Efficient Solutions to Traffic Congestion Externalities: More Complicated Than You Might Think

A problem-based learning exercise developed by Leah H. Palm-Forster and Joshua M. Duke

Model Specification and Welfare Calculations

General Model:

Marginal net benefit (MNB): \( MNB = 20 - x \)

Marginal private congestion cost (MPCC):

\[
MPCC = 0.5 \max\{0, (x - 8)\}
\]

\[
MPCC = \begin{cases} 
0 & \text{if } x \leq 8 \\
0.5(x - 8) & \text{if } x > 8 
\end{cases}
\]

Marginal social congestion cost (MSCC):

\[
MSCC = \begin{cases} 
0 & \text{if } x \leq 8 \\
x[0.5(x - 8)] & \text{if } x > 8 
\end{cases}
\]

Scenario #1: No Congestion; No Toll

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting

Marginal Net Benefit (MNB) = Marginal Private Congestion Cost (MPCC)(=0).

\( 20 - x = 0 \rightarrow x = 20 \); therefore, up to 20 drivers will enter the road

Calculate the socially optimal number of drivers:

Assuming that there are 18 students playing the role of drivers,

\[
\max \int (20 - x) \text{ st. } 0 \leq x \leq 18
\]

\( x = 18 \)
Compute Social Net Benefit (SNB) with the equilibrium number of drivers:

\[
\text{SNB} = \int_{0}^{18} (20 - x) = 198
\]

Which can also be solved with simple geometry by adding the area of a triangle and rectangle such that \(\frac{1}{2}(18)(20 - 2) + (2 \times 18) = 198\).

**Scenario #2: No Congestion; $9 Toll**

**Predict the equilibrium number of drivers:**

Solve for the equilibrium number of drivers by setting MNB = MPCC + toll

\(20 - x = 9 \Rightarrow x = 11\); therefore, 11 drivers will enter the road

**Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:**

\[
\text{SNB} = \int_{0}^{11} (20 - x) = 159.5 \quad \text{(assuming that the revenue is redistributed)}
\]

or,

\[
\frac{1}{2}(11)(20 - 9) + (9 \times 11) = 159.5
\]

**Scenario #3: Congestion; No Toll**

**Predict the equilibrium number of drivers:**

Solve for the equilibrium number of drivers by setting MNB = MPCC

\(20 - x = 0.5(x - 8) \Rightarrow x = 16\); therefore, 16 drivers will enter the road

**Calculate the socially optimal number of drivers:**

\[
\max \int (20 - x - x[0.5(x - 8)]) \quad \text{st. } 0 \leq x \leq 18
\]

\((2 + 0.5x)(10 - x) = 0 \Rightarrow x = 10\)
Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

\[ SNB = \int_{0}^{16} (20 - x) - \int_{8}^{16} x(0.5(x - 8)) = [20 \times 16 - 0.5(16)^2] - \left[ \left(\frac{1}{6}(16)^3 - 2(16)^2\right) - \left(\frac{1}{6}(8)^3 - 2(8)^2\right) \right] = 192 - (170.67 + 42.67) = 192 - 213.34 = -21.34 \]

Scenario #4: Congestion; $9 Toll

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting MNB = MPCC + toll

\[ 20 - x = 0.5(x - 8) + 9 \Rightarrow x = 10; \text{ therefore, } 10 \text{ drivers will enter the road} \]

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

\[ SNB = \int_{0}^{10} (20 - x) - \left[ \int_{8}^{10} x(0.5(x - 8)) \right] = 150 - [-33.33 + 42.67] = 140.66 \]

assuming that the toll revenue is redistributed.

or,

\[ \sim \left[ \frac{1}{2}(10)(20 - 10) + (10 \times 10) \right] - \left[ \frac{1}{2}(10 - 8) \times 10 \right] = 150 - 10 = 140 \]

Scenario #5: Congestion; $10 Toll

Predict the equilibrium number of drivers:

Solve for the equilibrium number of drivers by setting MNB = MPCC + toll

\[ 20 - x = 0.5(x - 8) + 10 \Rightarrow x = 9.33; \text{ therefore, } 9 \text{ drivers will enter the road} \]

Calculate Social Net Benefit (SNB) with the equilibrium number of drivers:

\[ SNB = \int_{0}^{9} (20 - x) - \left[ \int_{8}^{9} x(0.5(x - 8)) \right] = 139.5 - [-40.5 + 42.67] = 137.33 \]

assuming that the toll revenue is redistributed.