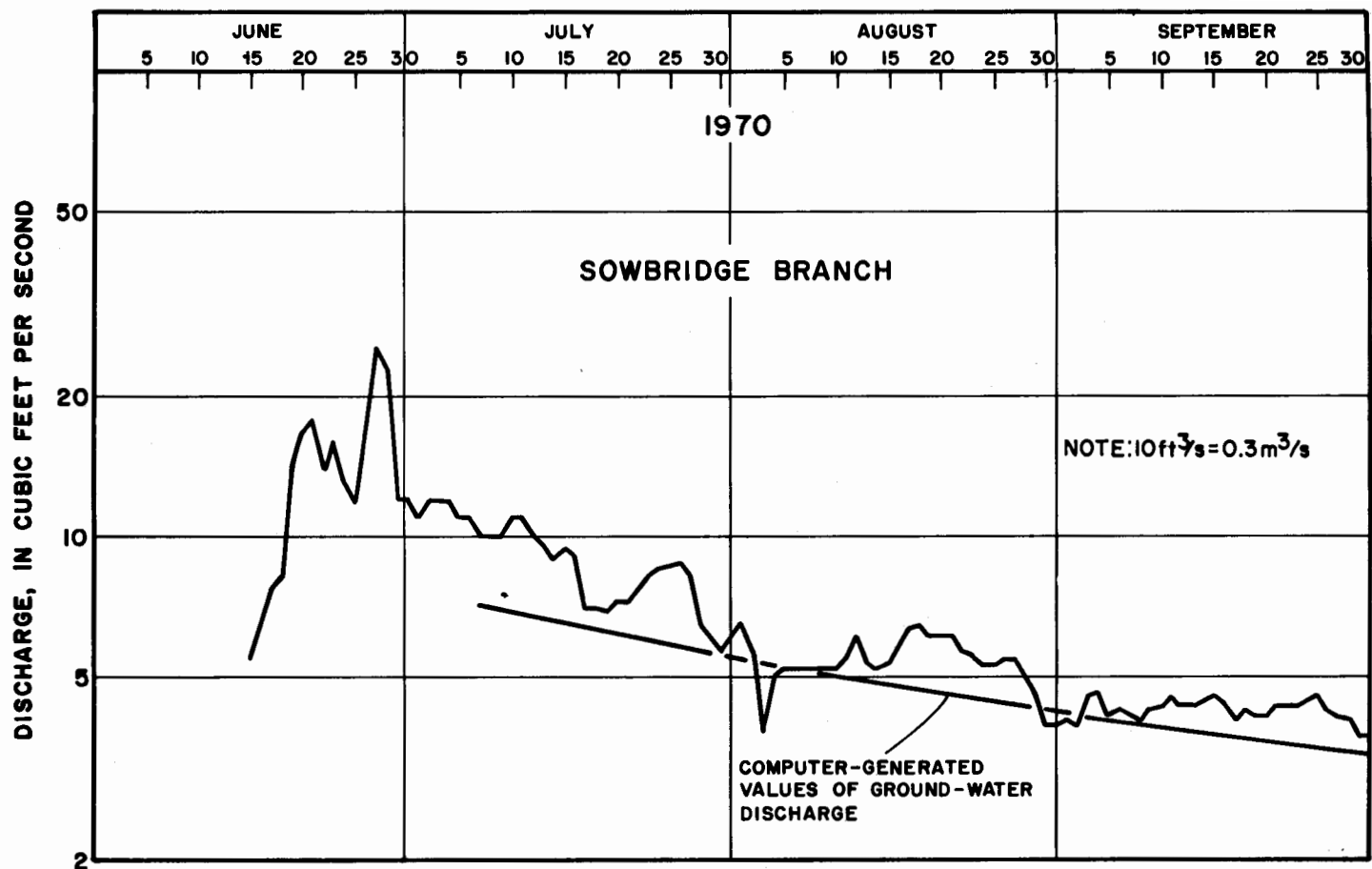


FIGURE II. COMPARISON OF STREAMFLOW HYDROGRAPHS FOR SOWBRIDGE BRANCH NEAR MILTON, DEL. AND STOCKLEY BRANCH AT STOCKLEY, DEL. AND COMPUTED VALUES OF GROUND-WATER DISCHARGE USING THE DIGITAL MODEL.

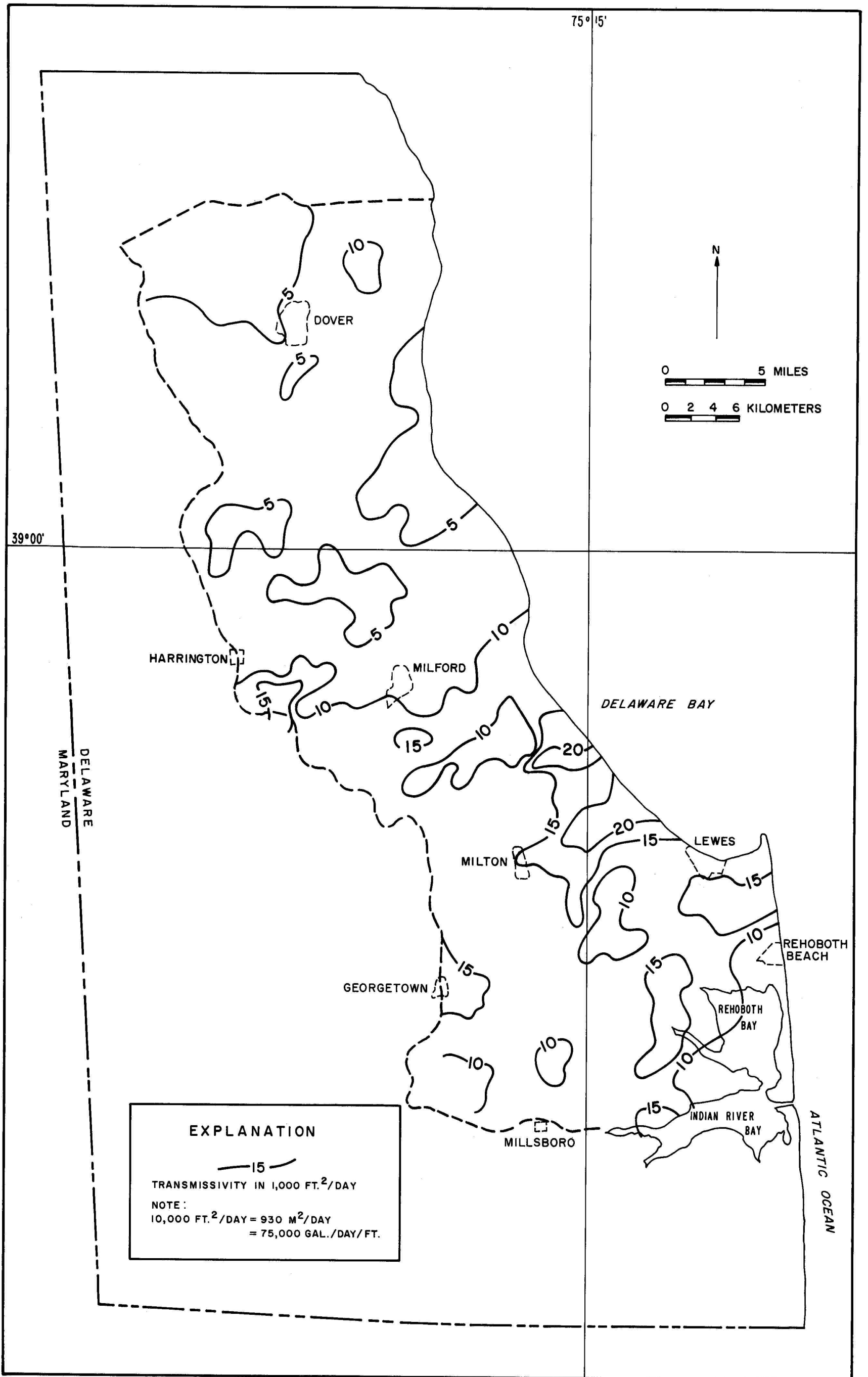


FIGURE 12. TRANSMISSIVITY OF THE UNCONFINED AQUIFER BASED ON CALIBRATION OF THE DIGITAL MODEL.



from some of the well data. The fact that many of the T values estimated from specific capacity are lower than the T values required for model calibration is not surprising. Specific capacity is affected by well construction and development and therefore may not represent a valid measure of aquifer transmissivity. The degree of well development is impossible to evaluate from reported data on well yields and pumping levels. However, well construction was evaluated to the extent that no wells of small diameter or short well screens were used in preparing the T map shown in Figure 2. Nevertheless the T values estimated from specific capacities, even for large-diameter, fully penetrating wells are not always reliable.

Noteworthy is the close agreement between T values obtained from aquifer tests (involving observation wells) with T values required for model calibration. At four of the test sites shown in Figure 2 (southeast of Harrington, east of Milton, and near Lewes and Rehoboth Beach), the T values derived from lengthy aquifer tests are almost identical with T values required for final model calibration. At one aquifer test near Dover (Figure 2) the field T value is about one-half of the T value required for calibration. However, this T value was obtained by analysis of data from a short pumping test and is considered suspect.

In summary, the two T maps (Figures 2 and 12) are in agreement where reliable aquifer test data exist but disagree where T values are based on specific-capacity data only. Although individual T values may vary as much as 100 percent, both maps show a southward increase in transmissivity across Delaware. The T map based on model calibration is considered more reliable and should supersede the earlier published map.

#### Estimated Ground-water Discharge and Net Fresh-Water Flow in Tidal Streams

The calibrated steady-state model was used to estimate ground-water discharge at ungaged streams, particularly in tidal rivers. Rough estimates of total fresh-water flow at the mouths of tidal rivers were also made. The measurement of net fresh-water flow in tidal streams is difficult and expensive, particularly in the tidal marshes near the coastline. There are no field data to check model-computed values in the tidal areas. However, the model reproduces winter base-flow at gaging stations reasonably well (Figure 6) and



closely reproduces the water-table configuration in both tidal and nontidal areas. Thus the model-computed values of base flow in tidal areas should be reliable.

In central and southern Delaware, the average winter base flow is about 90 percent of the average stream discharge at continuous record gaging stations (Johnston, 1973, Table 4). If this relationship is also true for the tidal areas, the net fresh-water flow is readily obtained from model-computed values of ground-water discharge.

Figure 13 shows the model-computed values of ground-water discharge in the ungaged and tidal reaches of the five major streams in the area. By combining these values with the winter base-flows at gaging stations, the total ground-water discharge for the five basins has been estimated (Figure 13).

The net fresh-water flows at the mouths of four tidal rivers in the model area are shown on Figure 13. These inferred flows must be used with caution in the absence of any field data. No value is shown for the St. Jones River because, as noted previously, substantial leakage to the underlying Cheswold aquifer occurs in the basin upstream from Dover. Thus, the relationship between winter base flow and average discharge observed at the gaging station in Dover, is different than the relationship in the tidal area where leakage is not significant.

### Water-Supply Potential of Selected Areas

The calibrated digital model was used to simulate substantial increases in ground-water withdrawals in five selected areas (see Figure 14). All of these areas are characterized by high ground-water-development potential. Each of the areas is located near small cities and represent future sources of moderate to large water supplies.

Two areas ( a seashore resort area and an irrigated farming area) have substantial pumpage during the summer and very light pumpage in the winter at present (1975). This pattern of seasonal pumpage is likely to continue, and therefore simulations of the conditions during a very dry summer were made to appraise the two areas realistically.

Water-level declines and streamflow depletion were computed for a 30-year period using the pumping rates shown in Figure 14. Steady-state conditions were reached in all

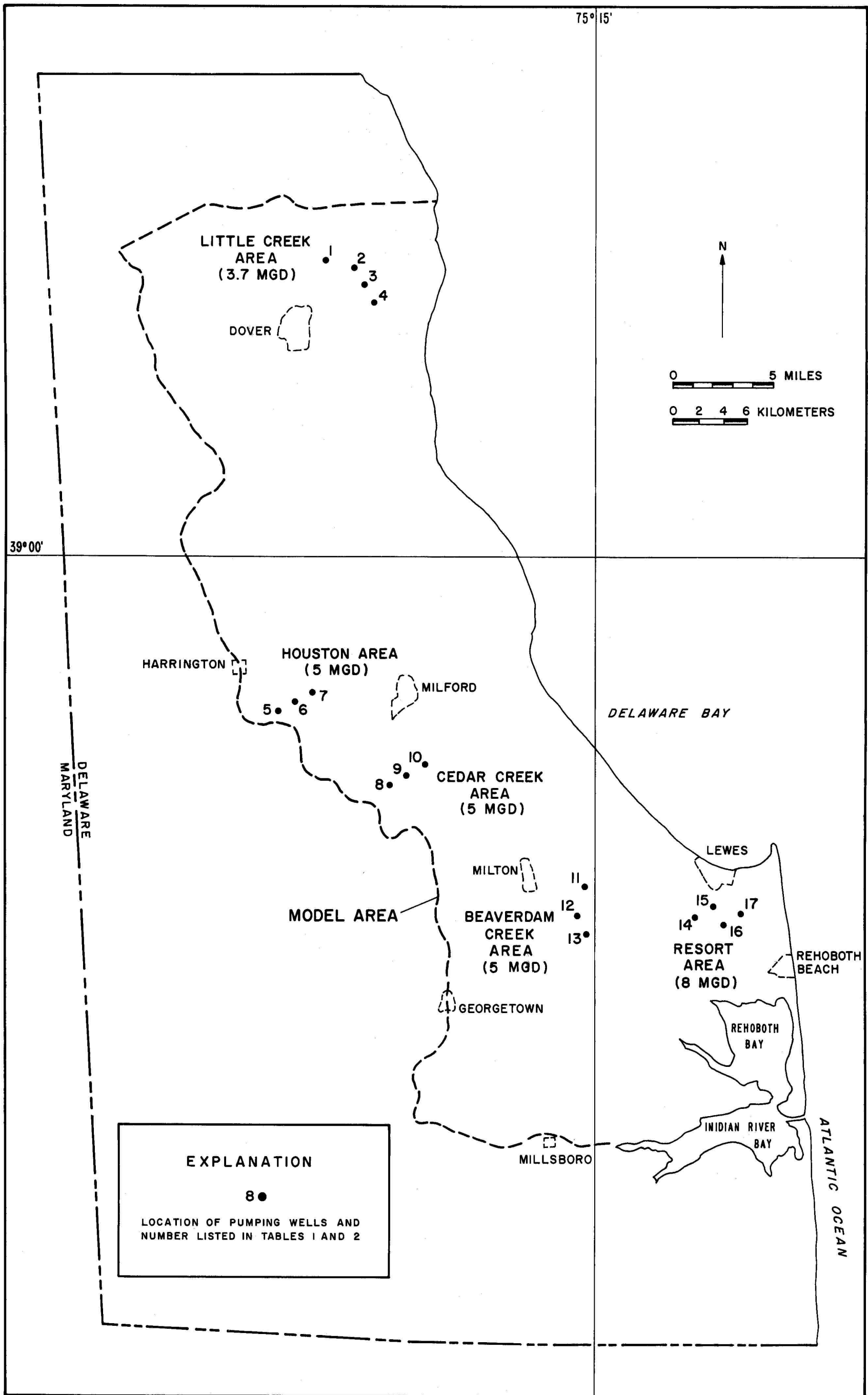


FIGURE 14. MAP SHOWING LOCATIONS OF HYPOTHETICAL WELLS USED FOR SIMULATING INCREASED PUMPING IN SELECTED AREAS.

TABLE 1. Projected water-level declines in five selected areas of high ground-water potential after 30-years of continuous pumping (based on transient simulation with average annual recharge rate of 16 inches). All wells have 12-in diameters.

Well No. (Figure 14)	Continuous pumping rate, in gallons per minute	Drawdown at pumping well in feet	Effective drawdown 1000 feet from pumping well, in feet
Little Creek area (3.9 Mgal/d)			
1	400 <sup>*</sup>	24	3.8
2	700	25	5.3
3	700	32	1.7
4	700	41	5.9
Houston area (5.2 Mgal/d)			
5	1,200	30	6.0
6	1,200	30	5.8
7	1,200	41	3.5
Cedar Creek area (5.2 Mgal/d)			
8	1,200	35	1.8
9	1,200	27	1.6
10	1,200	32	1.5
Beaverdam Creek area (5.2 Mgal/d)			
11	1,200	16	<1.7
12	1,200	22	<2.4
13	1,200	21	<1.9
Lewes-Rehoboth Beach area (7.8 Mgal/d)			
14	1,350	36	10.9
15	1,350	29	<sup>1</sup> 9.0
15	1,350	29	<sup>2</sup> 7.8
17	1,350	30	<sup>3</sup> 4.6

MSL = mean seal level

<sup>1</sup> Head is 2.7 ft below MSL

<sup>2</sup> Head is 1.6 ft below MSL

<sup>3</sup> Head is 2.9 ft below MSL

five areas within 9 years. Table 1 shows the projected water-level declines at individual pumping wells and at distances of 1,000 feet (305 m) from the wells after reaching steady-state conditions. The drawdown values are based on an average recharge rate of 14 inches (356 mm) per year, and therefore actual drawdowns will be somewhat greater in dry years and somewhat less in wet years.

The model was used to determine a near-maximum pumping rate which could be sustained indefinitely at each well. These pumping rates, together with their projected effects on water levels and streamflow in the surrounding area, are used as a basis for discussion in this section.

A uniform specific yield (0.15) was used in the model for simulating increased pumpage; the same value was used for the "quasi-transient" state calibration (no recharge). Small changes in specific yield (within the known range of 0.1 to 0.2) do not cause significant changes in computed drawdowns. For general planning purposes, the projected water levels and streamflow depletions given in this section are considered adequate.

The starting heads used for simulating the 30-year pumping are the computed heads obtained from the measured mean water-table altitudes. These heads incorporate a small part of present pumpage in one of the five areas, as discussed later.

Steady radial flow is assumed in the computation of the drawdowns within the pumping wells and drawdowns 1,000 feet (305 m) away from the wells, as listed in Table 1. The head at 1,000 feet (305 m) is approximately the same as the mean head for each node with a pumping well (for a finite-difference grid interval of 1 mile or about 1.6 km). The drawdowns or heads within the pumping wells were calculated from the "node head" using a variation of the equation for steady radial flow (Thiem formula), as described by Prickett and Lonngquist (1971, p. 61).

The criteria used to select the pumping rates listed in Table 1 was simply to use the maximum rate which could be sustained without the well "going dry." The spacing of the wells in the five areas was arbitrary and was dictated by the grid spacing of the model. Experiments were not made using variable finite-difference intervals to determine optimum well spacing. As development occurs, available digital models can be used to predict actual water-level and streamflow declines at proposed well sites.

The Little Creek area (Figure 14) is the only known locality near Dover where the unconfined aquifer is sufficiently thick and transmissive (Figure 12) to provide moderately large water supplies. At Dover municipal and industrial supply wells obtain water from the deeper Cheswold and Piney Point aquifers. Water levels have declined substantially in both aquifers in recent years and regionally extensive cones of depression have developed. Thus, pumping from the unconfined aquifer near Little Creek is an alternative to increased pumping from the artesian aquifers.

At present (1975), withdrawals from the unconfined aquifer near Little Creek occur mainly during the summer; the water is used mostly for irrigation of potatoes. A recent pumpage inventory by Frederick Robertson of the University of Delaware Water Resources Center (oral commun., October, 1975) indicates that about 100 million gallons ( $380,000 \text{ m}^3$ ) was used during the summer of 1974. Pumpage is highly variable in this area, depending upon summer rainfall. During a wet summer, pumpage is negligible. However, during a dry summer, such as 1974, pumpage is substantial.

The starting heads used for modeling the Little Creek area are steady-state water-table altitudes (as measured in 1959) and do not incorporate the effects of the present summer pumpage.

The 30-year transient simulation indicates that a total potential pumpage of  $3.6 \text{ Mgal/d}$  ( $14,000 \text{ m}^3/\text{d}$ ) can be obtained indefinitely from 4 wells in the Little Creek area spaced as shown in Figure 14. If the average summer withdrawal rate at present is  $1 \text{ Mgal/d}$  ( $4,000 \text{ m}^3/\text{d}$ ), an additional  $2.6 \text{ Mgal/d}$  ( $10,000 \text{ m}^3/\text{d}$ ) is available for development. Throughout the remainder of the year, the potential for  $3.6 \text{ Mgal/d}$  ( $14,000 \text{ m}^3/\text{d}$ ) additional development exists.

The simulation of conditions during a very dry summer is probably more significant in evaluating the ground-water potential of the Little Creek area. Table 2 shows projected drawdowns after 90 days to continuous pumping with no recharge. A comparison of Tables 1 and 2, indicates that there is little difference between projected drawdowns after 90 days pumping with no recharge, and drawdowns at the end of the long-term steady-state simulation with 14 inches (356 mm) of recharge annually.

Because of the proximity of the Little Creek area to Delaware Bay and its bordering tidal marshes, the possibility of salt-water encroachment into the unconfined aquifer must

TABLE 2. Projected water-level declines in Little Creek and Lewes-Rehoboth Beach areas after 90 days continuous pumping with no recharge. (All wells have 12-in diameters. MSL - mean sea level).

Well No. (Figure 14)	Continuous pumping rate, in gallons per minute	Water level in pumping well		Water level 1000 ft from pumping well	
		Drawdown in feet	Pumping level below MSL, in feet	Drawdown, in feet	Head with reference to MSL, in feet
Little Creek area (3.6 Mgal/d)					
1	400	22	-11	2.8	+7.7
2	700	23	-13	3.6	+5.0
3	700	32	-30	1.8	+0.2
4	700	35	-29	3.2	+1.7
Lewes-Rehoboth Beach area (7.8 Mgal/d)					
14	1,350	29	-19	5.1	+5.2
15	1,350	24	-18	5.1	+1.2
16	1,350	25	-19	4.3	+1.9
17	1,350	28	-26	3.1	-1.4



be considered. Table 2 indicates that all pumping levels will be below sea level, nevertheless, heads in the aquifer are above sea level within 1,000 ft (305 m) of the wells. All the wells shown in Figure 14 are at least 2,600 ft (790 m) from tidal creeks or marshes, so that lateral movement of salty water to the pumping wells will not occur. However, there remains the possibility of upward intrusion of salty water below the pumping wells. The base of fresh water occurs at 500 feet (152 m) below sea level in this area according to Cushing and others (1973, Plate 12). There is little possibility of upward movement of salty water across the thick section of silty confining beds and fresh-water artesian aquifers into the unconfined aquifer.

The Houston area has the potential for supplying moderately large supplies of ground water. In this area, the unconfined aquifer consists of about 90 feet (27 m) of coarse sand with transmissivities ranging up to 22,000 ft<sup>2</sup>/d or 2,000 m<sup>2</sup>/d (Johnston, 1973, p. 21). At present (1975), pumping is very light in this rural area. A long-term water-level record for an observation well indicates that there has been no decline of the water table within the past 20 years. Starting heads for the 30-year pumping simulation are the mean water-table altitudes.

The 30-year aquifer simulation suggests that at least 5.2 Mgal/d (20,000 m<sup>3</sup>/d) can be pumped indefinitely from 3 high-capacity wells spaced as shown in Figure 14. The draw-down will be relatively small both areally and at the well sites, even though each well would be pumping 1,200 gal/min 76 V/s) continuously (Table 1).

With continuous pumpage of 5.2 Mgal/d (20,000 m<sup>3</sup>/d), the model indicates that the average flow of Beaverdam Branch will be reduced by 6 ft<sup>3</sup>/s (0.17 m<sup>3</sup>/s) or 3.9 Mgal/d (15,000 m<sup>3</sup>/d). Upstream from gaging station 01484100 (Figure 6), the stream would receive no ground-water discharge and would, therefore, be dry except for short periods of overland runoff. Immediately downstream from the gage, substantial reductions of base flow would occur along the main stem and two small tributaries. Thus, any plan to withdraw 5.2 Mgal/d (20,000 m<sup>3</sup>/d) continuously from the unconfined aquifer must consider that the stretch of Beaverdam Branch upstream from the gaging station would be dried up and the average flow into Silver Lake at Milford would be reduced by 6 ft<sup>3</sup>/sec (0.17 m<sup>3</sup>/s).

The Cedar Creek area located between the towns of Milford and Milton (Figure 14), has the potential for developing a moderately large ground-water supply. At present (1975)

the area is completely rural and withdrawals of ground water are very small. The unconfined aquifer consists of fine to coarse sand with a saturated thickness exceeding 80 feet (24 m) locally. Little is known of the hydraulic characteristics of the aquifer in this area, however, calibration of the model suggests that the transmissivity is high, about  $15,000 \text{ ft}^2/\text{d}$  ( $1,400 \text{ m}^2/\text{d}$ ). Transient model simulations indicate  $5.2 \text{ Mgal/d}$  ( $20,000 \text{ m}^3/\text{d}$ ) can be pumped indefinitely with relatively small area declines in the water table (Table 1).

The average discharge of Cedar Creek would be reduced by about  $7 \text{ ft}^3/\text{s}$  ( $0.2 \text{ m}^3/\text{s}$ ) with pumpage from 3 wells spaced as shown in Figure 14. Upstream from the measuring site shown on Figure 5, the ground-water discharge would be reduced by  $4 \text{ ft}^3/\text{s}$  ( $0.11 \text{ m}^3/\text{s}$ ) which is equivalent to about one-third of the average winter base flow. During periods of low base flow, particularly during summer, Cedar Creek would be expected to go dry under continuous withdrawal of  $5.2 \text{ Mgal/d}$  ( $20,000 \text{ m}^3/\text{d}$ ).

The area southeast of Milton, along Beaverdam Creek (Figure 14), is rural with very little ground-water pumpage. Because of its location several miles inland from Lewes and Rehoboth Beach, it represents an alternative source of water for the developing seashore resort area. The saturated thickness of the unconfined aquifer ranges from 75 feet (23 m) to at least 110 ft (34 m). The transmissivity, as determined by a 4-day aquifer test at the site of well 12 (Figure 14), is  $14,000 \text{ ft}^2/\text{d}$  ( $1,300 \text{ m}^2/\text{d}$ ).

A noteworthy feature of the Milton area is that Beaverdam Creek has the highest average base flow in Delaware - about  $1.65 \text{ (ft}^3/\text{s)/mi}^2$  ( $0.018 \text{ (m}^3/\text{s)/km}^2$ ). In addition, there is little difference between the mean summer and mean winter base flow ( $11.8 \text{ ft}^3/\text{s}$  or  $0.33 \text{ m}^3/\text{s}$  versus  $12.1 \text{ ft}^3/\text{s}$  or  $0.34 \text{ m}^3/\text{s}$ ). These values represent about 90 percent of the total streamflow (during 1968-70) and are considered to be accurate because of the excellent records at the gaging station and the relative ease of separating streamflow hydrographs into the large ground-water runoff and small overland runoff components. The very high average base flow suggests either: (1) a very high recharge rate; about 22 inches per year ( $560 \text{ mm/year}$ ) or (2) significant upward leakage from the underlying Manokin artesian aquifer.

The calibrated steady-state model was able to closely reproduce the measured water-table altitudes in Beaverdam Creek basin using the transmissivity determined from the aquifer test and the average areal recharge rate (14 inches

per year or 356 mm/year). However, the model-computed value of ground-water discharge ( $9 \text{ ft}^3/\text{s}$  or  $0.25 \text{ m}^3/\text{s}$ ) is less than the field base flow value ( $12 \text{ ft}^3/\text{s}$  or  $0.34 \text{ m}^3/\text{s}$ ). Upward leakage from the Manokin aquifer (not considered by the model) rather than a high recharge rate probably accounts for the very high base flow of Beaverdam Creek. In view of the fact that water-table contours were matched but the model value of discharge is lower than the field value, the value of transmissivity used in the model may be low.

Inasmuch as the model does not consider leakage from deeper aquifers and the transmissivity of the modeled unconfined aquifer may be low, the model cannot be used to predict water-level declines and streamflow depletion accurately in Beaverdam basin. Note that increased pumpage from the unconfined aquifer would lower the water-table altitudes and thereby increase the rate of upward flow from the Manokin aquifer. Furthermore, head declines will be less than those computed by the digital model of the unconfined aquifer because  $T$  may be higher and because the model neglects the effect of upward flow. Thus the projected water-level declines based on a withdrawal of  $5.2 \text{ Mgal/d}$  ( $20,000 \text{ m}^3/\text{d}$ ) given in Table 1 are somewhat greater than would actually occur with this rate of withdrawal. In summary, Beaverdam Creek basin has the potential for the development of ground-water supplies in excess of  $5.2 \text{ Mgal/d}$  ( $20,000 \text{ m}^3/\text{d}$ ). However, the effects of this withdrawal on water levels and streamflow cannot be accurately predicted because the assumption of two-dimensional flow used in the model is clearly invalid for this basin.

The area south of Lewes (Figure 14) has the potential to supply considerably more ground-water than is currently withdrawn. At the Lewes municipal well field (well 15 in Figure 14), the aquifer consists of about 140 feet (43 m) of coarse sand with a relatively high transmissivity ( $15,000 \text{ ft}^2/\text{d}$  or  $1,400 \text{ m}^2/\text{d}$ ). Pumpage at Lewes was  $443 \text{ Mgal}$  ( $1,680,000 \text{ m}^3$ ) in 1974, according to the pumpage inventory made by Frederick Robertson of the University of Delaware Water Resources Center (oral commun., October, 1975). Pumpage varies seasonally with the largest withdrawal being in the summer months.

At nearby Rehoboth Beach, pumpage was  $251 \text{ Mgal}$  ( $950,000 \text{ m}^3$ ) in 1974 with most of the withdrawal occurring in the summer months. The transmissivity is considerably lower ( $7,000 \text{ ft}^2/\text{d}$  or  $650 \text{ m}^2/\text{d}$ ) at Rehoboth Beach than at Lewes.

Starting heads specified in the 30-year pumpage simulation are based on water-level measurements made in 1960. Those heads incorporate the effects of pumpage estimated to be about 1.1 Mgal/d (4,200 m<sup>3</sup>/d) in 1960. Therefore the 30-year simulation represents the effects of pumpage increases above the 1960 rates.

Transient model simulation suggests that at least 7.8 Mgal/d (30,000 m<sup>3</sup>/d) could be withdrawn indefinitely from the aquifer using the well spacing shown in Figure 14. Projected water-level declines range from 5 to 10 feet (1.5 to 3 m) at a distance of 1,000 feet (305 m) from the pumping wells (Table 1) when equilibrium conditions are reached.

The simulation of conditions during the dry summer with no recharge is probably more pertinent to an appraisal of the Lewes-Rehoboth area because of the highly seasonal nature of pumpage. Table 2 indicates that the head declines at the 4 pumping wells, after 90 days continuous pumping (no recharge), would be slightly less than the declines after the 30-year steady-state simulation with recharge. More important, heads will still be above sea level relatively close to the pumping wells (except for well 17) at the end of the dry summer simulation.

Salt-water encroachment into the unconfined aquifer has been a problem at Lewes and Rehoboth Beach in the past. The appearance of salty water in former public supply wells at both towns necessitated abandonment of the wells and construction of the present well fields farther inland. At each town, the abandoned wells were located relatively close to salt-water bodies. The Lewes-Rehoboth Canal was the probable source of salty water at Lewes and the ocean was the source at Rehoboth Beach (Rasmussen and others, 1960).

The model study did not include an investigation of the projected changes in the position of the saltwater-freshwater interface which would result from future pumping. However, certain inferences pertinent to the problem can be made using the heads computed by the 30-year (steady-state) simulation and the 90-day (dry summer) simulation. Results of the steady-state simulation indicate that heads would be 1 to 3 feet (0.3 to 0.9 m) below sea level at about 1,000 feet (305 m) from wells 15, 16, and 17 (Table 1). Potential danger for lateral movement of salty water exists at well 17, which is only one-half mile (0.8 km) from the Lewes-Rehoboth Canal. However, wells 15 and 16 are more than 1 mile (1.6 km) from saltwater bodies, and, because computed heads are above sea level at this distance, lateral movement of salty water to these wells is unlikely.

The 90-day simulation of summer conditions (Table 2) indicates that heads would be above sea level 1,000 feet (305 m) from wells 14, 15, and 16 but slightly below sea level at the same distance from well 17. To avoid the possibility of salty water moving to well 17, a conservative approach would be to pump 5.8 Mgal/d (22,000 m<sup>3</sup>/d) using wells 14, 15, and 16. Transient simulation of this reduced withdrawal rate from the Lewes-Rehoboth area indicates that drawdown both at the pumping wells and 1,000 feet (305 m) from the wells would be a few-tenths of a foot less than that shown in Table 2. At the site of well 17, the head would be about 1.5 ft (0.5 m) above sea level.

The possibility of upconing of salty water from deeper aquifers is remote. The base of fresh water is about 500 feet (152 m) below sea level, according to Cushing and others (1973, Plate 12). The fresh-water section below the unconfined aquifer is mostly silt and clay, particularly the lower 150 ft (76 m). Considering the relatively small head declines in the vicinity of the pumping wells, rates of upward movement of salty water across the confining beds would be extremely slow.

The results of transient simulation in the Lewes area are presented only as a rough guide to the area's ground-water potential. The model results suggest that the area west of Lewes (including the present municipal well field) has the potential of yielding at least 5.8 Mgal/d (22,000 m<sup>3</sup>/d) indefinitely, or approximately three times the present combined pumpage rate of Lewes and Rehoboth Beach. The chance of salt-water intrusion at the sites of wells 14, 15, and 16 (Figure 14) is minimal based on the heads computed with both the 30-year simulation and the dry summer (no recharge) simulation. A multi-aquifer digital model study of the Delaware seashore area is in progress at present (1976) and should provide a more quantitative evaluation of this area's ground-water potential.

## SELECTED REFERENCES

- Cushing, E. M., Kantrowitz, I. H., and Taylor, R. K., 1973, Water Resources of the Delmarva Peninsula: U.S. Geol. Survey Prof. Paper 822, 58 p.
- Jacob, C. E., 1943, Correlation of ground-water levels and precipitation in Long Island, New York: Am. Geophys. Union Trans., v. 24, pt. 2, p. 564-573.
- Johnston, R. H., 1973, Hydrology of the Columbia (Pleistocene) deposits of Delaware: Delaware Geol. Survey Bull. 14, 78 p.
- \_\_\_\_\_, 1976, Relation of ground water to surface water in four small basins of the Delaware Coastal Plain: Delaware Geol. Survey Rpt. Inv. No. 24, 56 p.
- Johnston, R. H., and Leahy, P. P., 1977, Combined use of digital aquifer models and field base-flow data to identify recharge-leakage areas of artesian aquifers: U. S. Geol. Survey Jour. of Research, vol. 5 (in press).
- Jordan, R. R., 1962, Stratigraphy of the sedimentary rocks of Delaware: Delaware Geol. Survey Bull. 9, 51 p.
- \_\_\_\_\_, 1964, Columbia (Pleistocene) sediments of Delaware: Delaware Geol. Survey Bull. 12, 69 p.
- Jordan, R. R., and Talley, J. H., 1976, Guidebook: Columbia deposits of Delaware: Delaware Geol. Survey Open File Rpt. 8, 49 p.
- Lohman, S. W., 1972, Ground-water hydraulics: U. S. Geol. Survey Prof. Paper 708, 70 p.
- Mather, J. R., 1969, Factors of the climatic water balance over the Delmarva Peninsula: Univ. of Delaware Water Resources Center Publ., 129 p.
- Miller, J. C., 1971, Ground-water geology of the Delaware Atlantic seashore: Delaware Geol. Survey Rpt. Inv. No. 17, 33 p.
- Owens, J. P., and Denny, C. S., 1974, Provisional stratigraphic sequence in the Maryland-Delaware parts of the lower Delmarva peninsula: Geol. Soc. of America, Abs. with Programs, v. 5, no. 1, p. 61-62.

- Pinder, G. F., 1970, An iterative digital model for aquifer evaluation: U. S. Geol. Survey Open-File Rpt. 44 p.
- Pinder, G. F., and Bredehoeft, J. D., 1968, Application of the digital computer for aquifer evaluation: Water Resources Research, v. 4, no. 5, p. 1069-1093.
- Prickett, R. A., and Lonquist, C. G., 1971, Selected digital computer techniques for groundwater resource evaluation: Illinois Water Survey Bull. 55, 62 p.
- Rasmussen, W. C., and others, 1960, Water resources of Sussex County Delaware: Delaware Geol. Survey Bull. 8, 228 p.
- Rasmussen, W. C., and Slaughter, T. H., 1955, The groundwater resources of Somerset, Wicomico, and Worcester counties: Maryland Geol. Survey Bull. 16, 533 p.
- Remson, Irwin, Hornberger, G. M., and Molz, F. J., 1971, Numerical methods in subsurface hydrology: New York, Wiley-Interscience, 389 p.
- Stallman, R. W., 1962, Numerical analysis, in Ferris, J. G., Knowles, D. B., Brown, R. H., and Stallman, R. W., Theory of aquifer tests: U. S. Geol. Survey Water-Supply Paper 1536-E, p. 135-139.
- Sundstrom, R. W., and Pickett, T. E., 1968, The availability of ground water in Kent County, Delaware, with special reference to the Dover area: Univ. of Delaware Water Resources Center Rpt., 123 p.
- \_\_\_\_\_, 1969, The availability of ground water in eastern Sussex County, Delaware: Univ. of Delaware Water Resources Center Rpt., 136 p.
- Trescott, P. C., 1973, Iterative digital model for aquifer evaluations: U. S. Geol. Survey Open-File Rpt., 63 p.
- U. S. Geological Survey, 1964-65, Water-table, surface-drainage and engineering soils maps of quadrangles in central and southern Delaware: U. S. Geol. Survey Hydrol. Inv., Atlas HA-101, HA-102, HA-103, HA-108, HA-109, HA-119, HA-121, HA-133, HA-134, HA-136, HA-137, HA-139, HA-140, HA-141.
- Von Rosenberg, D. V., 1969, Methods for the numerical solution of partial differential equations: New York, American Elsevier Pub. Co., 128 p.

