Modeling of a Hexapod Robot; Kinematic Equivalence to a Unicycle

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Abstract

This report describes the kinematic and dynamic modeling of a hexapod robot. The 6-DOF (degrees of freedom) analytical kinematic and dynamic equations of motion are derived following the classical Newtonian mechanics. Under certain task-specific assumption, it is shown that the complex 6-DOF model can be simplified, resulting in an abstract model. Specifically, the motion of the robot on the horizontal plane in particular is described by the unicycle model with dynamic extension. The abstract unicycle model exhibits restricted behavior compared to the concrete hexapod model, but facilitates motion planning and control design and ensures that higher level control plans are implementable as low level control laws.

1 Introduction

The motion of animals has recently inspired researchers' intuition that legged locomotion has unique advantages in exploration of uneven, and highly environments. In this view, legged machines that capture some measure of animal mobility afford the best hope against inhospitable terrain conditions [1]. Thus, numerous biologically inspired legged robots, primarily 2-legged (androids), 4-legged and 6-legged robots have been developed.

A hexapod (6-legged), compliant robot, named RHex, which resembles the motion pattern of the cockroach, is presented in [2]. RHex has the advantage of high mobility on uneven terrain with minimal torque configuration, since only one rotary actuator per leg is required. Several modifications of the first RHex platform have been developed, see [3]. One commercial version of this hexapod platform is RespondBot RDK, available from Sandbox Innovations [4]. The use of a hexapod robot like RespondBot RDK for navigation in unstructured environments offers both the mobility capabilities of this robotic platform, ought to its compliant design, and the simplicity of torque configuration, ought to the minimal actuation design.

In order to design control algorithms for this class of robots, an analytical expression of the full kinematic and dynamic model is typically needed. However, the models obtained are usually too large and complicated, and do not lend themselves easily to analytical control design and stability verification methods. It is thus desirable for one to be able to preform the control design using simplified models of the system, which nonetheless capture the behaviors of interest, and be able to implement the designs on the actual system. This report aims at mapping a hexapod dynamic model of medium complexity to the kinematic equations of a unicycle. The development of a high fidelity dynamical model, suitable for accurate simulation and visualization, requires detailed modeling of the leg structure, identification of the mechanical parameters, consideration of possible gaits, and falls beyond the scope of this report.

2 Mathematical Modeling

In this section, the kinematic and dynamic equations of motion for the RespondBot RDK are derived. The robot is modeled as a rigid body with 6 compliant legs, following the design presented in [5].

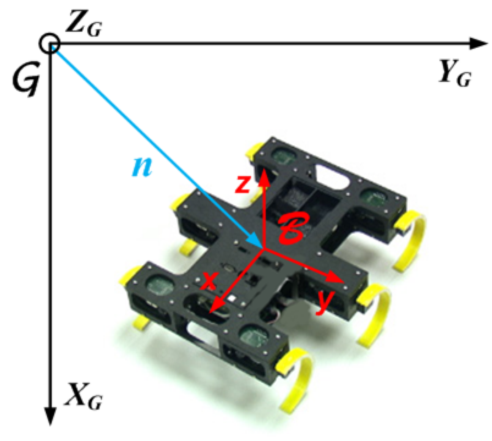

Figure 1: Global G and Body-fixed B Coordinate Frames (Reproduced from [4])

2.1 Kinematics

The kinematic equations of motion of the robot are developed using a global coordinate frame G and a body-fixed coordinate frame B, as depicted in Figure 1. The position and orientation vector of the robot with respect to the global frame G is defined as

$$\boldsymbol{n} = \begin{bmatrix} x & y & z & \rho & \pi & \gamma \end{bmatrix}^{\mathsf{T}}, \tag{1}$$

where x, y, z are the coordinates of the robot position and ρ, π, γ are the Euler angles (roll, pitch, yaw). The velocity vector is defined in the body-fixed frame B as

$$\boldsymbol{v} = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^{\mathsf{T}}, \tag{2}$$

where u, v, w are the linear and p, q, r are the angular velocities of the robot with respect to the body frame axes. Following [6] the kinematic equations are given in vectorial form as

$$\dot{\boldsymbol{n}} = \boldsymbol{J}(\boldsymbol{\Theta})\boldsymbol{v}, \tag{3}$$

where $\boldsymbol{\Theta} = \begin{bmatrix} \rho & \pi & \gamma \end{bmatrix}^{\mathsf{T}}$ and the transformation matrix $\boldsymbol{J}(\boldsymbol{\Theta}) \in \mathbb{R}^{6\times 6}$ is

$$\boldsymbol{J}(\boldsymbol{\Theta}) = \begin{bmatrix} \boldsymbol{R}_B^G(\boldsymbol{\Theta}) & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{T}(\boldsymbol{\Theta}) \end{bmatrix}, \tag{4}$$

with $\boldsymbol{R}_B^G(\boldsymbol{\Theta})$ and $\boldsymbol{T}(\boldsymbol{\Theta})$ the linear and the angular velocity transformation matrices from G to B frame, respectively

$$\boldsymbol{R}_B^G(\boldsymbol{\Theta}) = \begin{bmatrix} \cos\gamma\cos\pi & -\sin\gamma\cos\rho + \cos\gamma\sin\pi\sin\rho & \sin\gamma\sin\rho + \cos\gamma\cos\rho\sin\pi \\ \sin\gamma\cos\pi & \cos\gamma\cos\rho + \sin\gamma\sin\pi\sin\rho & -\cos\gamma\sin\rho + \sin\gamma\cos\rho\sin\pi \\ -\sin\pi & \cos\pi\ \sin\rho & \cos\pi\cos\rho \end{bmatrix}$$

$$\boldsymbol{T}(\boldsymbol{\Theta}) = \begin{bmatrix} 1 & \sin\rho\tan\pi & \cos\rho\tan\pi \\ 0 & \cos\rho & -\sin\rho \\ 0 & \sin\rho/\cos\pi & \cos\rho/\cos\pi \end{bmatrix}.$$

The following section presents a preliminary approach to modeling of the hexapod's legs, for the purpose of gaining insight into the type of motion they induce, rather than the construction of a detailed leg model.

2.2 Leg Modeling

In order to derive the analytical dynamic equations of motion for the RespondBot RDK, the model of the forces resulting from the contact of each leg with the ground should be initially considered.

RespondBot RDK is a 6-legged robot, with C-shaped legs (see Figure 2). Its mechanical design is based on the RHex hexapod robot [5]. RHex-like robots, as well as the RespondBot RDK, are able to walk, run and turn in place, using appropriate gaits for each operation. In order to move forward, RespondBot uses the alternating tripod gait, which is a biologically common walking pattern; the robot lifts three legs (one middle leg on one side and two non-adjacent legs on the other side of the body) at a time, whereas the other three legs remain on the ground and keep the its body statically stable.

Following [5], each leg i is described using a spherical coordinate frame $\left[l_i, \phi_i, \theta_i\right]^\mathsf{T}$ with origin at $\boldsymbol{\alpha_i}$. Each leg is actuated only along the revolute ϕ_i degree of freedom (DOF) and is compliant along the unactuated l_i and θ_i DOF. Compliance along l_i and θ_i is enhanced by the mechanical design and the C-shape of the leg. Thus, each leg is considered as a massless linear spring of length l_{0i} at rest and stiffness k_i.

The attachments of the legs and the joint orientations are all fixed with respect to the robot's body. The geometry of the robot implies that the legs lie on the $(x-z)^B$ plane when the robot is not moving, i.e. $\theta_i = 0$. Moreover, despite the compliance of the unactuated θ_i DOF, it can be assumed that the angle θ_i always remains very close to zero during the motion of the robot, $\theta_i \approx 0$.

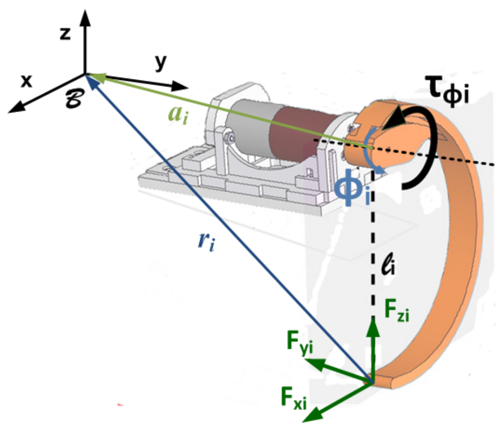

Figure 2: Modeling of Leg Forces (Reproduced from [7] with permission)

When a leg i is in contact with the ground, it experiences the ground reaction forces F_{xi}, F_{yi}, F_{zi}, expressed in the B frame, as shown in Figure 2. These forces result from the actuation force $F_{Ai} = \frac{\tau_{\phi i}}{l_i}$ that is exerted to the ground by the leg i, where $\tau_{\phi i}$ is the motor torque along ϕ_i DOF and l_i is the length of the spring, see also Figure 3 (left). Under the assumption that $\theta_i \approx 0$, i.e., that the actuation force F_{Ai} always lies on the $(x-z)^B$ plane, one can conclude that the lateral force F_{yi} can be considered as negligible, $F_{yi} \approx 0$. Thus, the leg i experiences the ground force F_{Gi}, (Figure 3) where $\|F_{Gi}\|$=$\|F_{Ai}\|$.

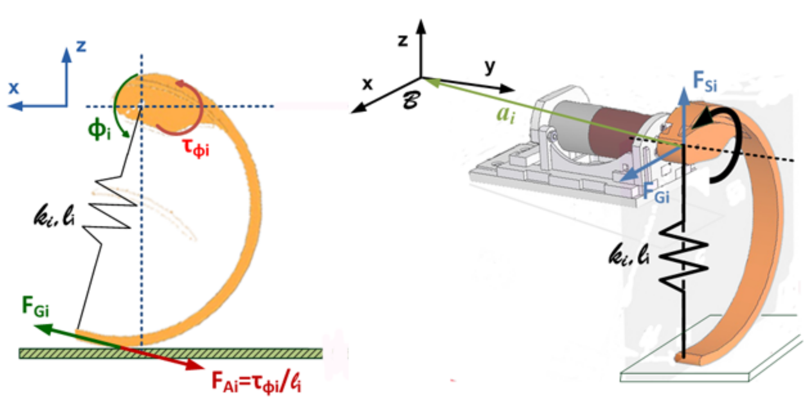

Figure 3: Leg Forces on the $(x-z)^B$ plane (Reproduced from [7] with permission)

Consequently, the forces exerted on the robot's body are

- ground forces F_{Gi} and spring forces F_{Si} by each leg i that is on the ground, (Figure 3), and
- gravitational forces and moments.

The spring force F_{Si} and the ground force F_{Gi} are expressed in the B frame using the following coordinate transformation

$$\boldsymbol{F}_i^B = \begin{bmatrix} F_{xi} \\ F_{yi} \\ F_{zi} \end{bmatrix} = \begin{bmatrix} -\sin\phi_i & -\cos\phi_i \\ 0 & 0 \\ -\cos\phi_i & \sin\phi_i \end{bmatrix} \begin{bmatrix} F_{Si} \\ F_{Gi} \end{bmatrix}, \tag{5}$$

where $F_{Si} = -k_i(l_i - l_{0i})$ and $F_{Gi} = \tau_{\phi i}/l_i$. Thus, the vector $\boldsymbol{F}^B$ of leg (actuation) forces F_i^B that apply to the robot's body, expressed in the B frame is given as

$$\boldsymbol{F}^B = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}^B = \sum_{i=1}^{6} \mathrm{leg}_i \boldsymbol{F}_i^B, \tag{6}$$

whereas the vector of the resulting actuation moments with respect to the center of gravity CG of the body is

$$\boldsymbol{\tau}^B = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}^B = \sum_{i=1}^{6} \mathrm{leg}_i \boldsymbol{\tau}_i^B, \tag{7}$$

in which

$$\mathrm{leg}_i = \begin{cases} 1 & \text{leg } i \text{ is in stance,} \\ 0 & \text{leg } i \text{ is in flight,} \end{cases} \tag{8}$$

and $\boldsymbol{\tau}_i^B = \boldsymbol{\alpha}_i \times \boldsymbol{F}_i^B$ (Figure 3). Recall that

$$\boldsymbol{\lambda} \times \boldsymbol{F} = \boldsymbol{S}(\boldsymbol{\lambda})\boldsymbol{F} = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix} \boldsymbol{F} \ ,$$

where $\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}^\mathsf{T}$.

Thus, in order to determine the actuation moments $\boldsymbol{\tau}_i^B$, one needs to consider the geometry of the robot and the allocation of actuation forces $\boldsymbol{F}_i^B$ of each leg i (Figure 4). Given $\alpha, \beta, \gamma > 0$ the vectors $\boldsymbol{\alpha}_i$, $i = 1 \ldots 6$ are

$$\begin{aligned} &\boldsymbol{\alpha}_1 = \begin{bmatrix} \alpha & \beta & 0 \end{bmatrix}^\mathsf{T}, && \boldsymbol{\alpha}_2 = \begin{bmatrix} \alpha & -\beta & 0 \end{bmatrix}^\mathsf{T}, && \boldsymbol{\alpha}_3 = \begin{bmatrix} 0 & \gamma & 0 \end{bmatrix}^\mathsf{T}, \\ &\boldsymbol{\alpha}_4 = \begin{bmatrix} 0 & -\gamma & 0 \end{bmatrix}^\mathsf{T}, && \boldsymbol{\alpha}_5 = \begin{bmatrix} -\alpha & \beta & 0 \end{bmatrix}^\mathsf{T}, && \boldsymbol{\alpha}_6 = \begin{bmatrix} -\alpha & -\beta & 0 \end{bmatrix}^\mathsf{T}, \end{aligned}$$

and the actuation moments $\boldsymbol{\tau}_i^B$ are

$$\begin{aligned} &\boldsymbol{\tau}_1^B = \begin{bmatrix} 0 & 0 & \beta \\ 0 & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} F_{x1} \\ 0 \\ F_{z1} \end{bmatrix}, && \boldsymbol{\tau}_2^B = \begin{bmatrix} 0 & 0 & -\beta \\ 0 & 0 & -\alpha \\ \beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} F_{x2} \\ 0 \\ F_{z2} \end{bmatrix}, && \boldsymbol{\tau}_3^B = \begin{bmatrix} 0 & 0 & \gamma \\ 0 & 0 & 0 \\ -\gamma & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{x3} \\ 0 \\ F_{z3} \end{bmatrix}, \\ &\boldsymbol{\tau}_4^B = \begin{bmatrix} 0 & 0 & -\gamma \\ 0 & 0 & 0 \\ \gamma & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{x4} \\ 0 \\ F_{z4} \end{bmatrix}, && \boldsymbol{\tau}_5^B = \begin{bmatrix} 0 & 0 & \beta \\ 0 & 0 & \alpha \\ -\beta & -\alpha & 0 \end{bmatrix} \begin{bmatrix} F_{x5} \\ 0 \\ F_{z5} \end{bmatrix}, && \boldsymbol{\tau}_6^B = \begin{bmatrix} 0 & 0 & -\beta \\ 0 & 0 & \alpha \\ \beta & -\alpha & 0 \end{bmatrix} \begin{bmatrix} F_{x6} \\ 0 \\ F_{z6} \end{bmatrix} . \end{aligned}$$

The alternating tripod gait implies that at the same time the robot lifts the legs 2-3-6, while legs 1-4-5 remain on the ground, producing thus the actuation forces and moments, whereas at the next cycle, legs 2-3-6 will be on the ground while 1-4-5 will be in flight, and so forth. According to this, the vector of actuation forces and moments $\boldsymbol{\tau_A}$ is

$$\boldsymbol{\tau_A} = \begin{bmatrix} \boldsymbol{F}^B \\ \boldsymbol{\tau}^B \end{bmatrix} = \begin{cases} \begin{bmatrix} \boldsymbol{F}_1^B + \boldsymbol{F}_4^B + \boldsymbol{F}_5^B \\ \boldsymbol{\tau}_1^B + \boldsymbol{\tau}_4^B + \boldsymbol{\tau}_5^B \end{bmatrix}, & \text{legs 1-4-5 on the ground;} \\ \begin{bmatrix} \boldsymbol{F}_2^B + \boldsymbol{F}_3^B + \boldsymbol{F}_6^B \\ \boldsymbol{\tau}_2^B + \boldsymbol{\tau}_3^B + \boldsymbol{\tau}_6^B \end{bmatrix}, & \text{legs 2-3-6 on the ground.} \end{cases} \tag{9}$$

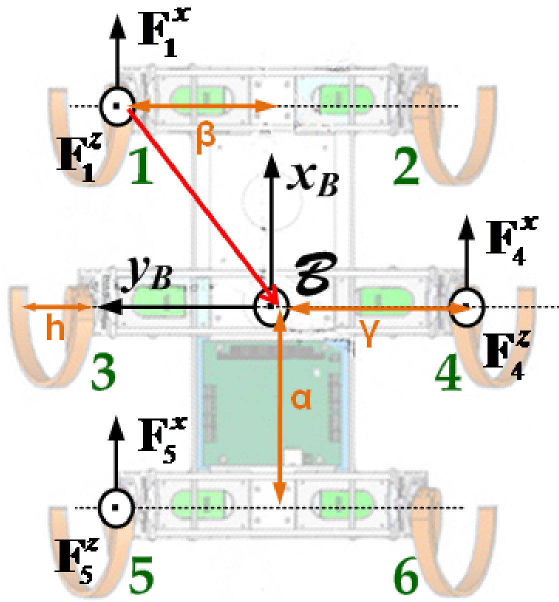

Figure 4: Allocation of Leg Forces (Reproduced from [7] with permission)

2.3 Dynamics

Following [6], the rigid-body dynamics are written in vectorial form as

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_{RB}, \tag{10}$$

where $\boldsymbol{M}_{RB}$ is the inertia matrix of the rigid-body, $\boldsymbol{C}_{RB}$ is the matrix of Coriolis and centripetal forces, $\boldsymbol{\tau}_{RB}$ is the vector of generalized external forces and $\boldsymbol{\nu}$ is the linear and angular velocity vector in the body-fixed frame B.

In order to derive the (simplified) analytical expression of (10), we consider the following assumptions:

1. The origin of B frame is located at the center of gravity CG of the robot, $\boldsymbol{r}_{CG} = \mathbf{0}$.
2. The axes of B frame coincide with the principle axes of inertia of the robot's body.

Under these, the analytical form of the inertia matrix is

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m\boldsymbol{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \boldsymbol{I}_0 \end{bmatrix}, \tag{11}$$

where m is the mass of the robot, $\boldsymbol{I}_{3\times3}$ is the 3×3 identity matrix and $\boldsymbol{I}_0 = \mathrm{diag}(I_x, I_y, I_z)$ is the matrix of moments of inertia.

Moreover, in [6] it is proved that the Coriolis matrix can be represented in the skew-symmetric form

$$\boldsymbol{C}_{RB}(\boldsymbol{\nu}) = \begin{bmatrix} \mathbf{0}_{3\times3} & -m\boldsymbol{S}(\boldsymbol{\nu}_1) \\ -m\boldsymbol{S}(\boldsymbol{\nu}_1) & -\boldsymbol{S}(\boldsymbol{I}_0\boldsymbol{\nu}_2) \end{bmatrix}, \tag{12}$$

where $\boldsymbol{\nu}_1 = \begin{bmatrix} u & v & w \end{bmatrix}^\mathsf{T}$, $\boldsymbol{\nu}_2 = \begin{bmatrix} p & q & r \end{bmatrix}^\mathsf{T}$,

$$-m\boldsymbol{S}(\boldsymbol{\nu}_1) = \begin{bmatrix} 0 & mw & -mv \\ -mw & 0 & mu \\ mv & -mu & 0 \end{bmatrix}, \qquad -\boldsymbol{S}(\boldsymbol{I}_0\boldsymbol{\nu}_2) = \begin{bmatrix} 0 & I_z r & -I_y q \\ -I_z r & 0 & I_x p \\ I_y q & -I_x p & 0 \end{bmatrix}.$$

Finally, the vector of generalized forces and moments is given as

$$\boldsymbol{\tau}_{RB} = \boldsymbol{\tau}_{\boldsymbol{A}} - \boldsymbol{g}(\boldsymbol{\eta}), \tag{13}$$

where $\boldsymbol{\tau_A} = \begin{bmatrix} \boldsymbol{F}^B \\ \boldsymbol{\tau}^B \end{bmatrix}$ is the vector of actuation forces and moments and $\boldsymbol{g}(\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{f}_{\boldsymbol{g}}^B \\ \boldsymbol{r}_{CG} \times \boldsymbol{f}_{\boldsymbol{g}}^B \end{bmatrix}$ is the vector of gravitational forces and moments with $\boldsymbol{f}_{\boldsymbol{g}}^B = \boldsymbol{R}_B^G(\boldsymbol{\Theta})^{-1} \begin{bmatrix} 0 & 0 & -mg \end{bmatrix}^{\mathsf{T}}$.

Consequently, the analytical form of the 6-DOF dynamic equations of motion for the RespondBot RDK are

$$m\dot{u} + mwq - mvr + mg\sin\pi = F_x \tag{14a}$$
$$m\dot{v} - mwp + mur - mg\sin\rho\cos\pi = F_y \tag{14b}$$
$$m\dot{w} + mvp - muq - mg\cos\rho\cos\pi = F_z \tag{14c}$$
$$I_x\dot{p} + I_z rq - I_y qr = \tau_x \tag{14d}$$
$$I_y\dot{q} - I_z rp + I_x pr = \tau_y \tag{14e}$$
$$I_z\dot{r} + I_y qp - I_x pq = \tau_z \tag{14f}$$

The dynamic equations of motion (14) for RespondBot RDK can be further simplified by considering that:

1. The rotational motion along x_B, y_B axes (roll and pitch DOF, respectively) is negligible in practice, since for the walking patterns that the robot uses (e.g. alternating tripod gait with low joint velocities), the angular velocities p, q remain always very close to zero, $p \approx 0$ and $q \approx 0$.
2. The lateral force F_y in the B frame is negligible, as it was justified in 2.2, i.e. $F_y = 0$.
3. For the motion of the robot on the horizontal plane, the angles roll ρ and pitch π remain always very close to zero, i.e. $\rho \approx 0$ and $\pi \approx 0$.

Then, the motion of the robot on the horizontal plane is described by the 3-DOF dynamics

$$m\dot{u} - mvr = F_x \tag{15a}$$
$$m\dot{v} + mur = 0 \tag{15b}$$
$$I_z\dot{r} = \tau_z \tag{15c}$$

whereas the 3-DOF kinematics describing the motion on the horizontal plane are derived from (3) as

$$\dot{x} = u\cos\psi - v\sin\psi \tag{16a}$$
$$\dot{y} = u\sin\psi + v\cos\psi \tag{16b}$$
$$\dot{\psi} = r \tag{16c}$$

Finally, one can take into consideration that the allocation of actuation forces, along with the gaits utilized, does not significantly excite the DOF along y_B axis (lateral motion), resulting thus in the assumption that the linear velocity along y_B axis is zero, $v = 0$. Therefore, the corresponding dynamic equation (14b) can be neglected. Consequently, the motion of the RespondBot RDK on the horizontal plane can be approximated by the unicycle model with dynamic extension

$$\dot{x} = u\cos\psi \tag{17a}$$
$$\dot{y} = u\sin\psi \tag{17b}$$
$$\dot{\psi} = r \tag{17c}$$
$$m\dot{u} = F_x \tag{17d}$$
$$I_z\dot{r} = \tau_z \tag{17e}$$

3 Conclusion

In this report, the kinematic and dynamic equations of motion for a 6-legged robot are derived. It is shown that under certain, task-specific assumptions, the motion of the robot on the horizontal plane can be adequately described by the equations of a unicycle with dynamic extension. This result facilitates the development of navigation and motion planning algorithms for this class of robots, by allowing control design on a higher level based on the simple models while at the same time ensuring that the control plans are implementable at the lower level.

4 Acknowledgements

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