HIERARCHICAL MARKOV CHAIN MONTE CARLO

AND PAVEMENT ROUGHNESS MODEL

by

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ABSTRACT

Traditionally, pavement roughness has been modeled to mimic heterogeneity across pavement sections. Modeling heterogeneity is challenging and can generate models that are unable to reflect true pavement conditions. Heterogeneity is fundamental to modeling pavement roughness and describes how road conditions change continuously with corresponding time change. However, road conditions are unpredictable and this feature raises inherent challenges when modeling heterogeneity across pavement sections. This thesis seeks to model the roughness of road pavements in Kansas using hierarchical Markov Chain Monte Carlo (MCMC) simulation. The aim is to investigate how efficient this technique is at estimating and predicting pavement roughness without neglecting inherent heterogeneity across pavement sections. Hierarchical MCMC models use Bayesian approach in their estimation process which allows them to account for heterogeneity in pavement roughness. Models easily lend themselves to validation and can be examined to see if they reflect roughness conditions on a specified length of roadway or a network of roads. Using individual lengths of pavements and a nineteen year time span, a hierarchical MCMC model is used to predict the IRI value for the twentieth year. Estimated IRI values are then compared with original IRI values to see how well they correlate and if they reflect prevailing road conditions. Once proven to be successful, this technique can be incorporated into pavement management systems and used as a basis for making sound decisions about the level of roughness on a given road network.

Chapter 1

INTRODUCTION

1.1 Background

Road pavements form an integral part of the transportation infrastructure of any nation. As part of a nation's transportation network, they contribute largely to the economic, social and individual well-being of citizens within the nation. Their significance emerges from the essential role they play in the transportation of people, goods and services. Real-life examples include how road transport is vital to the supply and demand logistics for just-in-time operations of the business world; and in transporting prompt health care services to emergency victims of a natural or man-made disaster. In doing this, roads serve as the primary mode of transport or as a secondary mode by linking other primary modes of transport. Roads can be classified into many groups according to use, surface type, pavement structure, location, type of distress etc. The basis of classification aids in identifying pavement families and the characteristics associated with a particular pavement family. Like other civil infrastructure, road pavements are susceptible to deterioration which adversely affects the performance of these pavements.

Deterioration of road pavements is inevitable and occurs as a result of using pavements with time or due to the presence of distresses. Examples of pavement distresses include rutting, bleeding, cracking, roughness, pot-holes etc. When deterioration occurs, the capacity of the roadway to function at its optimal level is reduced tremendously and the transportation of people, goods and services as well as their accompanying benefits to a nation is adversely impacted. To safeguard against the consequences of deterioration, road pavement distresses need to be monitored on a regular basis so as to intervene when necessary to reduce the propagation of these distresses.

This research specifically looks at one type of pavement distress: roughness and its assessment in the context of pavement management performance. Roughness refers to irregularities in the pavement surface that adversely affect ride quality, safety and vehicle dynamics. To assess the state of roughness on their road network, many Departments of Transportation (DOT's) use pavement management systems. Pavement management system is a process that assists in roadway monitoring and decision making. It serves as a means by which the performance of road pavements at both the network and project levels can be effectively monitored, and the appropriate maintenance and rehabilitation techniques tailored to meet their subsequent deterioration. Pavement management systems comprise the following activities: collecting road inventory data of all roads in a network; analyzing the data to assess the condition of road pavements and to identify any trends in pavement condition; and selecting the most economic and feasible alternative that will cater for the needs of the pavement and maximize its capacity to perform efficiently. However, doing this requires the use of pavement performance models that are able to reflect past pavement attributes and predict future pavement conditions.

Pavement performance models can be developed for a family of road pavements at the network or project level. At the network level, performance models are used for condition forecasting, budget planning, inspection scheduling and work planning. On the other hand, performance models at the project level are used to select specific rehabilitation alternatives to meet expected traffic and climatic conditions. This assists in comparing the economics of various maintenance and rehabilitation strategies which are aimed at improving the life and performance of road pavements.

1.2 Statement of Problem

Roughness characteristics of a particular stretch of roadway or network of roads are not stationary, but change incrementally with corresponding change in time. At times these changes can be dramatic as seen when the road pavement undergoes massive rehabilitation or suffers severe damage to its surface as a result of man-made or natural disaster. Heterogeneity is the term used to describe this sequence of changing pavement condition with time, and stimulates the development of models that attempt to characterize the inherent uncertainty associated with the degree of roughness along road pavements. Over the years, pavement performance models have been developed to model roughness characteristics of road pavements. Most of these models are developed using the traditional deterministic or probabilistic approach. Roughness models based on the deterministic approach use empirical, mechanistic or mechanistic-empirical relationships to relate roughness to explanatory variables such as pavement age, surface distresses, and the environment amongst others. Researchers have however questioned the basis on which explanatory variables are selected to be included in the deterministic models (Prozzi and Madanat 2003) whilst other researchers have pointed to the inability of deterministic models to depict uncertainty (Butt et al. 1987). Probabilistic models use principles of probability to model the uncertainty associated with pavement roughness. A commonly used probabilistic model is the Markov process which makes use of a transition probability matrix to determine the probability that a pavement remains in its present condition or moves to another condition. Transition probability matrices can either be constant and termed as homogenous or can be simulated to meet changing conditions in which case they are described as nonhomogenous. Research has argued that where as homogenous Markov processes do not fit real conditions, nonhomogenous Markov processes lack the support of real data (Butt et al. 1987; Li et al. 1996). Thus a situation arises in which an ideal model is sought to mimic actual pavement conditions whilst predicting future conditions within a particular confidence interval. With regards to all pavement distresses and roughness in particular, this model must have the unique property of depicting roughness characteristics and being mathematically sound at the same time. And as mentioned above, most traditional models achieve one or the other and not both. The task here is to explore the use of hierarchical Markov Chain Monte Carlo simulation to examine how best it models heterogeneity associated with pavement roughness without distorting prevailing conditions on the roadway.

1.3 Objective

This research seeks to model roughness of road pavements using a hierarchical Markov Chain Monte Carlo (MCMC) technique. The aim is to investigate how efficient this technique will be at estimating and predicting pavement roughness. Using individual lengths of pavements and a nineteen year time span, roughness values will be replicated for each year using the model and compared with original values to determine how well they correlate. Based on the results obtained, prediction of roughness values for the twentieth year of each pavement section will then be made. This will give an indication of the present condition and potential future condition of individual roads found in the network.

1.4 Scope of Study

A review of literature of past model forms and experiences at estimating pavement roughness will first be conducted. The merits and potential pitfalls of these methods will serve as a guide in this new estimation process. Exploratory data analysis will be conducted to determine the accuracy, completeness and consistency of data to be used in the estimation process. Statistical tools such as time series and distribution plots will be used in this regard. The hierarchical Markov Chain Monte Carlo (MCMC) technique will then be developed and used in estimating values of pavement roughness for data obtained from Kansas. This technique will also be employed to predict roughness values for the subsequent year that falls immediately outside the dataset. Analysis will then be made using results from the hierarchical MCMC technique.

1.5 Thesis Organization

The thesis has been structured in such a way as to give the reader a clear understanding of the entire modeling process. Chapter 1 introduces the complexities associated with modeling pavement roughness. It illustrates how pavement distresses and roughness in particular affect the performance of a road pavement and presents why modeling pavement roughness has posed as a challenge in the past. In this stems our objective to use the hierarchical MCMC as a means to address this challenge. Chapter 2 presents a review of literature showing past models and recognizing how effectively they modeled roughness characteristics on the roadway. This served as a guide in the current estimation approach. An overview of the structure of Markov Chain Monte Carlo (MCMC) methods and hierarchical models is presented in Chapter 3. This chapter provides equations and diagrams explaining their basic concepts.

Chapter 4 consists of activities conducted during the development of the hierarchical MCMC models. These include data analysis and model formulation. Results obtained from these activities are then presented and discussed in Chapter 5. Chapter 6 outlines the conclusions obtained from the research and puts forth recommendations to guide in future work.

CHAPTER 2

BACKGROUND

2.1 Modeling Pavement Roughness

Pavement performance measures the adequacy of a pavement's functional and structural service over a specified design period. It describes how pavement conditions change or how they serve their intended function with accumulating use (George et al. 1989). Pavement performance can be expressed in terms of distresses such as rutting, cracking and roughness. Of these, pavement roughness is one key indicator which influences the acceptability of service provided by the roadway. The American Society of Testing and Materials defines pavement roughness as the deviation from a true planar surface with characteristic dimensions that affects vehicle dynamics, ride quality, dynamic loads, and drainage. Notably, roughness affects driving comfort, vehicle operating costs and safety. It also increases the dynamic loading imposed by vehicles on the surface, accelerating the deterioration of the pavement structure. Roughness can also have adverse effects on drainage, causing water to pond on the surface, with consequence adverse impacts on both the performance of the pavement and vehicle safety.

The International Roughness Index (IRI) is a standard measurement scale that is accepted widely in evaluating pavement roughness (Smith and Tighe 2004). Fundamentally, IRI values form the basis on which models are developed to reflect and predict the roughness characteristics of a given road pavement. As such these values must be analyzed in such a way to make inferences formed from them as statistically sound as possible without neglecting heterogeneity across pavement sections. Due to the importance of roughness in the performance of road pavements, sufficient effort has been put into modeling roughness so as to characterize and predict its occurrence as accurately as possible. Models for predicting roughness are pivotal in managing road network, pavement design, road pricing and regulation (Paterson 1989; Prozzi and Madanat 2003).

Generally, pavement performance models, which can be extended to include models for roughness, can be classified into two groups: deterministic and probabilistic models (George 1989; Ningyuan et al. 1997). Traditionally, roughness models have been developed based on these two groups. Other ways of modeling roughness have been done using functional network, multivariate adaptive regression splines (MARS), adaptive neural networks and artificial neural networks. All these were developed with the aim of modeling roughness as accurately as possible and by highlighting the characteristics of these models, this review will serve as a tool for knowing how far this field of research has traveled and the gains that have been made thus far.

2.1.1 Modeling Roughness Using Deterministic Models

Deterministic models can either be empirical, mechanistic or mechanistic-empirical (George 1989; Ningyuan et al. 1997). In these, statistical methods such as regression analysis and correlations are used to relate roughness to pavement age, environment, pavement structural strength and traffic loading. Multiple regression techniques have traditionally been used to predict pavement performance ever since the first conceptual pavement performance prediction model was developed by Carey and Irick 40 years ago (Carey and Irick 1960). Straight-line extrapolation and fitting polynomials to performance data are also deterministic techniques. In regression analysis, IRI is the dependent variable and is related to one or more explanatory or independent variables statistically with a set of mathematical equations (Prozzi and Madanat 2003). Several examples of deterministic roughness models have been documented in literature. Paterson in 1989 developed a model that combined structural effects, surface defects and environmental-age-condition influences to predict roughness progression (Paterson 1989). Prior to his research, traditional methods had predominantly modeled roughness as an independent mode of distress dependent entirely on traffic loading and pavement strength or age. Paterson however saw the need to include in his model the mechanistic association between roughness and other distress modes such as potholing and rutting that contribute considerably to roughness. Down the timeline, Saleh et al. developed a mechanistic roughness model to model the relation between roughness and axle load, number of load repetitions and asphalt layer thickness. The authors discovered the scale of dynamic vehicle loads played a role in the amount of roughness developed on a pavement surface (Saleh, Mamlouk and Owusu-Antwi 2000). An exponential regression equation was also developed to demonstrate how the progression of overlay roughness in Canada increased steadily overtime (Smith and Tighe 2004). In this study, it was also shown that though overlay thickness and climatic region have a significant impact on the progression of pavement roughness subgrade thickness had little influence. Statistically significant relationships have also been established between IRI and HMA thickness, base type and thickness and drainage characteristics of flexible pavements constructed in different site conditions (Haider and Chatti 2009). Previous research work have also used an empirical sigmoid curve to fit the pavement deterioration process (Garcia and Riggins

1984), whilst others have used a mechanistic approach to develop damage functions for rutting and fatigue cracking (Rauhut et al. 1983).

In as much as deterministic models are used extensively in predicting pavement roughness, concerns have been raised about their application (Prozzi and Madanat 2003). These relate to the practice in which for most empirical work the selection of explanatory variables on the basis of their statistical significance meant that relevant variables were often left out of the models whilst irrelevant variables could often time be incorporated into the model. The authors further stated that mechanistic models were usually developed under restricted conditions and lacked validation under a wide range of traffic and environmental conditions. For mechanistic-empirical models, they expressed caution about the reliability of making extrapolations out of the original range of data for which the models were calibrated. Prior research have also highlighted the deficiency of straight-line models to explain variability among data points as well as errors inherent in using polynomials fit on data to make extrapolations. In general, it cannot be guaranteed deterministic models will portray uncertainty associated with pavement roughness (Butt et al. 1987).

2.1.2 Probabilistic Models for Pavement Roughness

Probabilistic models include Markov processes and survivor curves (George et al. 1989). Survivor curves describe pavement deterioration in the form of a cumulative distribution which can then be employed to develop a transition probability matrix (TPM). Markov models are stochastic models that have successfully been used in modeling pavement performance (Way et al. 1982; Butt et al. 1987; Li et al. 1996; Abaza and Ashur 1999; Hong and Wang 2003; Abaza et al. 2004). Markov models make use of a transition-probability matrix (TPM) which specifies the probabilities that a pavement remains in its current condition state or changes to another one in the future. Various methods can be used to estimate transition probabilities based on historical data or engineering judgments (Jiang et al. 1988; Abaza et al. 2004). One major advantage of using the Markov model is that it has the capacity to integrate pavement deterioration rates and M&R improvement variables into a single entity which is the transition matrix. Another advantage is the ease with which the Markov process can be combined with dynamic programming to produce optimal solutions for any maintenance problem (Butt et al. 1987). Researchers have used both homogenous and nonhomogenous Markov chains in modeling pavement performance (Way et al. 1982; Butt et al. 1987; Li et al. 1996; Abaza and Ashur 1999; Hong and Wang 2003; Abaza et al. 2004). Homogeneous Markov processes may not fit real conditions since they assume the probability matrix remains constant (Butt et al. 1987). Elsewhere, attempts have been made using reliability analysis and Monte Carlo simulation to build TPM's for nonhomogeneous Markov processes, but the process lacks the support of real data and only traffic is considered for nonhomogeneity (Li et al. 1996).

Another form of probabilistic modeling adopts the Bayesian approach in model development. Hong and Prozzi used this approach in developing an incremental pavement deterioration model based on the AASHO road test data (Hong and Prozzi 2006). The Bayesian approach incorporates existing knowledge into the modeling process so that previous experience can be utilized rather than ignored (Zellner 1971). Obtaining the probabilistic distribution of the parameters to reflect performance heterogeneity is straightforward, and the resulting output is a density function, which can provide comprehensive statistics of the individual parameters (Hong and Prozzi 2006). The Bayesian approach involves a process which assesses the effects of unobserved heterogeneity on pavement performance model parameters. In addition factors such as structural properties, environmental effects and traffic loading that affect pavement performance are easily incorporated into the modeling process. By using this approach, the probabilistic parameter distributions are obtained through a combination of existing knowledge (prior) and information from the data collected. The Markov chain Monte Carlo simulation is used for estimating parameter distributions in the Bayesian approach.

2.1.3 Non-traditional Models for Predicting Roughness

Neural networks are relatively new techniques that have been developed to model pavement roughness (Roberts and Attoh-Okine 1998; Chou and Pellinen 2005). The neural network is a memory-based technology that can accumulate past experiences through the process of training to make human-like decisions and judgments (Chou and Pellinen 2005). Neural networks have the unique attribute of using developing non-linear functions without being constrained by the principle of linearity which strictly governs regression analyses (Roberts and Attoh-Okine 1998). Back-propagation neural networks have also been used in roughness prediction modeling (Fwa and Chen 1993; Attoh-Okine 1994, 1995a). Like neural networks, their advantage over traditional methods lies in their ability to generalize and offer real-time solutions to complex-pavement-performance-prediction problems. Another model developed to predict pavement roughness used adaptive neural network (Attoh-Okine 1995b). Adaptive neural networks help identify variables that determine pavement performance. In their study, Roberts and Attoh-Okine (Roberts and Attoh-Okine 1998) discovered that adaptive neural networks appeared to out-perform back-propagation networks. The multivariate adaptive regression splines (MARS) has also been used in the past to predict roughness of flexible pavements (Attoh-Okine et al. 2003). Their strength lies in their ability to reduce redundancy in the model by focusing on the relevant variables and eliminating inconsistencies in the roughness data. Work has also been conducted into using functional networks in roughness prediction (Attoh-Okine 2005). Their main advantage is in combining domain and data knowledge to develop prediction models. Where as neural networks have the ability to learn from data, functional networks possess the quality of reproducing physical properties as well as learning from data and domain knowledge (Castillo and Guitierrez 1998).

CHAPTER 3

MARKOV CHAIN MONTE CARLO (MCMC) METHODS AND HIERARCHICAL MODELS

3.1 Bayesian Inference

Markov Chain Monte Carlo Methods use Bayesian inference in estimation. The Bayesian inference is described as follows: Assuming y is a vector or matrix of data and θ is a vector or matrix that contains parameters that describe y , then from Bayes' theorem,

$$f(\theta|y) = \frac{f(y|\theta) f(\theta)}{f(y)} \alpha \ f(y|\theta) f(\theta) \tag{1}$$

Where $f(\theta|y)$ is the posterior distribution, $f(y|\theta)$ is the likelihood and $f(\theta)$ is the prior distribution. In Bayesian inference, θ is considered to be a quantity whose variation can be described by its probability distribution $f(\theta)$. $f(\theta)$ is a subjective description based on the experimenter's belief and is formulated before seeing the data. Specification of the prior distribution is important in Bayesian inference since it influences the posterior distribution. The prior mean provides a prior point estimate for the parameter of interest while the variance expresses uncertainty concerning this estimate. The likelihood is the distribution of the data conditional on the parameters. It is the data generating process. Of these, the posterior distribution is of fundamental interest. It summarizes all knowledge about θ after seeing the data. The main distinction between Bayesian inference and the classical approach lies with the parameter θ . Whilst the classical approach sees θ as a fixed but unknown quantity, Bayesian inference considers θ to be a random variable.

To make computation easier, natural conjugate priors are commonly used. A natural conjugate prior distribution is one which, when combined with the likelihood, yields a posterior that falls in the same class of distributions. In addition a natural conjugate prior has the same functional form as the likelihood function. In this way, the prior information can be interpreted the same way as the likelihood function information.

In some cases, no reliable prior information about θ may exist. In this situation a non-informative prior distribution could be used that contained "no information" about θ, in the sense that it did not favor one θ value over another. When this occurs, inferences made from the posterior distribution are regarded as objective rather than subjective.

3.2 Markov Chain Monte Carlo (MCMC) methods

Markov Chain Monte Carlo methods are simulation techniques through which posterior distributions can be obtained accurately by specifying the prior and likelihood distributions. They are flexible, general and most importantly tractable in comparison to direct simulation methods. The MCMC is an iterative process that is based on the construction of a Markov chain which eventually "converges" to a stationary, posterior distribution. Unlike direct simulation methods, the MCMC output is a dependent sample generated from a Markov chain.

A Markov chain is a stochastic process $\{\theta^{(1)}, \theta^{(2)}, \ldots\ldots, \theta^{(T)}\}$ such that

$$f\left(\theta^{(t+1)} \middle| \theta^{(t)}, \ldots\ldots \theta^{(1)}\right) = f\left(\theta^{(t+1)} \middle| \theta^{(t)}\right) \qquad (2)$$

that is, the distribution of θ at sequence $t+1$ given all the preceding θ values (for times $t, t-1, ...1$) depends only on the value $\theta^{(t)}$ on the previous sequence t.

The main idea behind an MCMC method is to simulate realizations from a Markov Chain which has a stationary distribution say $f(U)$. Given a vector random variable $U = (U_1, ...U_k)$ with joint distribution $f(U_1, ...U_k)$, then the expected value of some intractable function $h(U)$ can be approximated by obtaining independent random draws $U^{(t)}$, $t = 1, ..., n$, from the distribution $f(U)$. The desired expectation can be approximated using:

$$E[h(U)] \approx \frac{1}{n} \sum_{t=1}^{n} h(U^{(t)}) \text{ as } n \to \infty \tag{3}$$

The two most popular MCMC methods used for simulating realizations from the stationary distribution are Gibbs sampling and the Metropolis-Hastings algorithm.

3.2.1 Gibbs Sampling

Gibbs sampling is a Markovian updating scheme (Gelfand and Smith 1990). It is a technique by which random variables can be generated indirectly from a marginal distribution without having to calculate densities. Basically, sampling for each variable is done from a conditional distribution where all other variables are considered known and are given the values of the previous state of the chain. That is conditional distributions have a known form and random numbers can easily be simulated using standard functions in statistical and computing software. This makes Gibbs sampling a popular and convenient method by which the posterior distribution can be simulated from a given Markov chain. The process of the algorithm used in the Gibbs sampling is described as:

For a set of random variables $U_1, U_{2.}, ..., U_m$, the joint distribution is denoted as $f(U_1, U_{2.}, ..., U_m)$. With given arbitrary starting values of U_s's, say $U_1^{(0)}, U_{2.}^{(0w)}, ..., U_m^{(0)}$, the first iteration of random draws of U_s 's is obtained as

$$U_1^{(1)} \text{ from } f(U_1 | U_2^{(0)}, U_{3.}^{(0)}, ..., U_m^{(0)})$$

$$U_2^{(1)} \text{ from } f(U_2 | U_1^{(1)}, U_{3.}^{(0)}, ..., U_m^{(0)})$$

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$$U_m^{(1)} \text{ from } f(U_m | U_1^{(1)}, U_{2.}^{(0)}, ..., U_{m-1}^{(1)})$$

In a similar manner, the second set of random draws of U_s 's is obtained through the update process. After r iterations as shown above, the series of U_s 's is obtained as $(U_1^{(r)}, U_{2.}^{(r)}, ..., U_k^{(r)})$. It is shown that under mild conditions for each variable $U_s^{(r)} \to U_s \sim f(U_s)$ as $r \to \infty$ (Geman and Geman, 1984), which means that after enough iterations, r , $U_s^{(r)}$ can be regarded as a random draw from the distribution of $f(U_s)$.

3.2.2 Metropolis-Hastings algorithm

Unlike Gibbs sampling, conditional distributions for the Metropolis-Hastings algorithm have unknown forms. For each variable, a new value is generated from a proposed distribution which is then compared with the old value. The new value is accepted with a probability so that the draws are simulating from the posterior distribution. The variable retains its old value once a value is rejected. Complexities arise in the choice of the proposed distribution. Poor choices delay the convergence of the Markov chain towards its stationary distribution.

Due to its convenience as a simulation technique, this research employs Gibbs sampling for obtaining parameters from the posterior distribution.

3.3 Hierarchical Models

Hierarchical models refer more to a general set of modeling principles than to a specific family of models. They are also known as multilevel, repeated measures, mixed or longitudinal models (Ntzoufras 2009). Hierarchical modeling involves organizing models using a set of sequential statements of conditional relationships. Bayesian models portray an attribute of hierarchy because of the conditional structure of the posterior distribution which can be decomposed to the data likelihood multiplied by the prior distribution.

Hierarchical models are formed when random variables are modeled using a sequence of distributions placed in a hierarchy. Bayesian models have an inherently hierarchical structure that can be represented by a class of models that aid in having a better understanding of the statistical problem. The main concept in the formulation of a hierarchical model is the use parameters and priors to model the posterior distribution in stages. In essence, priors are specified using new parameters not indicated in the likelihood; and these new parameters themselves will require priors that may (or may not) depend on the new parameters. The process terminates when new parameters are no longer introduced into the modeling process.

Given a prior distribution $f(\theta|a)$ of the model parameters θ, prior parameters a can be considered as one level of hierarchy and the likelihood as the final stage of a Bayesian model resulting in the posterior distribution

$$f(\theta|y) \;\alpha\; f(y|\theta)\; f(\theta;a) \tag{4}$$

This is a case of a one level hierarchical model and is shown in Figure 1. (Credit: Ntzoufras 2009)

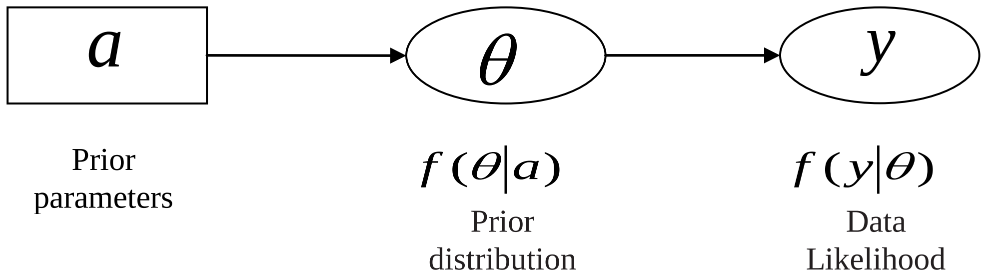


Figure 1 Graphical representation of standard Bayesian model

In other complex cases, a series of conditional distributions called hierarchical stages of the prior distribution can be used to generate the posterior distribution. An example of this is a two level hierarchical model where for parameters θ, a and b, the first level is characterized by $f(\theta|a)$ and the second level is characterized by $f(a|b)$. The distribution $f(a|b)$ is identified as the hyperprior and b are said to be the hyperparameters of the prior parameter a. The posterior distribution $f(\theta|y)$ is represented as

$$f(\theta|y)\ \alpha\ f(y|\theta)\ f(\theta;a)\ f(a;b) \tag{5}$$

$$\alpha\ f(y|\theta)\ f(\theta|a)\ f(a|b)$$

A two-stage hierarchical model is shown in Figure 2. (Credit: Ntzoufras 2009)

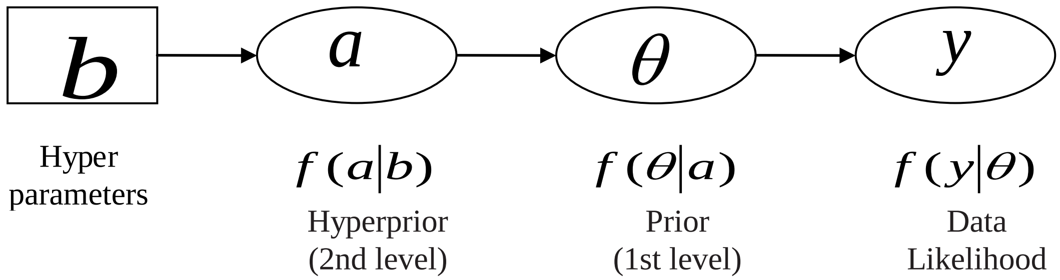


Figure 2 Graphical representation of a 2-stage Bayesian hierarchical model

Figure 3 is a representation of a hierarchical model showing priors and hyperpriors for a given dataset y. In this figure, prior parameters are represented by θ and hyperpriors by Θ.

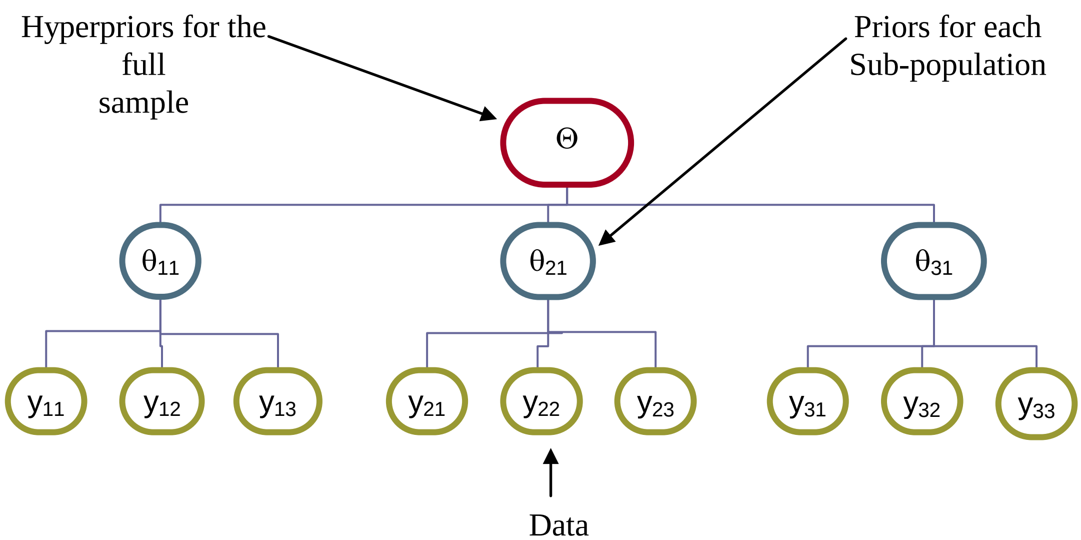
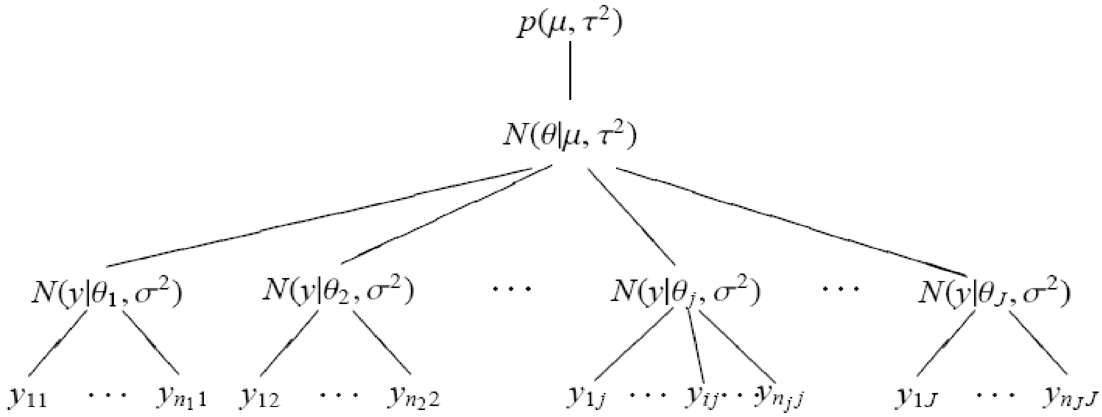


Figure 3 **Graphical illustration of a hierarchical model**

Hierarchical models can be considered as a large set of stochastic formulations that include popular models such as the random effects, the variance components, the multilevel and the generalized linear mixed models (GLMM) (Ntzoufras 2009). Specific examples of Bayesian hierarchical models include binomial-Poisson hierarchical model, gamma-Poisson hierarchical model and the normal hierarchical model. Figure 4 illustrates the structure of a normal hierarchical model.



With $\sigma_j^2 = \sigma^2/n_j$ known, the model is

$$p(y, \theta, \mu, \tau^2|\sigma^2) \propto \underbrace{\prod_j N(y_j|\theta_j, \sigma_j^2)}_{\text{Level 1}} \underbrace{\prod_j N(\theta_j|\mu, \tau^2)}_{\text{Level 2}} \underbrace{p(\mu, \tau^2)}_{\text{Level 3}}$$

where $y_j = \overline{y}_{\cdot j}$.

Figure 4 Hierarchical Normal Model

The significance of hierarchical models in MCMC methods are outlined below:

- Generally, hierarchical models are able to describe complex data sets by incorporating correlation or other important properties into the model. Thus in cases where multivariate or repeated responses are observed, correlation can be incorporated in the model via a common “random” effect for all measurements referring to the same individual. This introduces a marginal correlation between repeated data, whilst interpretation is based on the conditional means.
- Hierarchical models can be used to imply a complicated marginal distribution whilst keeping the conditional structure as simple as possible. In these,

random effects and the corresponding hierarchical structure are used to specify the marginal sampling distribution and this simplifies the MCMC scheme that can be used to estimate the posterior distributions of interest.

- Hierarchical models can be used to combine information between different observations or studies or introduce a clustering effect on observations within the data.
- In addition to the above, the use of hierarchical models is valuable in statistics since they are based on the logic of simple generalized linear models by including random terms in the linear predictor that can be used to introduce dependence or overdispersion or to simply change the marginal distribution of the data, that is the likelihood.

3.4 WinBUGS

WinBUGS is a Bayesian analysis software that uses Markov Chain Monte Carlo (MCMC) to fit statistical data. BUGS is an acronym for Bayesian inference Using Gibbs Sampling. The BUGS project was initiated by the MRC Biostatistics Unit in 1989. Different versions of BUGS software have been developed in the past, however, the first experimental version of BUGS for windows, WinBUGS was presented in 1997. The current version is WinBUGS 1.4.3.

WinBUGS has the ability to generate a random sample from the posterior distribution of the parameters of a Bayesian model. It can be used in statistical problems as simple as estimating means and variances or as complicated as fitting multilevel models, measurement error models and missing data models. It does all these using the Bayesian approach. WinBUGS 1.4.3 was used in this research to develop hierarchical models.

CHAPTER 4

HIERARCHICAL MCMC MODELS FOR PAVEMENT ROUGHNESS

4.1 Introduction

This research seeks to model roughness of road pavements in Kansas using a hierarchical Markov Chain Monte Carlo (MCMC) technique. The aim is to investigate how efficient this technique will be at estimating and predicting pavement roughness. Activities undertaken are data analysis, model development and discussion of results.

4.2 Source of Data

Data used for estimation was obtained from annual roughness of road pavements in Kansas. It spanned a period of 19years, from 1989 to 2007. Roughness values for 30 individual sections were provided. These are shown in Table 1. The first fifteen sections: Sections A to O were used to examine the process through which hierarchical MCMC models were fit to the data.

Table 1 Annual IRI values for selected Kansas pavement sections

ID	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
A	79	66	114	89	92	114	123	101	110	68	69
B	110	79	123	107	115	124	133	112	76	61	72
C	87	69	123	102	95	99	95	101	69	66	81
D	86	73	118	103	91	102	100	83	75	77	83
E	90	71	120	97	95	94	104	64	78	87	78
F	98	90	132	112	120	115	117	82	91	100	100
G	155	82	129	113	121	103	102	106	119	138	110
H	166	119	135	122	119	125	121	123	128	124	131
I	155	112	126	115	121	120	120	116	119	120	123
J	144	106	127	107	108	109	105	104	105	105	111
K	127	97	119	111	100	103	100	104	100	108	109
L	119	84	114	96	88	83	90	85	91	94	97
M	56	60	98	74	72	78	86	82	93	59	52
N	56	49	98	67	67	73	90	78	91	49	53
O	53	47	94	73	62	65	82	76	81	48	53
P	55	50	104	76	77	84	83	85	97	53	57
Q	90	73	116	90	87	93	88	92	115	51	47
R	55	60	104	76	71	79	80	86	100	52	49
S	87	70	107	90	96	149	144	94	113	61	64
T	79	72	110	90	124	167	150	96	94	58	62
U	87	83	109	88	106	192	167	124	131	56	62
V	86	75	113	89	107	166	162	131	134	50	56
W	82	72	108	92	96	187	154	125	124	52	56
X	85	77	108	83	59	89	108	53	61	63	50
Y	83	76	115	87	58	105	101	65	61	63	47
Z	103	84	119	77	53	110	99	54	58	63	48
AA	104	93	122	70	55	111	109	50	61	66	51
AB	127	108	106	71	55	99	107	56	69	72	56
AC	120	102	123	71	74	119	114	52	67	77	58
AD	73	82	112	70	101	110	107	66	70	73	67

Table 1 Continued

ID	2000	2001	2002	2003	2004	2005	2006	2007
A	91	92	92	88	90	94	105	115
B	83	85	88	81	88	94	93	104
C	76	86	86	77	81	86	84	96
D	82	87	96	85	91	93	90	109
E	94	103	109	82	93	96	90	106
F	101	102	110	89	106	114	110	153
G	125	75	82	83	80	89	91	89
H	131	65	67	68	81	72	78	88
I	127	75	79	79	78	82	88	96
J	116	65	70	71	75	75	75	78
K	106	58	64	68	71	66	85	95
L	98	66	67	70	73	71	76	113
M	52	66	61	64	70	76	58	69
N	57	61	69	51	81	62	54	65
O	53	57	64	53	72	64	59	81
P	58	62	72	62	83	73	58	75
Q	52	59	63	60	80	74	59	82
R	53	57	60	59	71	69	48	70
S	75	51	63	67	82	76	55	61
T	62	63	62	61	70	69	47	55
U	62	66	69	70	90	83	48	58
V	59	62	62	62	81	76	59	64
W	55	58	57	59	67	67	50	60
X	54	48	50	57	58	59	59	43
Y	49	58	60	52	61	57	59	47
Z	51	59	59	58	63	63	62	42
AA	53	54	54	56	67	72	63	47
AB	67	59	62	74	89	81	80	49
AC	67	54	59	66	79	78	77	54
AD	70	60	64	71	82	77	80	56

Source: Kansas Department of Transportation

4.3 Exploratory Data Analysis

STATSGRAPHICS Plus Version 5.1 was used in undertaking exploratory data analysis. By subjecting the data to rigorous statistical analysis, pertinent data features such as completeness, accuracy and consistency were evaluated. Summary statistics obtained included measures of central tendency, measures of variability and measures of shape. The averages and corresponding standard deviations show the wide variability with which IRI values change annually for a given pavement section. Summary statistics for the fifteen sections are shown in Table 2.

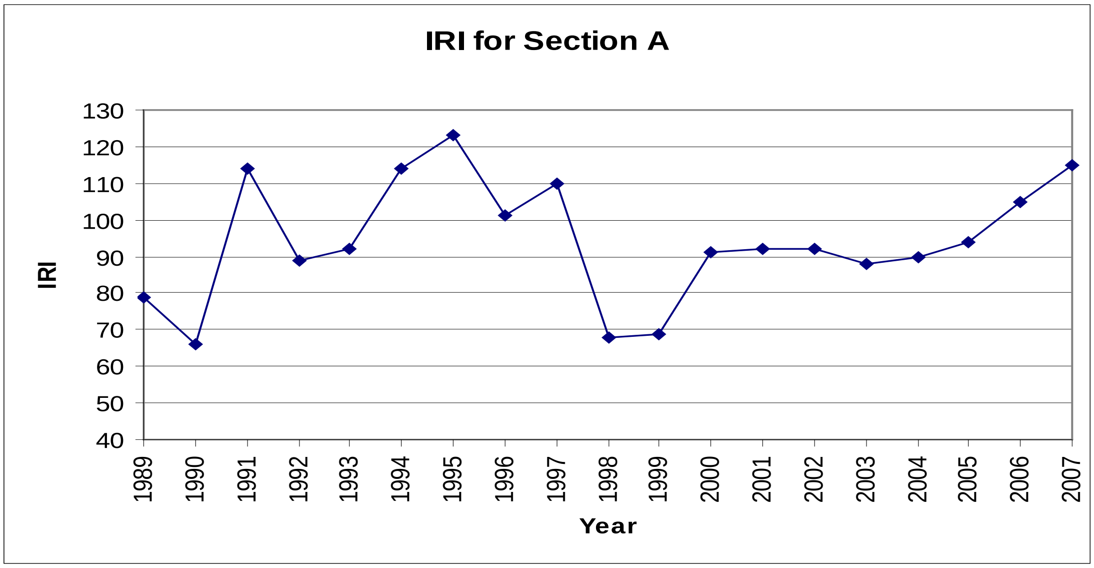
Time series plots were used to observe changes in pavement condition with time. Figures 5 to 19 show how roughness progressed annually along Sections A to O. The plots depict heterogeneity in the observed IRI values for these sections. Consequently, the roughness prediction model must attempt to model these changes without ignoring heterogeneity exhibited at these points. Also shown in the figures are normal probability plots for each section. The closer the points lie to the diagonal line, the better the plot is at predicting normality of the observed data for a particular pavement section.

Table 2 Summary statistics of Roughness for Sections A to O

SECTION ID	**A**	**B**	**C**	**D**	**E**
Count (Year)	18	18	18	18	18
Average	93.2	95.8	86.8	89.7	91.4
Standard Dev.	16.2	20.2	14.1	11.2	13.6
Minimum	66	61	66	73	64
Maximum	123	133	123	118	120
Range	57	72	57	45	56
Standard Skewness.	-0.045	0.48	1.37	1.38	-0.088
Standard Kurt.	-0.39	-0.76	0.99	0.85	0.32

SECTION ID	**F**	**G**	**H**	**I**	**J**
Count (Year)	18	18	18	18	18
Average	104.9	105.7	109.7	108.6	98.8
Standard Dev.	12.6	22.5	29.6	22.6	21.9
Minimum	82	75	65	75	65
Maximum	132	155	166	155	144
Range	50	80	101	80	79
Standard Skewness.	0.26	0.87	-0.51	-0.23	0.015
Standard Kurt.	-0.09	-0.35	-0.75	-0.54	-0.46

SECTION ID	**K**	**L**	**M**	**N**	**O**
Count (Year)	18	18	18	18	18
Average	94.2	86.8	69.8	67	64.2
Standard Dev.	20.5	15.1	13.7	15.3	13.1
Minimum	58	66	52	49	47
Maximum	127	119	98	98	94
Range	69	53	46	49	47
Standard Skewness.	-0.86	0.94	1.03	1.18	1.23
Standard Kurt.	-0.78	-0.04	-0.42	-0.53	-0.11



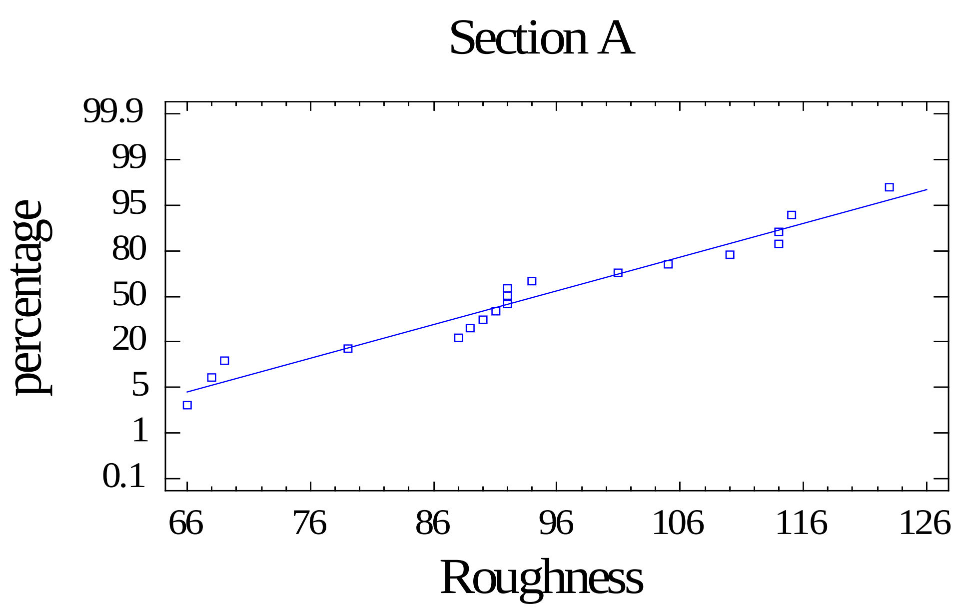
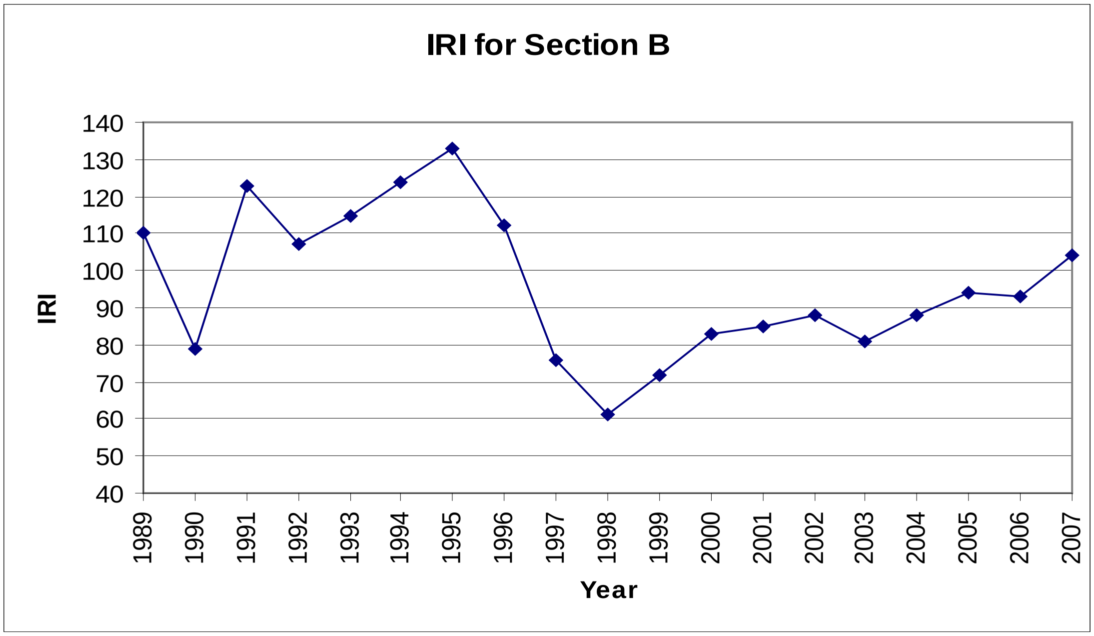


Figure 5 Time Series and Normal Plots for Section A

The plots show an extreme case of heterogeneity from years 1989 to 1998 characterized by alternate and significant changes in deterioration and rehabilitation The pavement shows incremental signs of deterioration and maintenance after 1998.



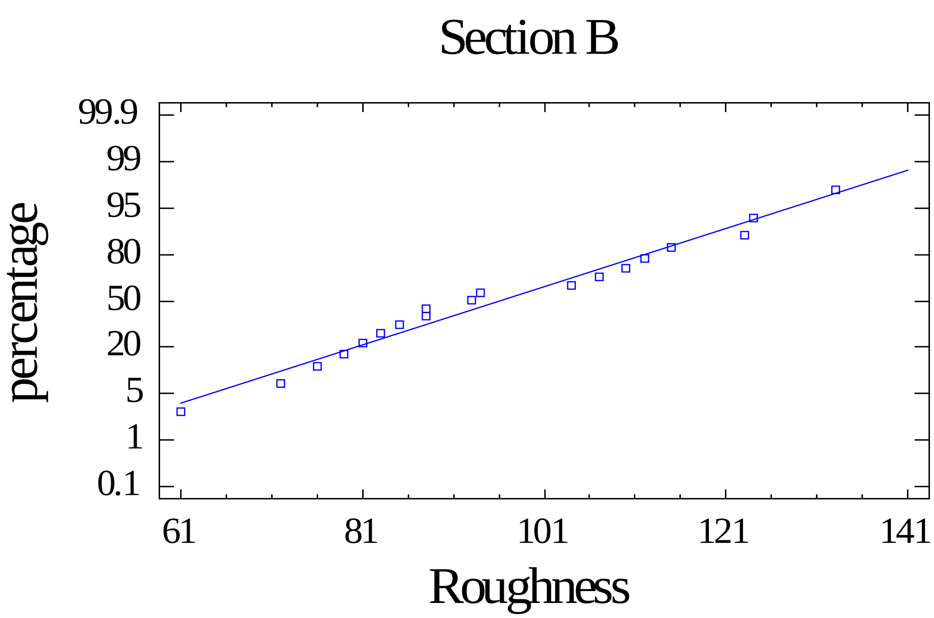
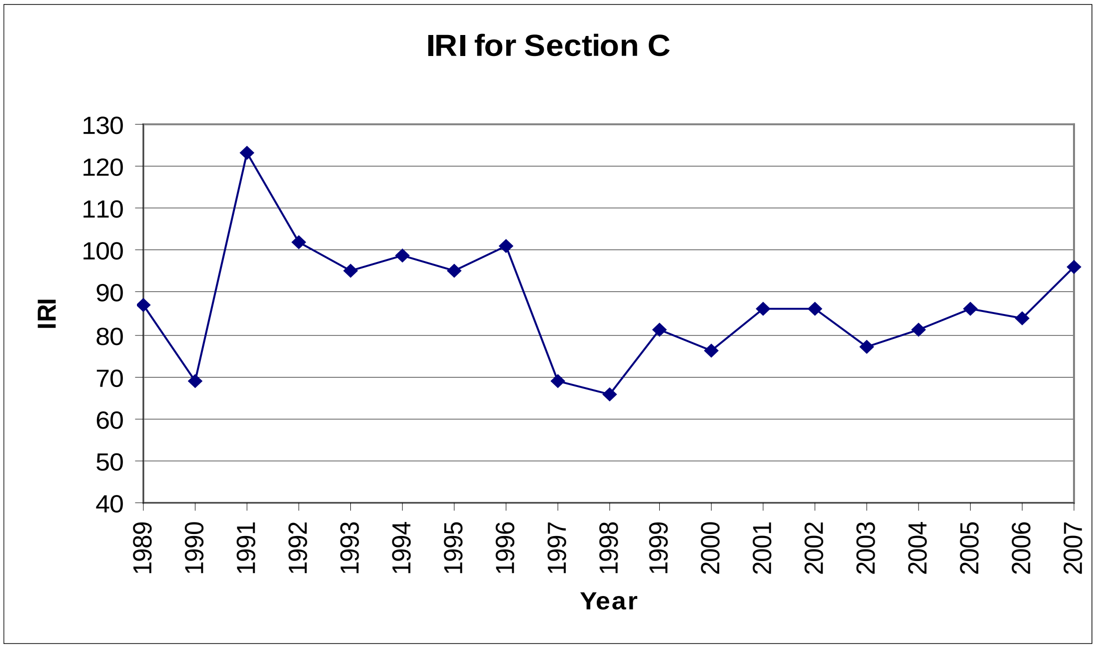


Figure 6 Time Series and Normal Plots for Section B

Steep changes in gradient are observed from 1989 to 1998. Subsequent years show gradual changes with regards to pavement deterioration and maintenance.



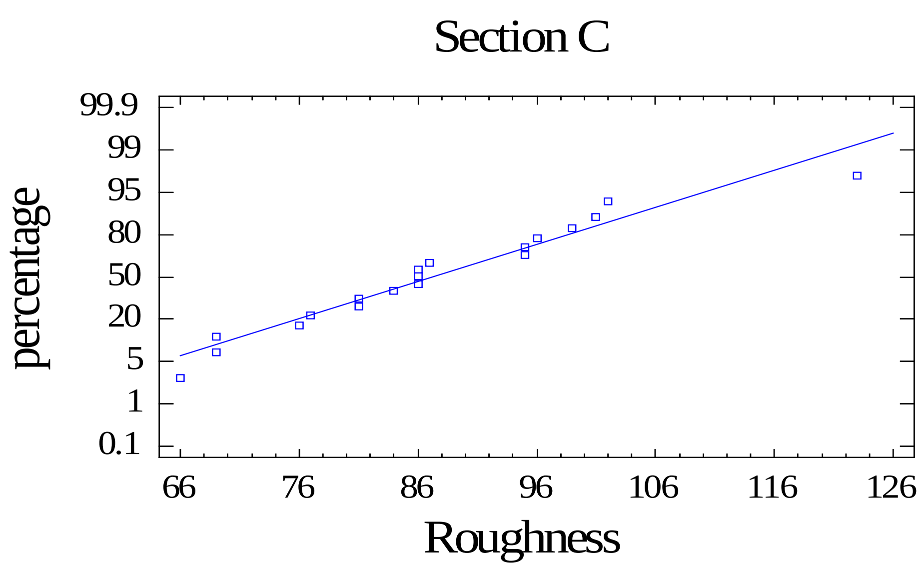
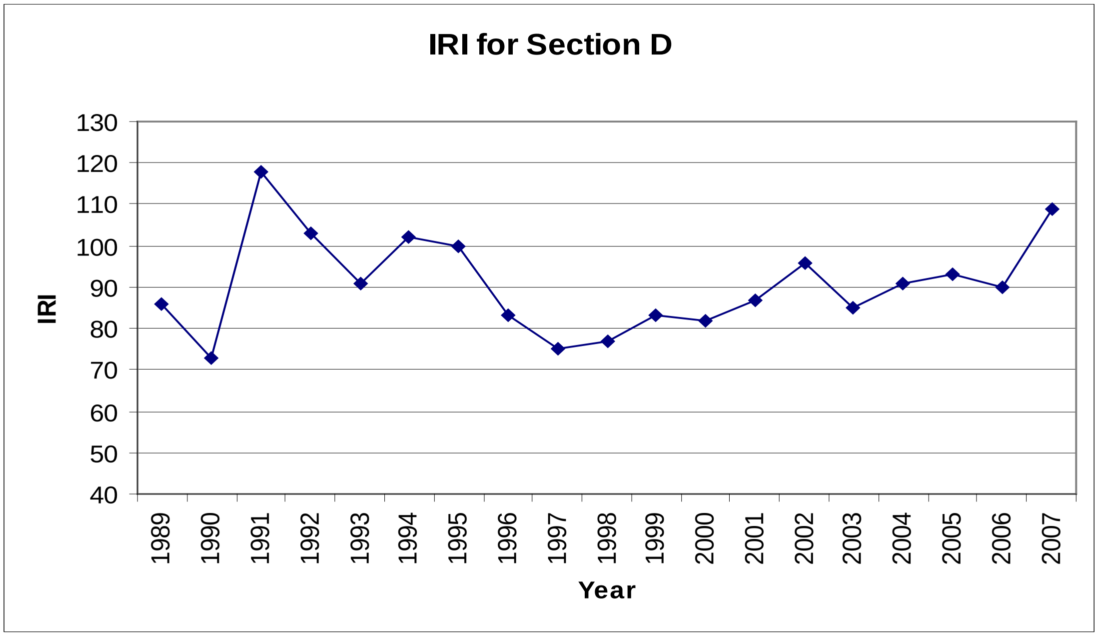


Figure 7 Time Series and Normal Plots for Section C

Apart from a huge jump from 1990 to 1991, and a large drop from 1996 to 1997, all other trends followed routine pavement maintenance and deterioration.



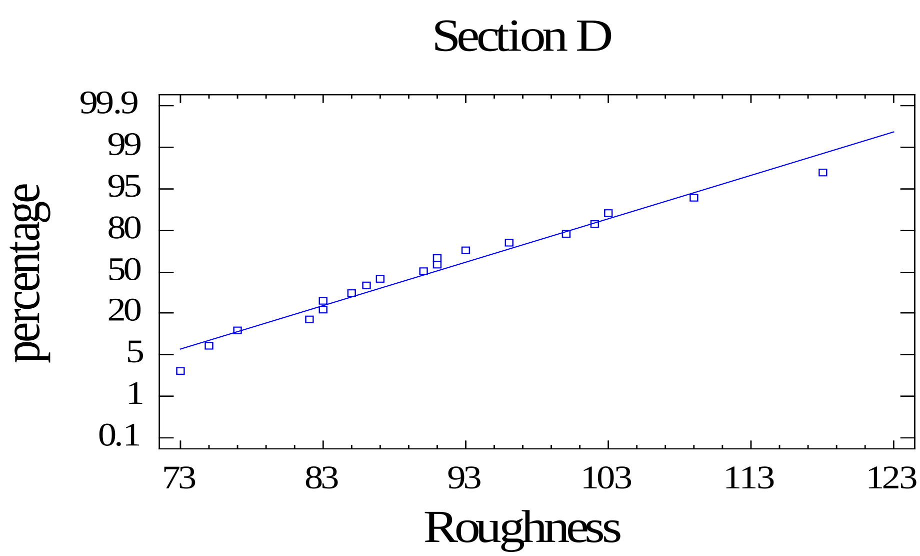
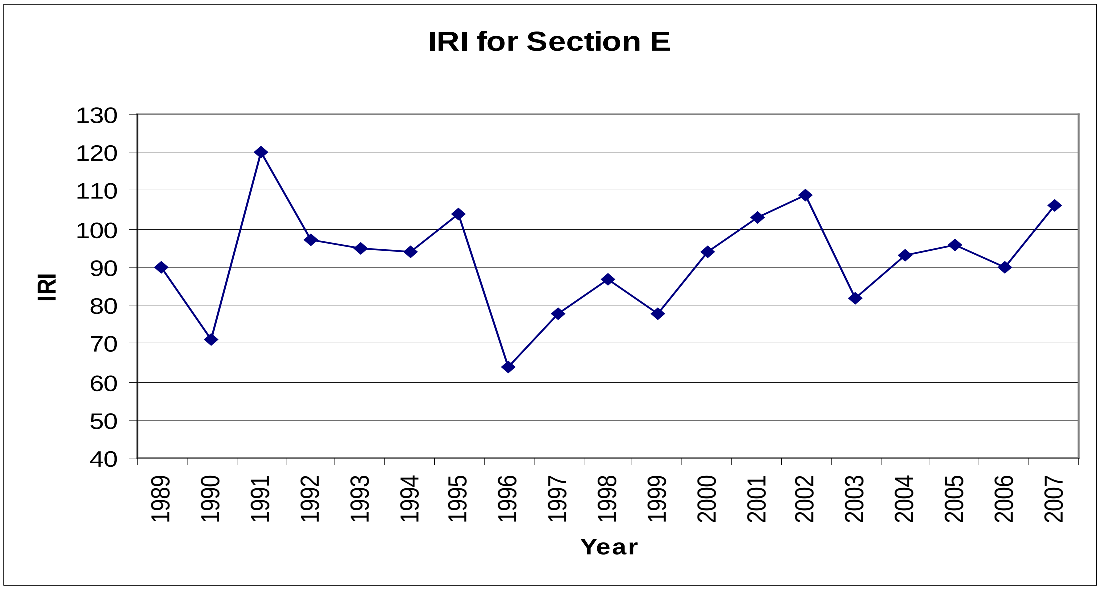


Figure 8 **Time Series and Normal Plots for Section D**

Apart from a sharp rise in deterioration from 1990 to 1991, the pavement section followed a general trend of deterioration and maintenance.



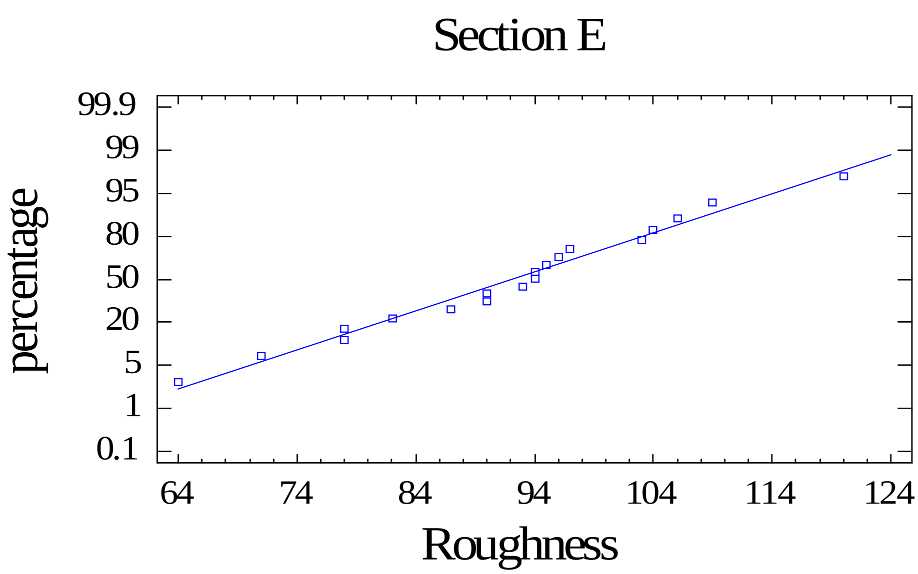
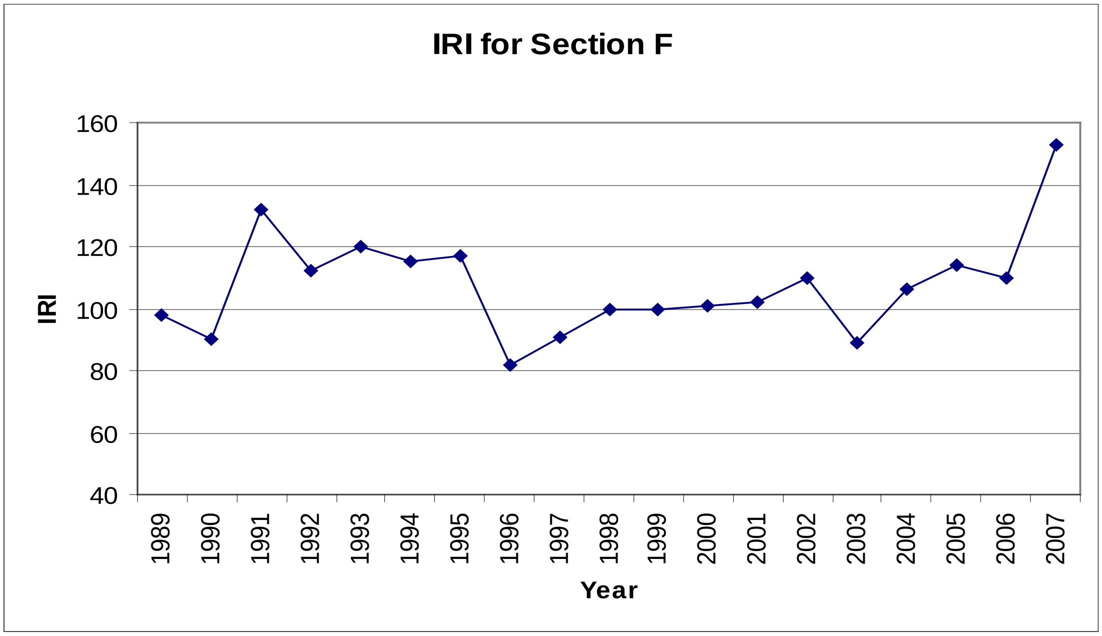


Figure 9 **Time Series and Normal Plots for Section E**

Where as roughness increased sharply to its highest value in 1991; two major rehabilitations took place in 1996 and 2003. All other points followed an incremental maintenance or deterioration cycle.



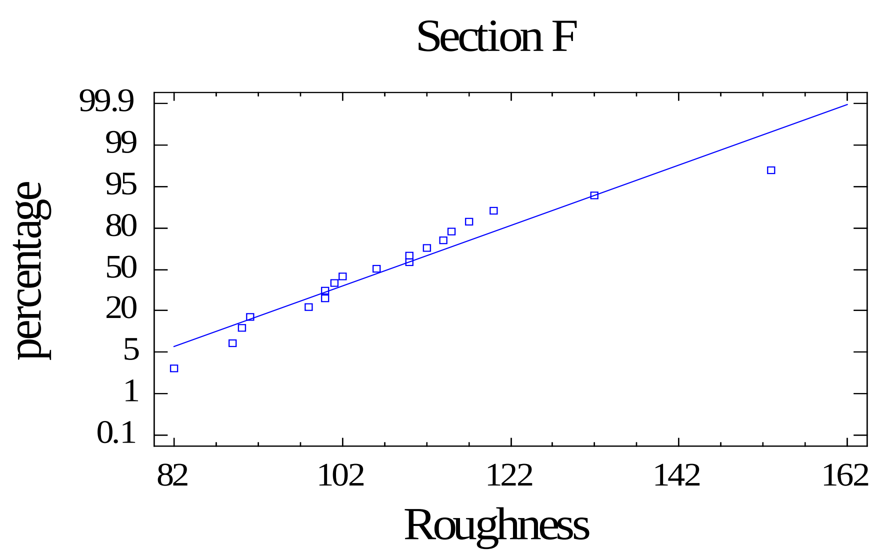


Figure 10 Time Series and Normal Plots for Section F

Plots generally show a gradual change in slope over time and represent a cycle of pavement deterioration and maintenance. Notable changes are however seen in 1991, 1996 and 2007.

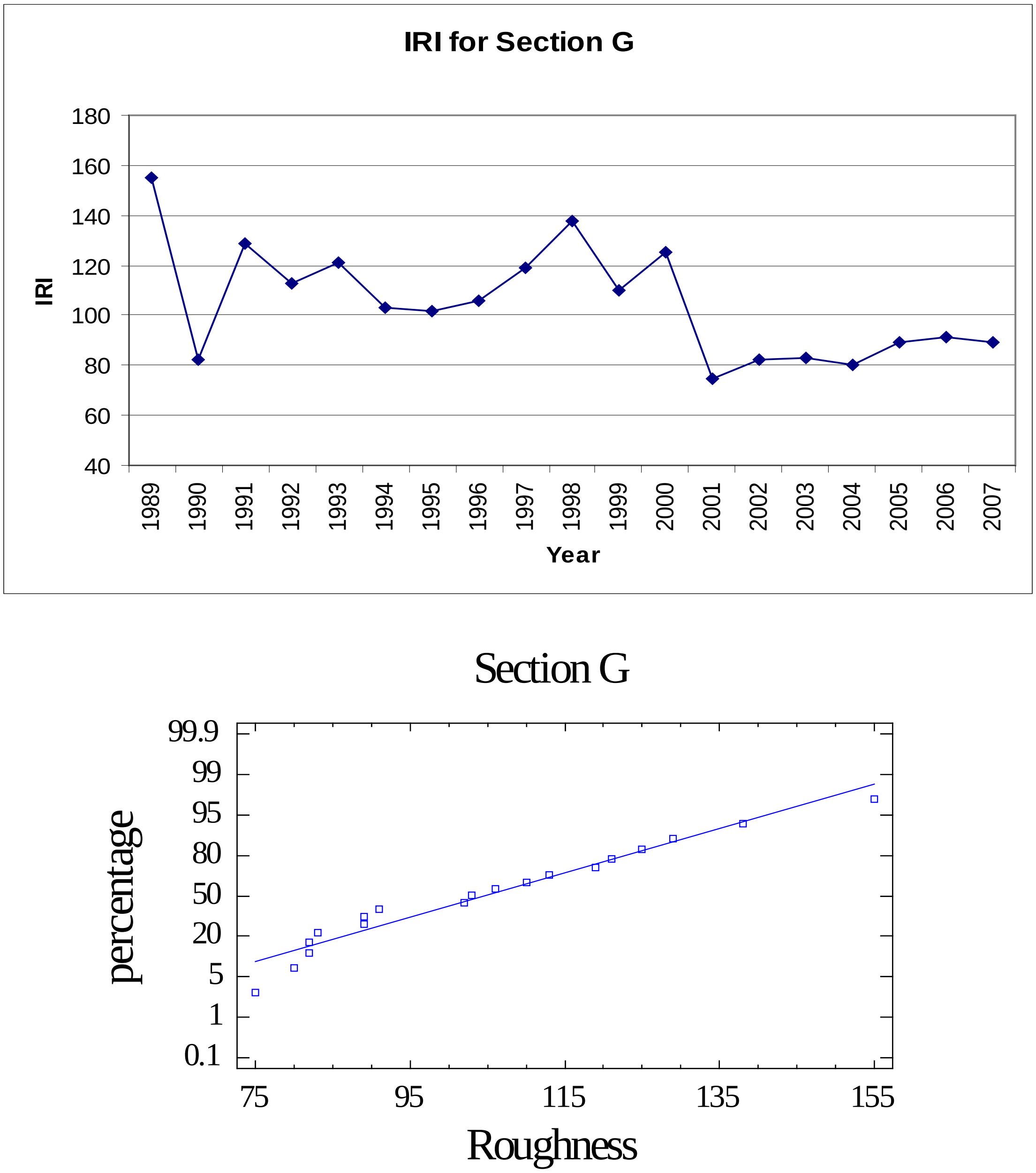
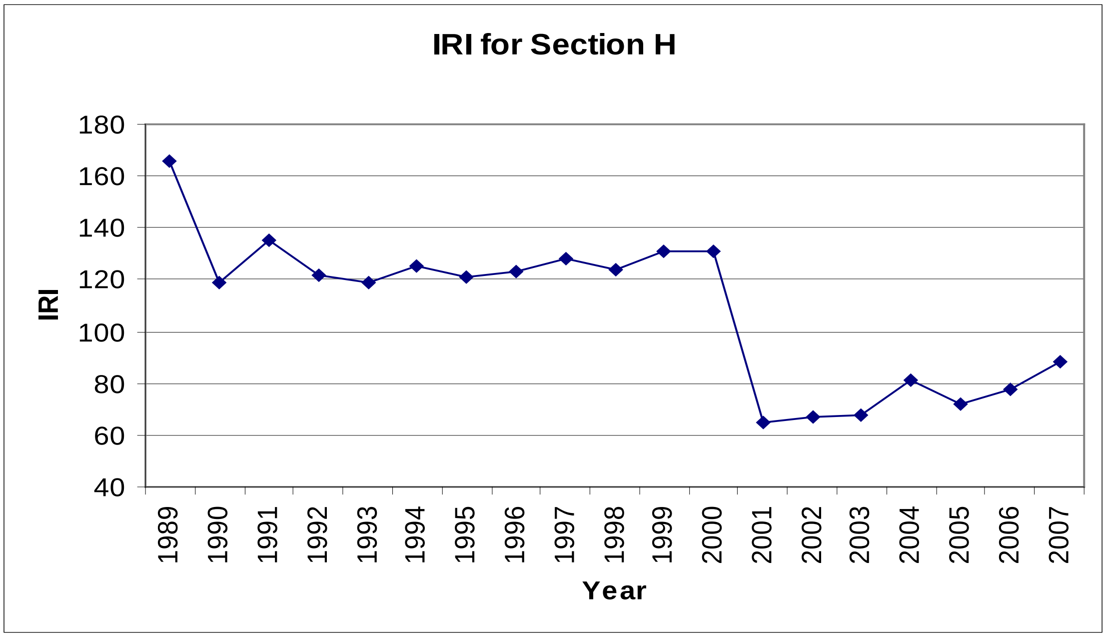


Figure 11 Time Series and Normal Plots for Section G

Apart from two sharp drops in 1990 and 2001, plots had gentle rising and falling slopes highlighting incremental change in roughness across the pavement section within the time interval.



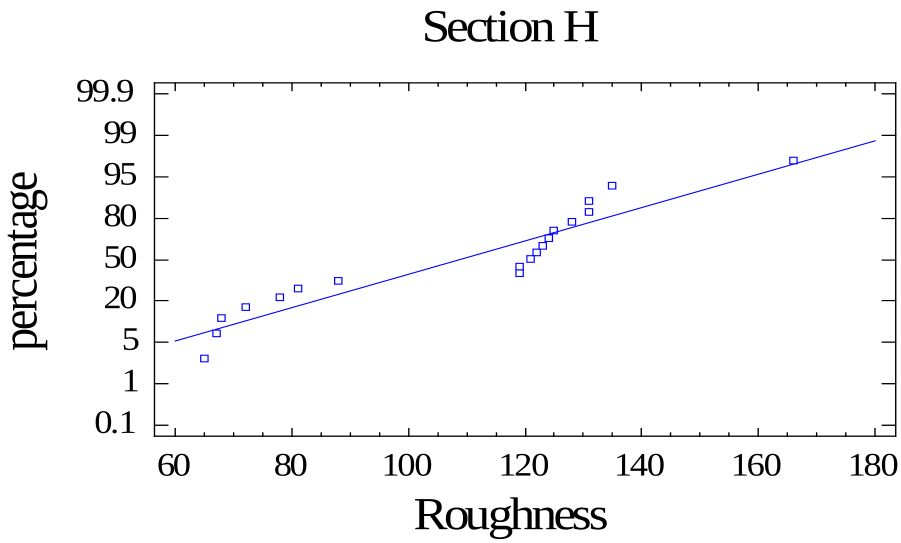
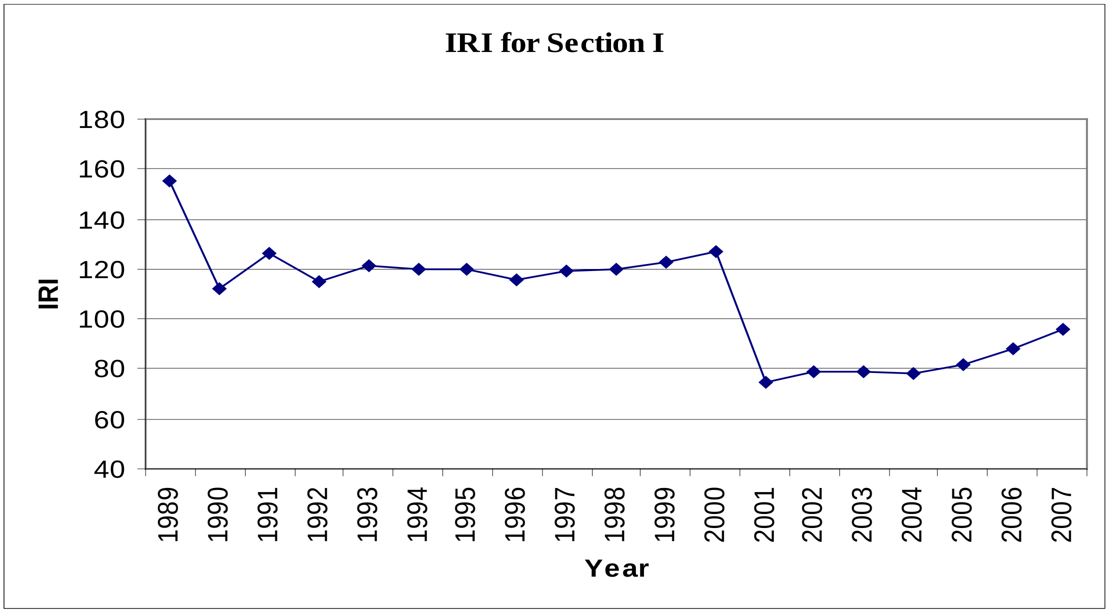


Figure 12 Time Series and Normal Plots for Section H

Apart from 2 major rehabilitation efforts in 1990 and 2001, plots followed a general cycle of incremental increase and decrease in pavement roughness. A stable trend in IRI values is generally seen from year 1992 to 2000.



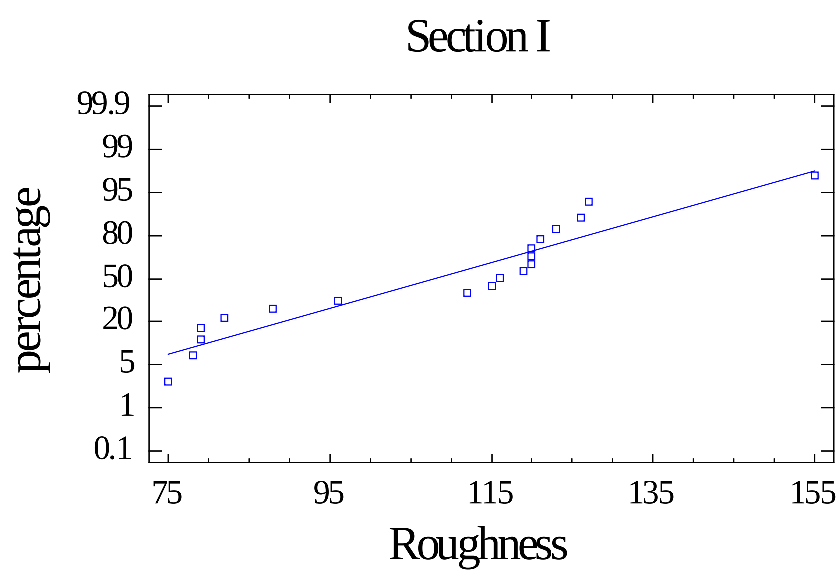
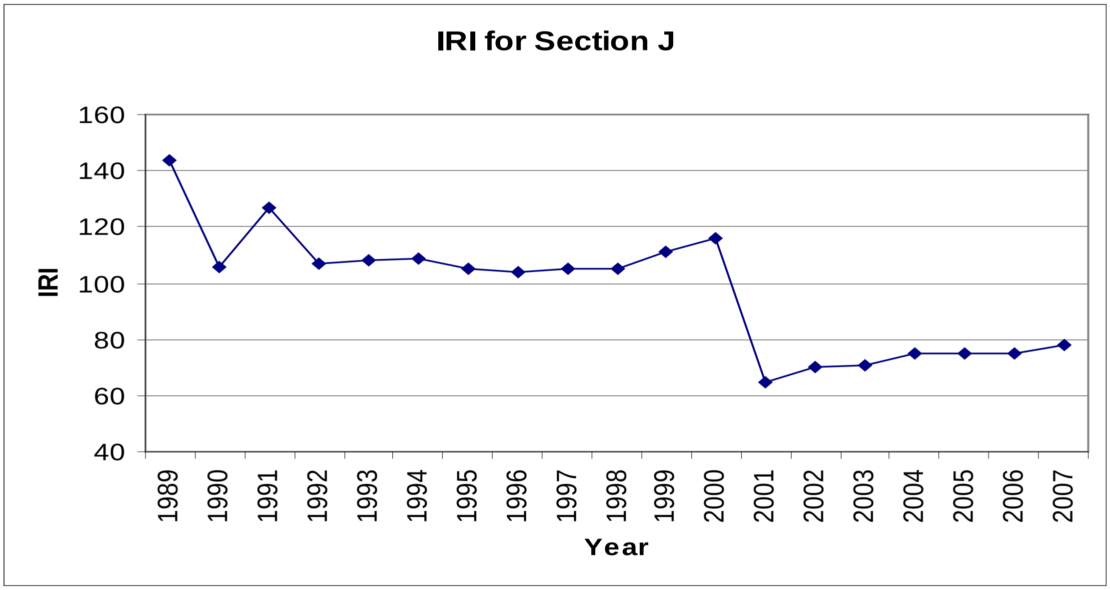


Figure 13 Time Series and Normal Plots for Section I

Two major rehabilitation efforts occurred in 1990 and 2001. A stable trend in IRI values is generally seen from year 1992 to 2000. The gentle slope from 2001 to 2007 is indicative of a gradual increase in roughness for this time window.



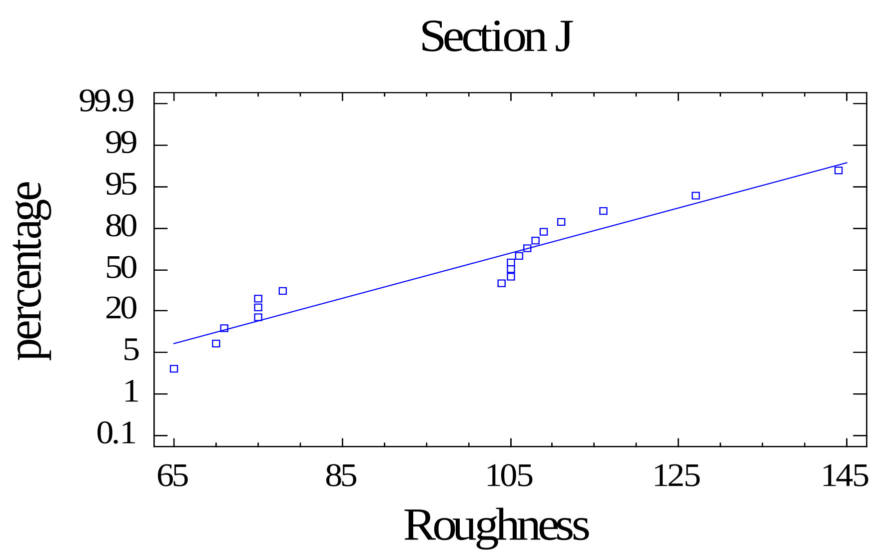
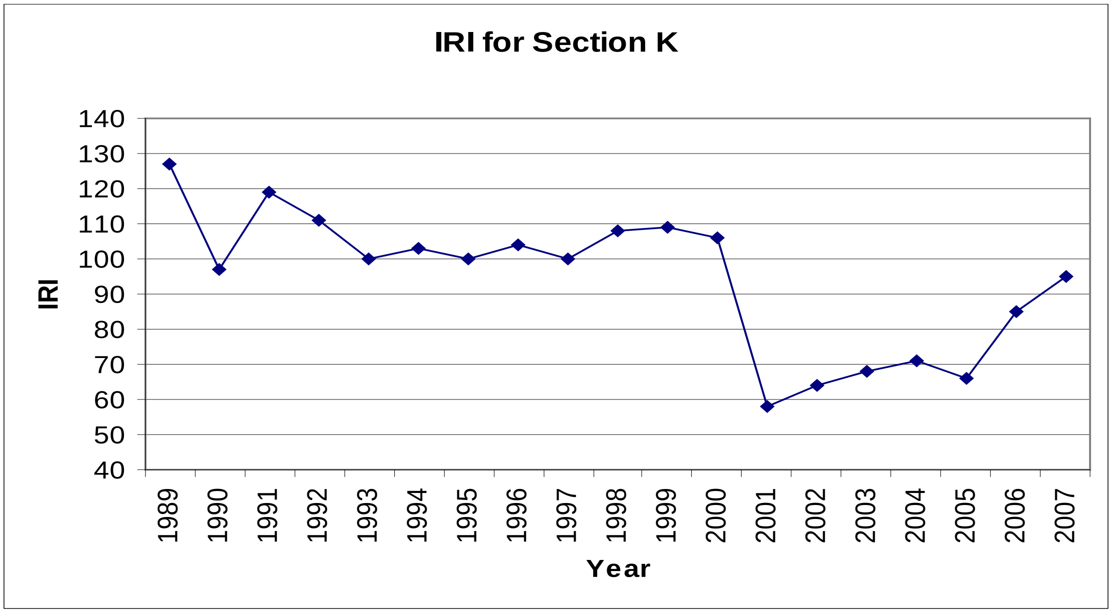


Figure 14 Time Series and Normal Plots for Section J

Two significant rehabilitation efforts took place in 1990 and 2001. Rehabilitation in 1990 was not sustainable as there was an increase in roughness in 1991. On the contrary, the major rehabilitation in 2001 produced a favorable trend in IRI values from 2002 to 2007



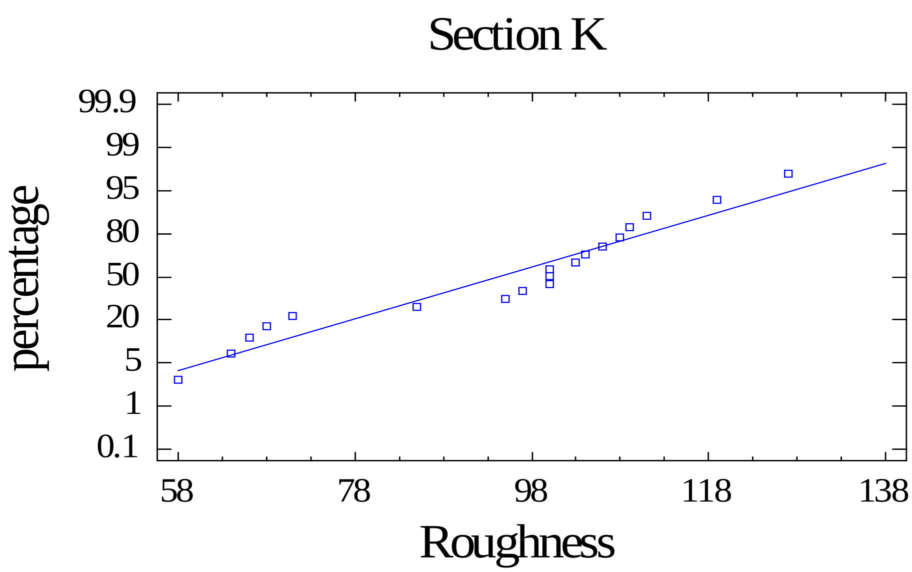
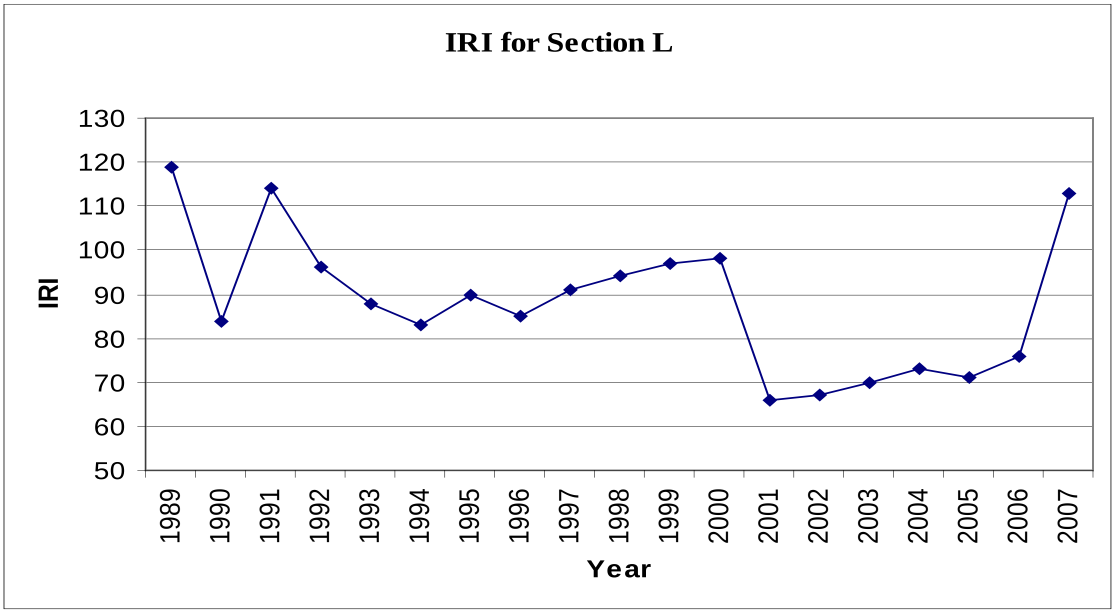


Figure 15 Time Series and Normal Plots for Section K

Major rehabilitation took place in 1990 and 2001, but that in 2001 was greater. Whereas rehabilitation works in 1990 was followed by a sharp increase in roughness in 1991; that in 2001 produced a gradual increase from 2002 to 2004.



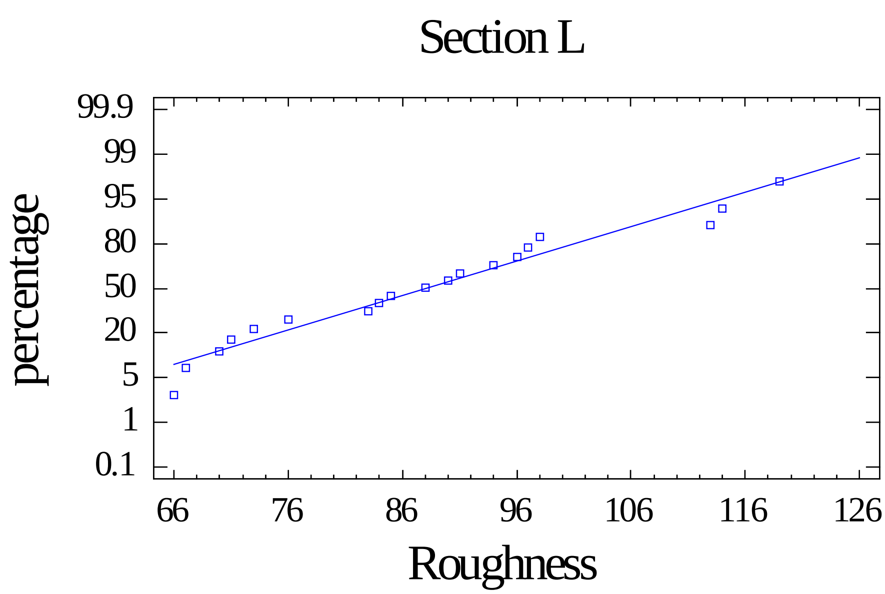
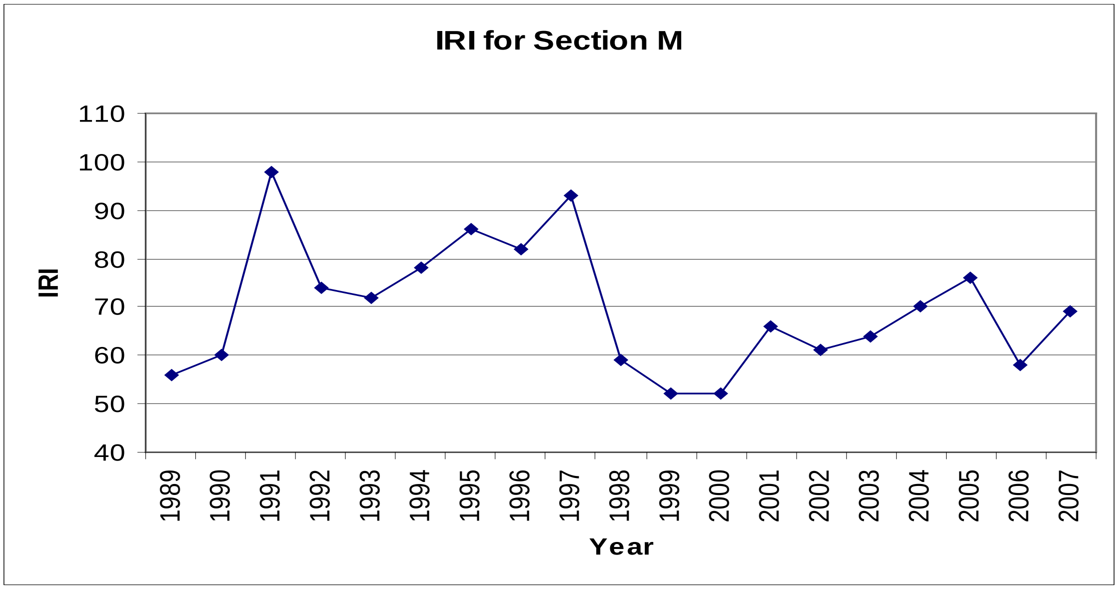


Figure 16 Time Series and Normal Plots for Section L

The pavement had similar rehabilitation efforts in 1990 and 2001. Rehabilitation in 1990 did not last and significant deterioration in roughness occurred in 1991. Rehabilitation in 2001 grew incrementally for 3 years, before a sudden rise in 2007.



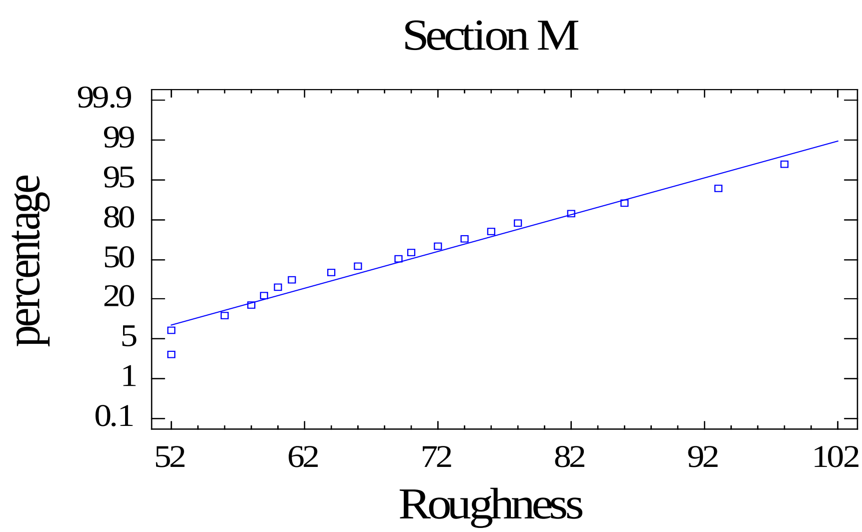
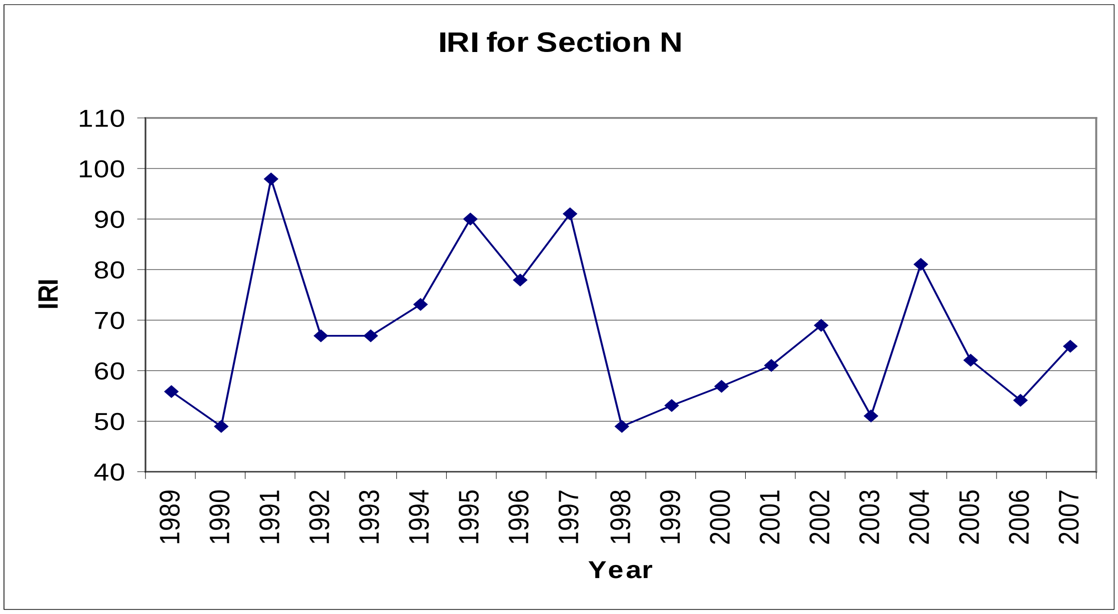


Figure 17 Time Series and Normal Plots for Section M

Significant changes occur in 1991 and 1998. In 1991 there is a sharp increase in pavement roughness from 1990. Rehabilitation is then undertaken and roughness then increases gradually from 1992 to 1995. In 1998, rehabilitation from 1997 preserved pavement till 2000 after which roughness increased gradually to 2005.



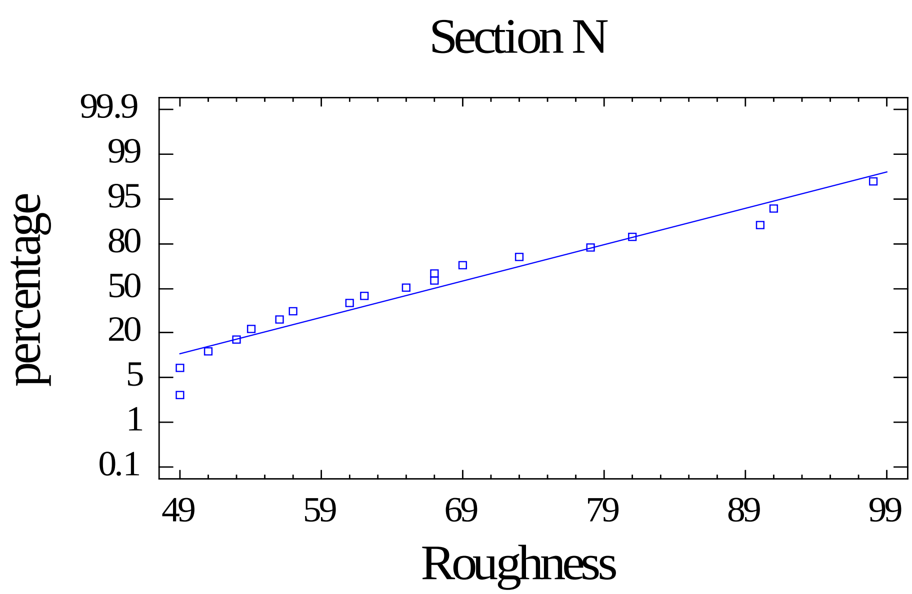
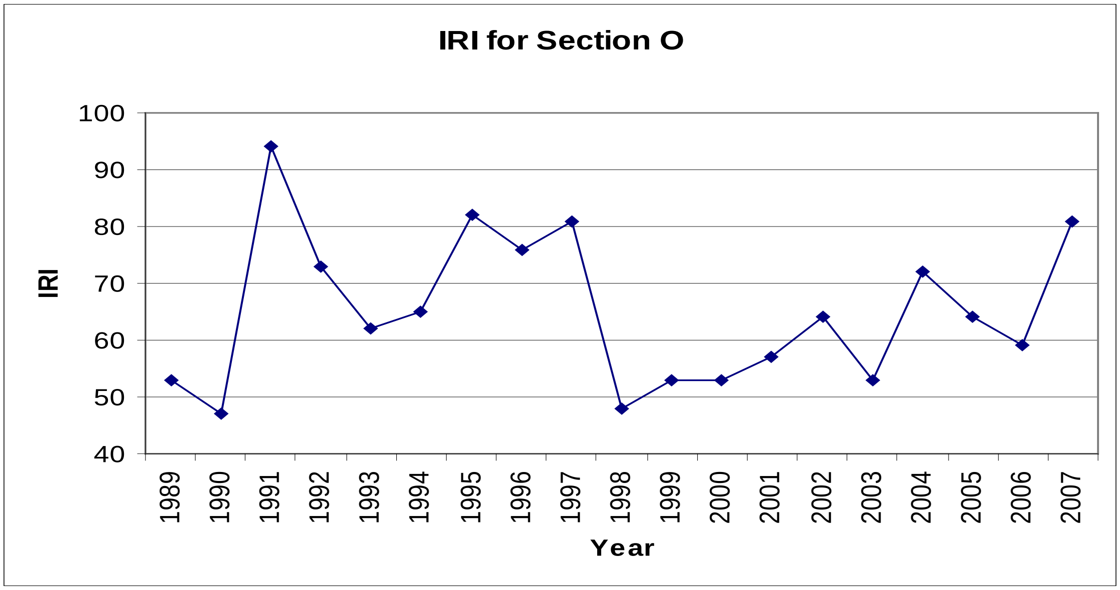


Figure 18 Time Series and Normal Plots for Section N

The only trend that produced a steady pattern was from 1998 to 2002. All other points behaved unpredictably which depicted extreme heterogeneity for this pavement section.



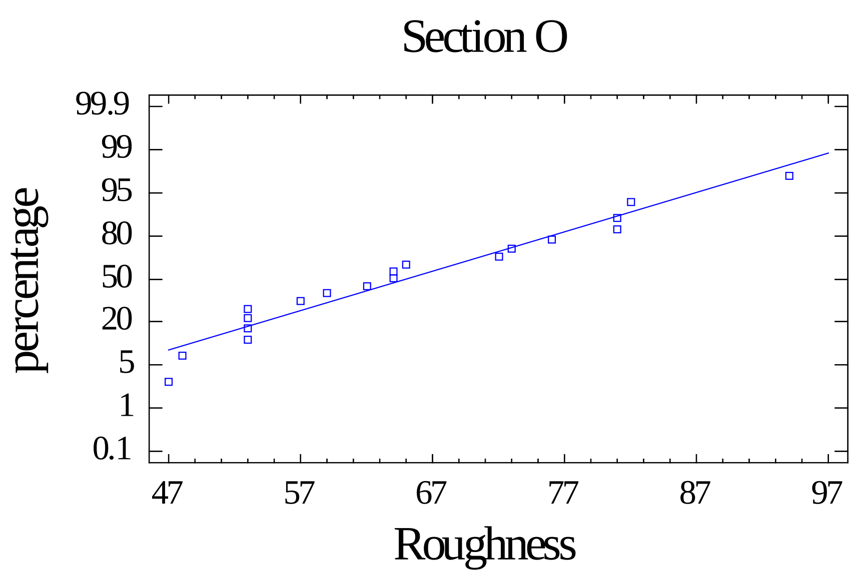


Figure 19 Time series and Normal Plots for Section O

The only trend that produced a steady pattern was from 1998 to 2002. All other points behaved unpredictably which depicted extreme heterogeneity for this pavement section. The deterioration in 1991 and the rehabilitation in 1998 were the most conspicuous.

4.4 Model Development

Normal hierarchical Markov Chain Monte Carlo (MCMC) model was used in estimating and predicting parameters for these sections. A diagrammatic representation of a normal hierarchical model is shown in Figure 20. The accompanying syntax in WinBUGS is shown in Figure 21.

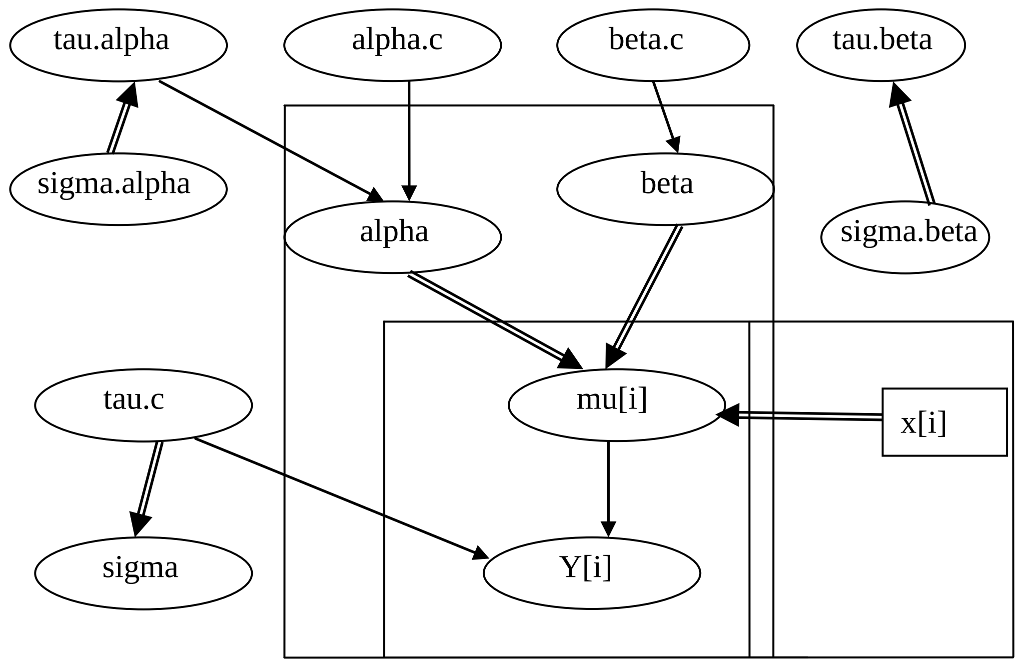


Figure 20 Graphical representation of normal hierarchical MCMC model

```
model
{
for(i in 1:N) {
Y[i]~dnorm(mu[i],tau.c)
mu[i]<-alpha+(beta*(x[i]))
}
alpha~dnorm(alpha.c,tau.alpha)
beta~dnorm(beta.c,tau.beta)
tau.c~dgamma(0.001,0.001)
sigma<-1/sqrt(tau.c)
alpha.c~dnorm(0.0,1.0E-6)
sigma.alpha~dunif(0,100)
sigma.beta~dunif(0,100)
tau.alpha<-1/(sigma.alpha*sigma.alpha)
tau.beta<-1/(sigma.beta*sigma.beta)
beta.c~dnorm(0.0,1.0E-6)
}
list(
Y=c(………………………),N=….,
x=c(………………………..))
INITS
list(alpha=…,beta=.., alpha.c=.., beta.c=.., tau.c=.., sigma.alpha=.., sigma.beta=..)
```

Figure 21 Syntax for normal hierarchical MCMC model in WinBUGS

CHAPTER 5

RESULTS OF NORMAL HIERARCHICAL MCMC MODELS

5.1 Estimating IRI values of sections

Estimation was done to determine the accuracy of the normal hierarchical model. This was done for each individual section. The process is illustrated as follows using three randomly selected pavement sections, Sections D, F, and H:

- From the data, choose any 5 arbitrary years. Omit their IRI values prior to running the model by replacing them with NA (not available).
- Run the model in WinBUGS and generate 18001 simulations. Ensure convergence by "burning the first 3000" simulations.
- Observe the results for the selected 5 years and see if they correspond to the original roughness values in the data. Once good results are obtained, estimation is done for all years spanning 1989 to 2007.

5.1.1 Estimation Results for Section D

IRI values obtained after running the model are shown in Table 3. A comparison is then made between IRI values in the dataset and those obtained from the run.

Table 3 Estimated IRI values of 5 arbitrary years for Section D

Year	Original IRI (prior to run)	Observed IRI (after run) - 2.5 percentile	Observed IRI (after run) – 50 percentile	Observed IRI (after run) – 97.5 percentile
1992	103	78.4	88.96	99.34
1995	100	81.27	89.65	97.77
1998	77	82.85	90.35	97.68
2001	87	82.43	90.97	99.37
2005	93	80.01	91.83	103.7

From Table 3, it can be seen that observed IRI values are in close proximity to the original IRI values. This shows the efficiency of the normal hierarchical model at estimating roughness values is high. Table 4 shows all the estimated IRI values for section D within the 19 year time frame. Parameters used in the normal hierarchical MCMC model are shown in Table 5.

Table 4 Estimated IRI values for Section D from 1989 to 2007

Year		**Mean**	**sd**	**MC error**	**2.5%**	**Median**	**97.5%**	**Observed**
89	mu[1]	88.28	6.782	0.1367	74.69	88.28	101.7	86
90	mu[2]	88.5	6.258	0.1241	75.99	88.49	100.9	73
91	mu[3]	88.73	5.757	0.1115	77.22	88.72	99.97	118
92	mu[4]	88.95	5.287	0.09907	78.4	88.96	99.34	103
93	mu[5]	89.18	4.856	0.08669	79.53	89.2	98.75	91
94	mu[6]	89.4	4.476	0.07444	80.45	89.4	98.2	102
95	mu[7]	89.62	4.16	0.0624	81.27	89.65	97.77	100
96	mu[8]	89.85	3.925	0.05073	81.94	89.88	97.55	83
97	mu[9]	90.07	3.784	0.03975	82.51	90.11	97.55	75
98	mu[10]	90.3	3.75	0.03022	82.85	90.35	97.68	77
99	mu[11]	90.52	3.824	0.02392	82.82	90.55	98.03	83

Table 4 Continued

Year		Mean	sd	MC error	2.5%	Median	97.5%	Observed
00	mu[12]	90.74	4.001	0.02363	82.77	90.75	98.59	82
01	mu[13]	90.97	4.268	0.02951	82.43	90.97	99.37	87
02	mu[14]	91.19	4.609	0.03886	81.95	91.18	100.3	96
03	mu[15]	91.42	5.009	0.04976	81.44	91.41	101.4	85
04	mu[16]	91.64	5.456	0.06138	80.77	91.63	102.5	91
05	mu[17]	91.86	5.938	0.07339	80.01	91.83	103.7	93
06	mu[18]	92.09	6.448	0.08563	79.2	92.06	104.9	90
07	mu[19]	92.31	6.98	0.098	78.35	92.29	106.2	109

Table 5 Parameters used in estimation for Section D

Node	Mean	sd	MC error	2.50%	Median	97.50%	Sample
tau.c	0.005986	0.002441	2.30E-05	0.002219	0.00566	0.01164	18001
alpha	88.06	7.325	0.1493	73.3	88.07	102.5	18001
beta	0.224	0.6412	0.01278	-1.045	0.2212	1.508	18001

Monte Carlo error (MC error) measures the variability of the estimate due to the simulation. A low MC error is required to calculate the parameter of interest with increased precision. In order to obtain a stationary posterior distribution, convergence was guaranteed by ‘burning’ the first 3000 samples that were generated. Figure 22 shows a density plot of the posterior distribution for monitored node mu[9]. The plot indicates that random values generated for this node are normally distributed. A time series plot displaying both original and observed IRI values is shown in Figure 23. As can be seen, only 5 out of the 19 values do not fall within the 95% confidence envelope.

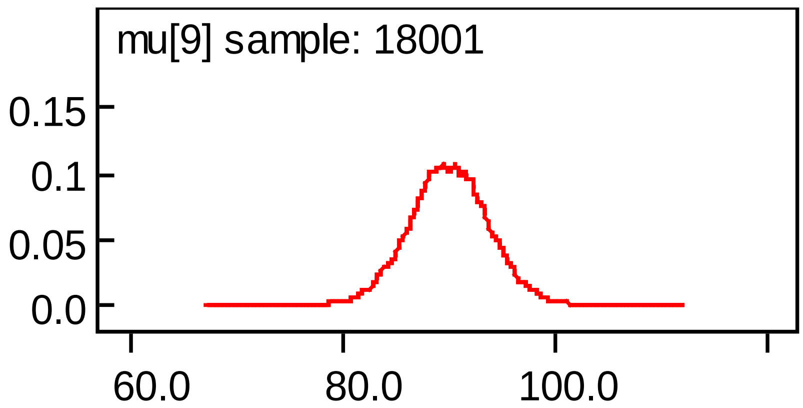


Figure 22 Density plot of monitored node for Section D

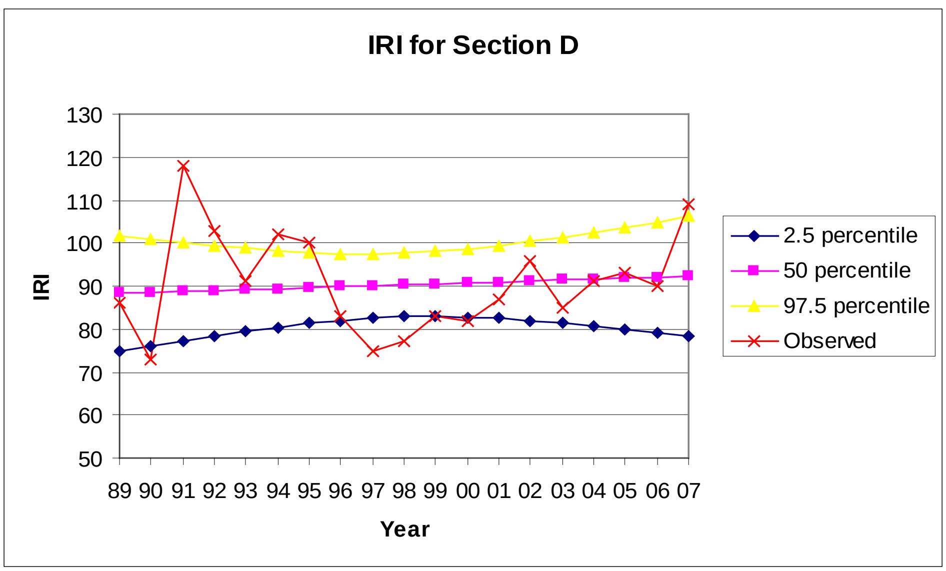


Figure 23 Plot of Estimated versus Actual IRI for Section D

5.1.2 Estimation Results for Section F

Like Section D, tables 6 and 7 show close proximity between observed and actual IRI values. Table 8 shows parameters used for estimation.

Table 6 Estimated IRI values of 5 arbitrary years for Section F

Year	Original IRI (prior to run)	Observed IRI (after run) - 2.5 percentile	Observed IRI (after run) – 50 percentile	Observed IRI (after run) – 97.5 percentile
1993	120	92.48	105.4	118.3
1996	82	96.79	107.5	117.9
1999	100	98.97	109.6	119.9
2002	110	98.6	111.6	124.6
2006	110	96.13	114.4	132.6

Table 7 Estimated IRI values for Section F from 1989 to 2007

Year		Mean	sd	MC error	2.5%	Median	97.5%	Observed
89	mu[1]	102.6	9.123	0.1855	84.45	102.6	120.7	98
90	mu[2]	103.3	8.402	0.1679	86.55	103.3	119.9	90
91	mu[3]	104	7.715	0.1503	88.63	104	119.2	132
92	mu[4]	104.7	7.074	0.1328	90.64	104.7	118.7	112
93	mu[5]	105.4	6.492	0.1155	92.48	105.4	118.3	120
94	mu[6]	106.1	5.986	0.09838	94.06	106.1	117.9	115
95	mu[7]	106.8	5.577	0.0816	95.51	106.8	117.7	117
96	mu[8]	107.5	5.287	0.06543	96.79	107.5	117.9	82
97	mu[9]	108.2	5.136	0.05046	97.87	108.2	118.3	91
98	mu[10]	108.9	5.138	0.03811	98.58	108.9	119	100
99	mu[11]	109.5	5.291	0.03165	98.97	109.6	119.9	100
00	mu[12]	110.2	5.584	0.03454	99.11	110.3	121.1	101
01	mu[13]	110.9	5.995	0.04502	98.95	110.9	122.8	102

Table 7 Continued

Year		Mean	sd	MC error	2.5%	Median	97.5%	Observed
02	mu[14]	111.6	6.503	0.05918	98.6	111.6	124.6	110
03	mu[15]	112.3	7.087	0.07497	98.21	112.3	126.5	89
04	mu[16]	113	7.729	0.09155	97.65	113	128.4	106
05	mu[17]	113.7	8.416	0.1086	96.9	113.7	130.4	114
06	mu[18]	114.4	9.138	0.1258	96.13	114.4	132.6	110
07	mu[19]	115.1	9.888	0.1432	95.31	115.1	134.8	153

Table 8 Parameters used in estimation for Section F

Node	Mean	sd	MC error	2.50%	Median	97.50%	Sample
tau.c	0.00319	0.001299	1.21E-05	0.001188	0.00301	0.00619	18001
alpha	101.9	9.872	0.2032	82.24	101.9	121.4	18001
beta	0.6917	0.8896	0.01792	-1.073	0.6876	2.474	18001

Monte Carlo error (MC error) measures the variability of the estimate due to the simulation. A low MC error is required to calculate the parameter of interest with increased precision. In order to obtain a stationary posterior distribution, convergence was guaranteed by ‘burning’ the first 3000 samples that were generated. Figure 24 shows generated IRI values from the posterior distribution for Section F are normally distributed. Node mu[15] is used in this case. The time series plot in Figure 25 shows that approximately 74% of values fell within the 95% confidence interval.

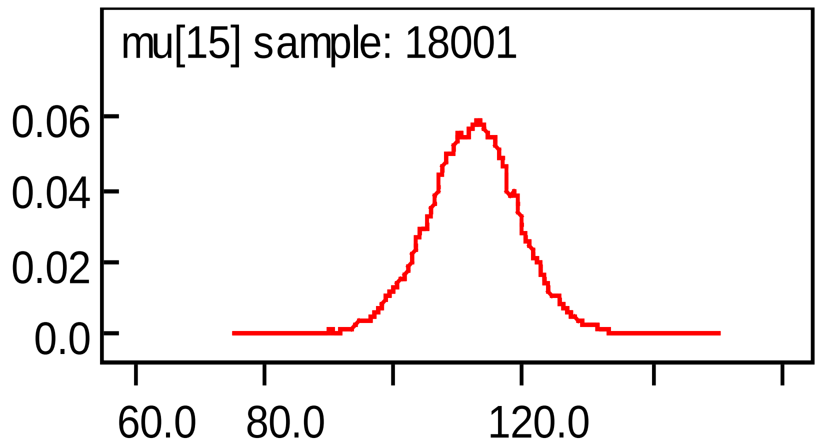


Figure 24 Density plot of monitored node for Section F

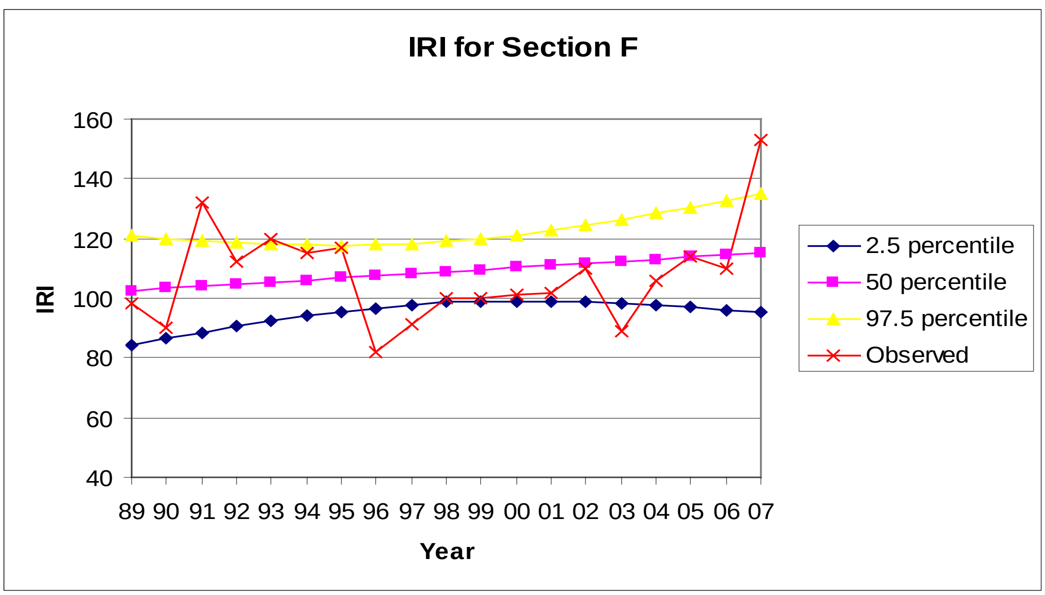


Figure 25 Plot of Estimated versus Actual IRI for Section F

5.1.3 Estimation Results for Section H

Tables 9 and 10 show close proximity between observed and original IRI values. Parameters used in estimation are shown in Table 11.

Table 9 Estimated IRI values of 5 arbitrary years for Section H

Year	Original IRI (prior to run)	Observed IRI (after run) - 2.5 percentile	Observed IRI (after run) – 50 percentile	Observed IRI (after run) – 97.5 percentile
1991	135	116.5	134.8	152.5
1994	125	108.1	122.2	136
1997	128	98.43	109.8	121.1
2000	131	85.63	97.26	108.9
2004	81	65.42	80.52	96.42

Table 10 Estimated IRI values for Section H from 1989 to 2007

Year		Mean	sd	MC error	2.5%	Median	95%	Observed
89	mu[1]	143	10.57	0.2087	121.4	143.2	163.9	166
90	mu[2]	138.9	9.76	0.19	119	139	158.2	119
91	mu[3]	134.7	8.98	0.1713	116.5	134.8	152.5	135
92	mu[4]	130.6	8.241	0.1527	113.8	130.6	146.7	122
93	mu[5]	126.4	7.554	0.1342	111	126.5	141.3	119
94	mu[6]	122.2	6.934	0.1161	108.1	122.2	136	125
95	mu[7]	118.1	6.402	0.09826	105.1	118.1	130.8	121
96	mu[8]	113.9	5.981	0.08108	101.9	113.9	125.9	123
97	mu[9]	109.7	5.695	0.06502	98.43	109.8	121.1	128
98	mu[10]	105.6	5.565	0.05114	94.53	105.6	116.7	124
99	mu[11]	101.4	5.603	0.04169	90.25	101.4	112.7	131
00	mu[12]	97.25	5.804	0.03995	85.63	97.26	108.9	131
01	mu[13]	93.08	6.153	0.04678	81	93.07	105.5	65

Table 10 Continued

Year		Mean	sd	MC error	2.5%	Median	95%	Observed
02	mu[14]	88.92	6.626	0.05929	75.95	88.9	102.3	67
03	mu[15]	84.76	7.2	0.07468	70.64	84.7	99.33	68
04	mu[16]	80.59	7.851	0.09151	65.42	80.52	96.42	81
05	mu[17]	76.43	8.563	0.1091	59.8	76.33	93.74	72
06	mu[18]	72.26	9.321	0.1272	54.22	72.18	91.14	78
07	mu[19]	68.1	10.12	0.1455	48.46	68	88.69	88

Table 11 Parameters used in estimation for Section H

Node	Mean	sd	MC error	2.50%	Median	97.50%	Sample
tau.c	0.00275	0.00113	1.12E-05	0.001018	0.002595	0.00537	18001
alpha	147.2	11.41	0.2276	123.9	147.4	169.6	18001
beta	-4.164	0.969	0.01917	-6.056	-4.168	-2.2	18001

Monte Carlo error (MC error) measures the variability of the estimate due to the simulation. A low MC error is required to calculate the parameter of interest with increased precision. In order to obtain a stationary posterior distribution, convergence was guaranteed by ‘burning’ the first 3000 samples that were generated. Fig. 26 shows generated IRI values from the posterior distribution for Section H are normally distributed. The time series plot in Fig. 27 shows that 16 out of 19 IRI values were either within or close to the 95% confidence band.

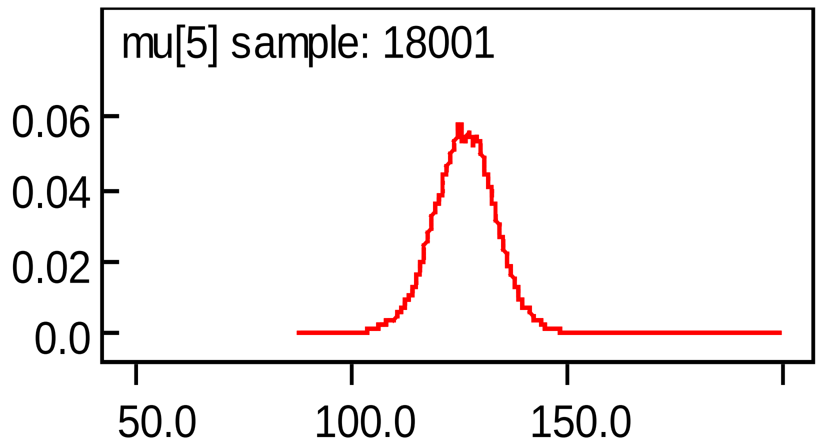


Figure 26 Density plot of monitored node for Section H

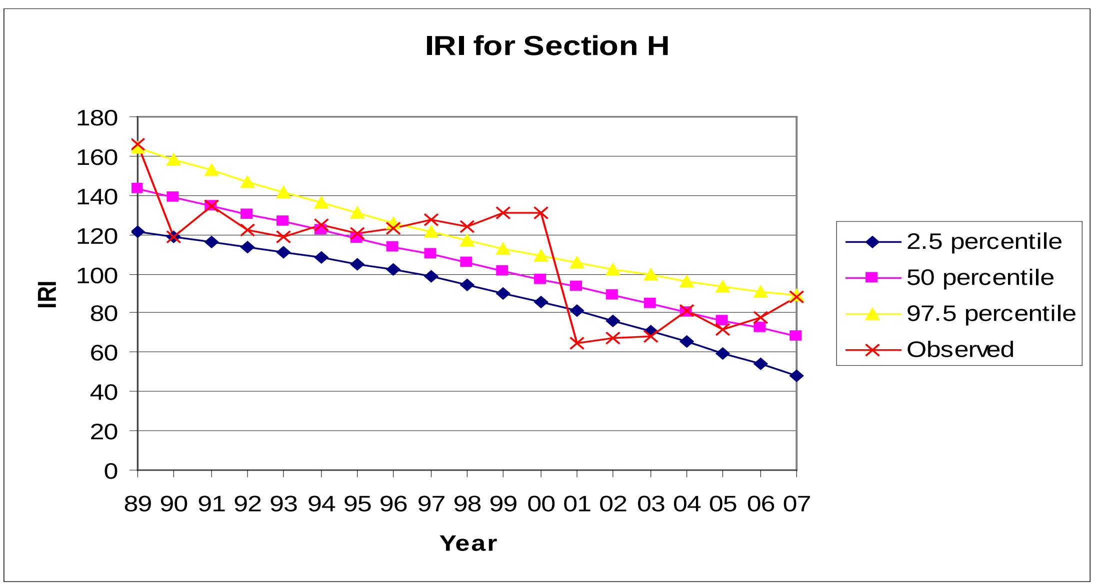


Figure 27 Plot of Estimated versus Actual IRI for Section H

5.2 Predicting IRI values of sections

After having certified the normal hierarchical model reflected roughness conditions of the selected roads, an attempt was made to predict the IRI value for the 20th year which is 2008. The results for individual section D, F and H are shown respectively in Tables 12, 13 and 14. Values for the 20th year are in bold caps.

Table 12 Prediction results for Section D

Year		Mean	sd	MC error	2.5%	Median	97.5%	Sample
89	mu[1]	88.18	7.087	0.1337	74.04	88.21	102.3	18001
90	mu[2]	88.42	6.536	0.1213	75.45	88.47	101.4	18001
91	mu[3]	88.65	6.009	0.1089	76.76	88.68	100.6	18001
92	mu[4]	88.88	5.512	0.0966	78.01	88.9	99.98	18001
93	mu[5]	89.11	5.055	0.08435	79.17	89.13	99.27	18001
94	mu[6]	89.34	4.649	0.07221	80.18	89.38	98.69	18001
95	mu[7]	89.58	4.309	0.06026	81.01	89.61	98.19	18001
96	mu[8]	89.81	4.052	0.04861	81.81	89.8	97.84	18001
97	mu[9]	90.04	3.894	0.03757	82.26	90.05	97.72	18001
98	mu[10]	90.27	3.847	0.02786	82.63	90.29	97.75	18001
99	mu[11]	90.5	3.915	0.02137	82.68	90.5	98.15	18001
00	mu[12]	90.74	4.093	0.0213	82.58	90.74	98.81	18001
01	mu[13]	90.97	4.367	0.0277	82.3	90.99	99.55	18001
02	mu[14]	91.2	4.72	0.03737	81.88	91.25	100.5	18001
03	mu[15]	91.43	5.136	0.0484	81.22	91.5	101.6	18001
04	mu[16]	91.66	5.601	0.06003	80.46	91.72	102.7	18001
05	mu[17]	91.9	6.104	0.07199	79.68	91.96	104	18001
06	mu[18]	92.13	6.636	0.08412	78.81	92.19	105.3	18001
07	mu[19]	92.36	7.191	0.09637	77.92	92.43	106.7	18001
08	**mu[20]**	**92.59**	**7.764**	**0.1087**	**77.05**	**92.64**	**108**	**18001**

Values of nodes mu[1] to mu[19] in Table 12 are approximately the same compared to their corresponding values in Table 4.

Table 13 Prediction results for Section F

Year		Mean	sd	MC error	2.5%	Median	97.5%	Sample
89	mu[1]	102.6	9.496	0.178	83.53	102.6	121.5	18001
90	mu[2]	103.3	8.743	0.161	85.8	103.3	120.7	18001
91	mu[3]	104	8.025	0.1441	87.97	104	119.9	18001
92	mu[4]	104.7	7.353	0.1272	90.12	104.7	119.4	18001
93	mu[5]	105.3	6.742	0.1104	92.05	105.4	118.9	18001
94	mu[6]	106	6.208	0.09389	93.78	106.1	118.5	18001
95	mu[7]	106.7	5.773	0.07766	95.32	106.8	118.3	18001
96	mu[8]	107.4	5.461	0.062	96.68	107.4	118.3	18001
97	mu[9]	108.1	5.295	0.04747	97.57	108.2	118.5	18001
98	mu[10]	108.8	5.287	0.0355	98.3	108.9	119.1	18001
99	mu[11]	109.5	5.438	0.02941	98.62	109.5	120.1	18001
00	mu[12]	110.2	5.736	0.03266	98.75	110.3	121.5	18001
01	mu[13]	110.9	6.159	0.0432	98.68	110.9	123	18001
02	mu[14]	111.6	6.685	0.05712	98.43	111.7	124.8	18001
03	mu[15]	112.3	7.289	0.0725	97.82	112.4	126.7	18001
04	mu[16]	113	7.956	0.08859	97.18	113.1	128.7	18001
05	mu[17]	113.7	8.67	0.1051	96.45	113.8	130.9	18001
06	mu[18]	114.4	9.42	0.1217	95.61	114.5	133.2	18001
07	mu[19]	115.1	10.2	0.1386	94.81	115.2	135.5	18001
08	**mu[20]**	**115.8**	**11**	**0.1555**	**93.9**	**115.9**	**137.7**	**18001**

Values of nodes mu[1] to mu[19] in Table 13 are approximately the same compared to their corresponding values in Table 7. This assigns credibility to the prediction run and the IRI value obtained in 2008.

Table 14 Prediction results for Section H

Year		Mean	sd	MC error	2.5%	Median	95%	Sample
89	mu[1]	143.1	10.41	0.2254	122.7	143.1	163.5	18001
90	mu[2]	138.9	9.62	0.2055	120	139	157.8	18001
91	mu[3]	134.8	8.864	0.1856	117.1	134.8	152.2	18001
92	mu[4]	130.6	8.149	0.1658	114.4	130.6	146.5	18001
93	mu[5]	126.4	7.488	0.1461	111.6	126.5	141.1	18001
94	mu[6]	122.3	6.895	0.1266	108.4	122.3	135.8	18001
95	mu[7]	118.1	6.39	0.1074	105.3	118.2	130.7	18001
96	mu[8]	113.9	5.995	0.08861	102	114	125.8	18001
97	mu[9]	109.8	5.734	0.07067	98.21	109.8	121.1	18001
98	mu[10]	105.6	5.623	0.05437	94.37	105.6	116.7	18001
99	mu[11]	101.4	5.674	0.04168	90.04	101.5	112.6	18001
00	mu[12]	97.26	5.88	0.03659	85.49	97.3	108.8	18001
01	mu[13]	93.09	6.227	0.04195	80.67	93.12	105.5	18001
02	mu[14]	88.92	6.693	0.05477	75.65	88.93	102.1	18001
03	mu[15]	84.75	7.255	0.07114	70.38	84.78	98.98	18001
04	mu[16]	80.58	7.892	0.08911	64.91	80.61	96.08	18001
05	mu[17]	76.41	8.589	0.1079	59.33	76.43	93.32	18001
06	mu[18]	72.25	9.33	0.1271	53.74	72.23	90.58	18001
07	mu[19]	68.08	10.11	0.1466	48.1	68.05	87.96	18001
08	**mu[20]**	**63.91**	**10.91**	**0.1663**	**42.31**	**63.85**	**85.46**	**18001**

Values of nodes mu[1] to mu[19] in Table 14 are approximately the same compared to their corresponding values in Table 10. This assigns credibility to the prediction run and the IRI value obtained in 2008.

Figures 28, 29 and 30 shows plots of prediction results for Sections D, F and H. These are graphical representations of Tables 12, 13 and 14.

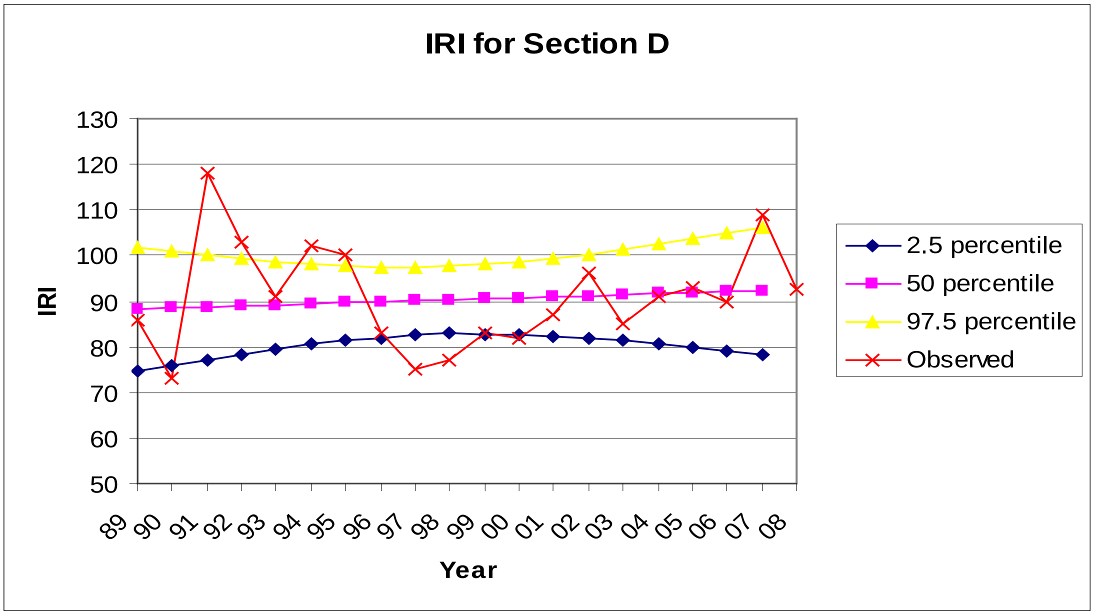


Figure 28 Plot of Predicted versus Actual IRI values for Section D

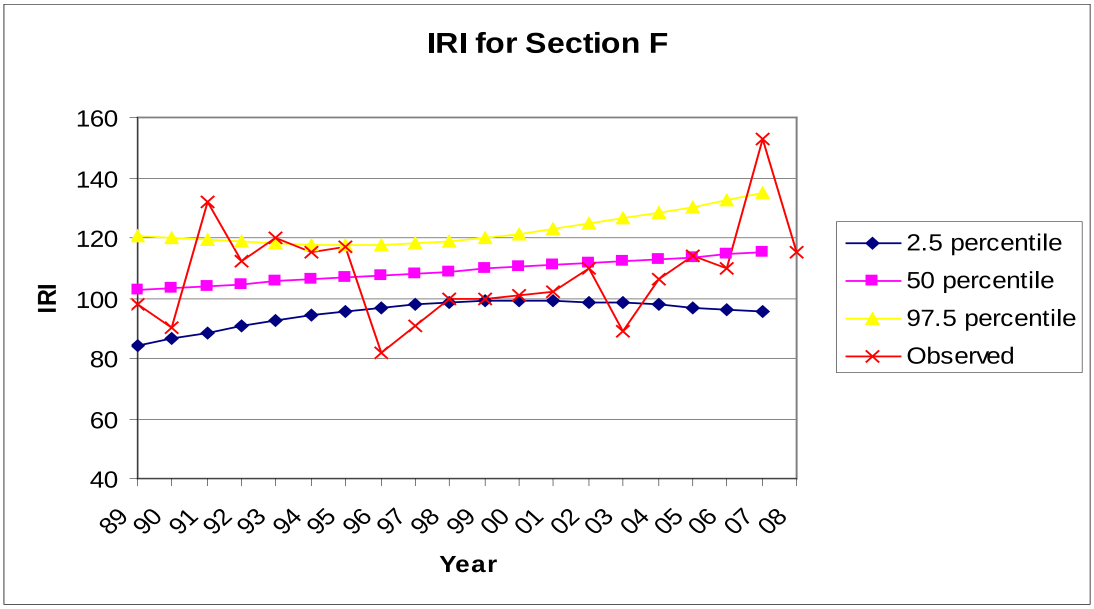


Figure 29 Plot of Predicted versus Actual IRI values for Section F

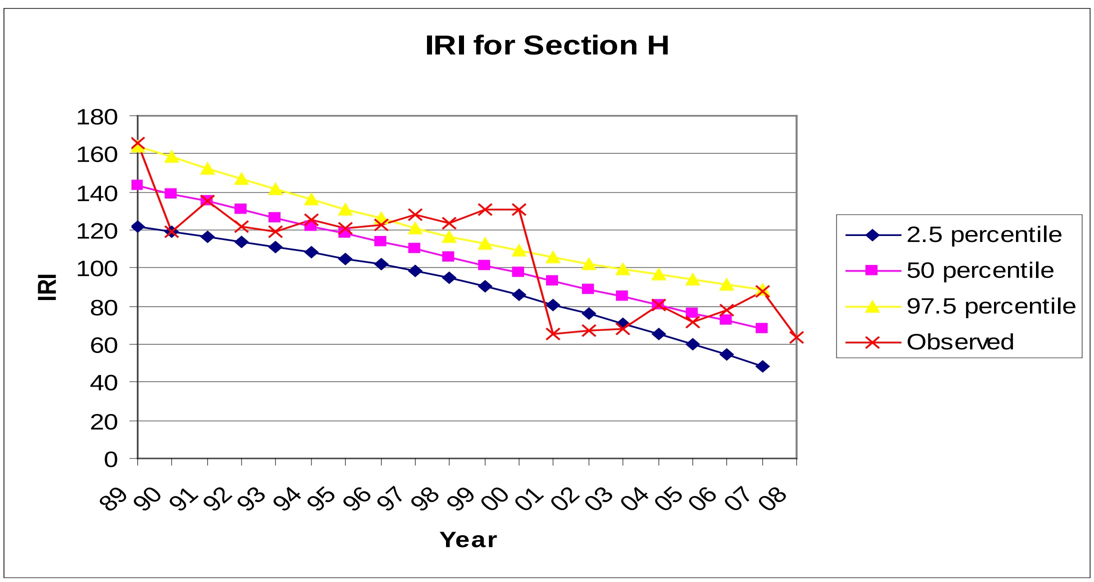


Figure 30 Plot of Predicted versus Actual IRI values for Section H

5.3 Discussion of Results

The normal hierarchical MCMC model that was developed was validated on its ability to model heterogeneity without ignoring roughness conditions on the roadway.

Using a confidence interval of 95%, estimated IRI values for pavement sections from the fitted model corresponded strongly with IRI values from the dataset. Approximately 70% of observed IRI values fell within the confidence intervals for all fifteen pavement sections used in this research. The models thus captured a significant proportion of the distribution of roughness for the individual pavement sections.

Prediction of IRI values for the twentieth year was also made for the sections using a second run of the model which was independent of the one used for estimation. Even though they were run separately, generated values from the estimation and prediction runs follow the same trends for the first nineteen years. Similarity between the two sets of simulated values serves as a check on prediction and validates the normal hierarchical MCMC model.

With regards to the 3 individual sections discussed in this chapter, the model, in the case of Sections D and H, best described recorded roughness conditions within the respective length of roadway. The replicated points for Section H within the 95% confident interval were all consistent with observed field data. This was also the case for Section D, except for 1991 where the sudden spike in roughness was too extreme to be accounted for. For Section F, the model was able to replicate for the most part all the IRI values within the section but could not reflect very well the peaks in 1991 and 2007 in addition to the dip in 1996.

CHAPTER 6

SUMMARY AND CONCLUSION

6.1 Summary

Heterogeneity is a feature of pavement distresses that traditional pavement performance models have always found difficult to characterize. With regards to pavement roughness, heterogeneity describes the continuous change in roughness in relation to a change in corresponding time. It is unpredictable and pavement performance models sometimes fail to address this form of uncertainty. Whereas some research have modeled heterogeneity mainly from a mathematical perspective by disregarding its influence in pursuit of sound mathematical parameters, other research focus entirely on heterogeneity as pertains to prevailing pavement conditions having no regard for the soundness of mathematical estimates used in the model. The aim of this research was to model roughness using sound mathematical estimates without disregarding inherent heterogeneity associated with it.

This research used hierarchical Markov Chain Monte Carlo models in estimating IRI values and in predicting values for a given data set. The data used was annual roughness for Kansas spanning a period of nineteen years. The model used the Gibbs sampler with MCMC simulation, and was able to reflect prevailing roughness conditions without neglecting heterogeneity. WinBUGS, a Bayesian analysis software that uses Markov Chain Monte Carlo (MCMC) to fit statistical data was used in simulation.

A chronological sequence of activities conducted as part of the research included literature review to investigate past model forms; exploratory data analysis to identify completeness and consistency in the dataset; formulation of the model; estimation of original IRI values and prediction of IRI values for the twentieth year. Out of the thirty pavement sections in the database, fifteen were used for this research and the results obtained were outstanding. Not less than 70% of generated IRI values from the model fell within the 95% confidence interval established for each pavement section. The soundness of the estimates obtained served as the premise on which prediction for the twentieth year was done. A second model run independent of the one used for estimation was used in prediction. Analysis of the results from the two independent runs showed that generated IRI values for both runs were similar. This confirmed the accuracy of the model used and served as a check on prediction.

6.2 Conclusion

Modeling pavement roughness using hierarchical Markov Chain Monte Carlo (MCMC) simulation is an efficient way to depict the roughness characteristics of a given road pavement or a network of roads. Its strength lies in its ability to characterize prevailing roughness characteristics on the basis of sound mathematical principles without neglecting apparent heterogeneity inherent in pavement roughness.

The soundness of estimates obtained and that of the model in general, can serve as a useful tool in the decision making process for any pavement management system. This will go a long way to reduce the amount of uncertainty encountered during decision making and can form the basis when justifying the use of resources in managing road pavements.

In future, the use of hierarchical MCMC models can be extended to larger datasets and other pavement distresses. Ultimately, its use in assessing the performance of other civil infrastructure can also be examined and if found successful, will serve as an innovative technique in the asset management toolbox.

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